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MULTIDIMENSIONAL EDGE AND LINE DETECTION BY HYPERSURFACE FITTING USING BASIS FUNCTIONS

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ABSTRACT

This paper considers the problem of fitting hypersurfaces to the multidimensional picture function for edge and line detection. The multidimensional greytone surface is expanded as a weighted sum of basis functions. Using multidimensional orthogonal polynomial basis functions, expressions are developed for the coefficients of fitted hyperquadric and hypercubic surfaces. Assuming noise is Gaussian, statistical tests are devised for the detection of significant edges and lines. For directional edge and line detection, parameters of the fitted surfaces are obtained under rotation of the coordinate system. Direction isotropic properties of the fitted surfaces are derived. For computational efficiency, recursive relations are obtained between the parameters of the fitted surfaces of successive neighborhoods. Furthermore, experimental results are presented by applying the developed theory to multichannel Landsat imagery data.

I. INTRODUCTION

Edge and line detection is an important operation in a number of image processing applications such as scene analysis, mapping of lineaments from remotely sensed data (example, Landsat data) to aid geological ground surveys, detection of seismic horizons in the automation of geological interpretations, etc. The digital picture function in general is a sampling of the underlying reflectance function of the objects in the scene with noise added to the true function values. The edges or lines refer to the places in the image where there are jumps in the values of the function or its derivatives.

A variety of operators[1] are proposed in the literature for the detection of edges and lines by fitting a surface to a picture function in the neighborhood of each image point and taking the derivatives of the function at that point. Most of these operators work on single band image and the selection of thresholds is a difficult task. Recently Haralick[2] introduced the noise term and devised statistical tests for the detection of significant edges by fitting a plane in the neighborhood of a point for a single-channel image.

Several authors[3-5] treated the surface fitting using an orthogonal basis of two dimensional functions. O'Gorman[3] used the two-dimensional walsh functions, Hueckel[4] used the polar version of the orthonormal Fourier basis and Haralick[5] used the orthogonal polynomials. All these operators work on a plane or on a single band image.

In multispectral images such as those acquired by Landsat, different objects respond differently in different bands and hence it is advantageous to use information from all the bands in edge and line detection.

Recently Morgenthaler and Rosenfeld[6] generalized the Prewitt[1] operators to n-dimensions by fitting a hyperquadric surface. However the noise is not introduced into the formulation. It is the purpose of this paper to develop the multidimensional edge and line detection theory by fitting a hypersurface to the picture function in the neighborhood of an image point using basis functions. Also, statistical tests are devised for the detection of significant edges and lines and the properties of the operators are studied for rotational invariance. A simple thinning algorithm is proposed for thinning the detected significant edges and lines, and experimental results are presented.

II. HYPERSURFACE FITTING USING ORTHOGONAL BASIS FUNCTIONS

Let $X = (x_0\ x_1\ \ldots\ x_n)^T$ be a point in the n-dimensional space. Let r_0 be a hyperrectangular region. Without loss of generality, we choose our coordinate system so that the center of the region is at the origin. Because of the symmetry in the chosen coordinate system, we can write

$$\sum_{x_i \in r_0} x_i = 0 \qquad \text{for } \forall i . \tag{2-1}$$

Let $\{S_i(X),\ 0 \leq i \leq N\}$ be a set of n-dimensional orthogonal basis functions defined over the region r_0. Let f(X) be the digital picture function. Let g(X) be an estimate of f(X) and is estimated as a weighted sum of the basis functions. That is

$$g(X) = \sum_{i=0}^{N} a_i \, S_i(X) \qquad (2\text{-}2)$$

where $\{a_i,\ 0 \leq i \leq N\}$ are a set of coefficients. The total squared estimation error, ε^2, can be written as

$$\varepsilon^2 = \sum_{X \varepsilon r_0} \left(f(X) - g(X)\right)^2 . \qquad (2\text{-}3)$$

Using the orthogonal properties of the basis functions, from equations (2-2) and (2-3), the coefficients a_i that minimize ε^2 can be obtained as

$$a_i = \frac{\sum_{X \varepsilon r_0} f(X)\ S_i(X)}{\mathrm{S}_{X \varepsilon r_0} \ S_i^2\ (X)} . \qquad (2\text{-}4)$$

From equation (2-4), it is seen that the masks that estimate the coefficients a_i are orthogonal since $S_i(X)$ are orthogonal. Let the picture function f(X) can be written as

$$f(X) = \sum_{i=0}^{N} \alpha_i \ \ S_i(X) + \eta(X) \qquad (2\text{-}5)$$

where $\eta(X)$ is a noise term and is assumed to be Gaussian with zero mean and variance σ^2. The noise $\eta(X)$ is also assumed to be independent from pixel to pixel. Putting equation (2-4) in equation (2-5) yields

$$a_i = \alpha_i + \frac{\Sigma\ \eta(X)\ S_i(X)}{\Sigma\ S_i^2\ (X)} . \qquad (2\text{-}6)$$

From equation (2-6), it is seen that the estimates a_i are unbiased. The variance of the estimates a_i are given by the following:

$$\mathrm{Var}(a_i) = E\ (a_i - \alpha_i)^2 = \frac{\sigma^2}{\sum_{X \varepsilon r_0} S_i^2(X)} . \qquad (2\text{-}7)$$

Since $\eta(X)$ is normally distributed, the coefficients a_i are also normally distributed. For $\ell \neq m$, we get

$$E\left[(a_\ell - \alpha_\ell)\ (a_m - \alpha_m)\right] =$$

$$\sigma^2 \frac{\sum_X S_\ell(X)\ S_m(X)}{\left(\sum_X S_\ell^2(X)\right)\ \left(\sum_X S_m^2(X)\right)} = 0 . \qquad (2\text{-}8)$$

Since a_i are normally distributed, from equation (2-8), a_i are also independent. The total squared error, ε^2, can be written as follows:

$$\varepsilon^2 = \sum_X \eta^2(X) - \sum_{i=0}^{N} (a_i - \alpha_i)^2 \left(\sum_X S_i^2(X)\right) . \qquad (2\text{-}9)$$

Since $\eta(X)$ is normally distributed with zero mean and variance σ^2, the quantity $\left(\sum_X \eta^2(X)/\sigma^2\right)$ is distributed as a chi-squared variate with $\sum_X(1)$ degrees of freedom. Since a_i are independent normals, $\left(\sum_{i=0}^{N} (a_i - \alpha_i)^2 \left(\sum_X S_i^2(X)\right)/\sigma^2\right)$ is distributed as a chi-squared variate with (N+1) degrees of freedom. Therefore, (ε^2/σ^2) is distributed as a chi-squared variate with $\left(\sum_X(1) - (N+1)\right)$ degrees of freedom. From this it follows that to test the hypothesis that the coefficients a_{ℓ_i}, i = 1, 2, ⋯, m (m < N) are in fact zero, we use the ratio

$$F = \frac{\left(\sum_{i=1}^{m} a_{\ell_i}^2 \ \sum_X S_{\ell_i}^2\ (X)\right)/m}{\varepsilon^2 \ / \ \left(\sum_X(1) - (N+1)\right)} \qquad (2\text{-}10)$$

which has F distribution with $\left[m,\ \left(\sum_X 1 - (N+1)\right)\right]$ degrees of freedom and reject the hypothesis for large values of F.

III. DISCRETE ORTHOGONAL POLYNOMIALS AS BASIS FUNCTIONS

Let χ_i be the domain of x_i. Let $\left[\{P_{ij}(x_i),\ 0 \leq j \leq M\}\right]$ be a set of discrete orthogonal polynomials on χ_i, $1 \leq i \leq n$. The set of n-dimensional basis functions $\{S_\ell(X),\ 0 \leq \ell \leq N\}$, can be constructed using one-dimensional discrete orthogonal polynomials $P_{ij}(x_i)$ as follows.

$$\left[\{P_{10}(x_1)\ P_{20}(x_2)\ \ldots\ P_{n0}(x_n)\},\ \ldots,\right.$$

$$\left.\{P_{1i_1}(x_1)\ P_{2i_2}(x_2)\ \ldots P_{ni_n}(x_n)\}\right] \qquad (3\text{-}1)$$

For $n \geq 0$, the discrete one-dimensional orthogonal polynomials can be written as

$$P_{2n}(x) = x^{2n} + \sum_{i=1}^{n} a_{2n,2(n-i)}\ x^{2(n-i)} \qquad (3\text{-}2)$$

$$P_{2n+1}(x) = x^{2n+1} +$$

$$\sum_{i=1}^{n} a_{2n+1,2(n-i)+1}\ x^{2(n-i)+1}\ . \qquad (3\text{-}3)$$

where $P_n(x)$ is a polynomial of degree n, all the exponents of x being even or odd with n. From equation (3-3), we get

$$P_0(x) = 1,\ P_1(x) = x\ . \qquad (3\text{-}4)$$

The discrete orthogonal polynomials can be recursively generated using the following theorem.[7]

A. THEOREM 1

The discrete orthogonal polynomial set $\{P_k(x),\ 0 \leq k \leq n\}$ satisfies the relationship given in the following:

$$P_k(x) = x\ P_{k-1}(x) - C_2 P_{k-2}(x) \qquad (3\text{-}5)$$

where $$C_2 = \frac{\sum_x x P_{k-1}(x)\ P_{k-2}(x)}{\sum_x P_{k-2}^2(x)}\ . \qquad (3\text{-}6)$$

Let $\mu_{ik} = \Sigma x_i^k$, be the kth moment of x_i over the domain χ_i. A few discrete orthogonal polynomials constructed using equations (3-4), (3-5), and (3-6) are given in the following:

$$P_{i0}(x_i) = 1,\ P_{i1}(x_i) = x_i$$

$$P_{i2}(x_i) = x_i^2 - \frac{\mu_{i2}}{\mu_{i0}}$$

$$P_{i3}(x_i) = x_i^3 - \frac{\mu_{i4}}{\mu_{i2}}\ x_i\ . \qquad (3\text{-}7)$$

IV. FITTING OF HYPERSURFACES USING DISCRETE ORTHOGONAL POLYNOMIALS

This section concerns with fitting of hyperquadric and hypercubic surfaces to the picture function in the neighborhood of an image point. Also, statistical tests are developed for the detection of significant edges and lines.

A. HYPERQUADRIC SURFACE

In terms of one-dimensional discrete orthogonal polynomials, the n-dimensional hyperquadric surface can be written as follows:

$$g(X) = a_0 + \sum_{i=1}^{n} a_i P_{i1}(x_i) + \sum_{i=1}^{n} a_{ii} P_{i2}(x_i) +$$

$$\sum_{\substack{i,j=1\\ i<j}}^{n} a_{ij}\ P_{i1}(x_i)\ P_{j1}(x_j)\ . \qquad (4\text{-}1)$$

Let r_0 be the hyperrectangular neighborhood of the image point under consideration. Let the coordinate system be positioned at the center of the r_0 region. From equations (2-4) and (4-1), the coefficients a's, that minimize the sum of squares of errors, are given in the following:

$$a_0 = \frac{\sum_{r_0} f(X)}{\sum_{r_0} (1)}\ ,\ a_{ij} = \frac{\sum_{r_0} x_i x_j\ f(X)}{\sum_{r_0} (x_i x_j)^2}\ ,\ \begin{array}{c} 1 \leq i,j \leq n \\ i < j \end{array}$$

$$a_i = \frac{\sum_{r_0} x_i\ f(X)}{\sum_{r_0} x_i^2}\ , \qquad (4\text{-}2)$$

$$a_{ii} = \frac{\sum_{r_0} P_{i2}(x_i)\ f(X)}{\sum_{r_0} P_{i2}^2(x_i)}\ ,\ 1 \leq i \leq n\ .$$

In terms of the coordinates $(x_1, x_2, \ldots, x_n)$, the hyperquadric surface can be written as follows.

$$g(x) = b_0 + \sum_{i=1}^{n} b_i x_i +$$

$$\sum_{i=1}^{n} b_{ii}\ x_i^2 + \sum_{\substack{i,j=1\\ i<j}}^{n} b_{ij}\ x_i x_j \qquad (4\text{-}3)$$

From equations (4-1), (4-2), and (4-3) the a's and b's are related as follows:

$$b_0 = a_0 - \sum_{i=1}^{n} \frac{\mu_{i2}}{\mu_{i0}} a_{ii} \ , \ b_{ij} = a_{ij} \quad 1 \leqslant i,j \leqslant n, \ i < j$$

$$b_i = a_i \ , \ b_{ii} = a_{ii} \ , \ 1 \leqslant i \leqslant n \ . \tag{4-4}$$

It is seen that the solution that is obtained for the gradients by fitting the hyperquadric surface is the same as that would have obtained by fitting the hyperplane. However, there is a change in the constant term.

Since the coefficients a's of equation (4-2) are unbiased, the coefficients b's of equation (4-3) are unbiased. The variances of b_i, b_{ii}, and b_{ij} are given by the variances of a_i, a_{ii}, and a_{ij}, respectively. Since the a's are independent and the noise $\eta(X)$ is independent from pixel to pixel, the variance of b_0 is given by the following[7]:

$$\mathrm{Var}\,(b_0) = \left[\frac{1}{\Sigma\,(1)} + \sum_{i=1}^{n} \left(\frac{\mu_{i2}}{\mu_{i0}}\right)^2 \frac{1}{\Sigma\, P_{i2}^2(x_i)}\right] \cdot \sigma^2 \ . \tag{4-5}$$

The Laplacian of equation (4-3) at the center of the region r_0 can be written as

$$L = \sum_{i=1}^{n} \frac{\partial^2 g}{\partial x_i^2} \Bigg|_{X=0} = 2 \sum_{i=1}^{n} b_{ii} \ . \tag{4-6}$$

From equations (2-6), (4-4), and (4-6), it is seen that the Laplacian is unbiased and its variance is given by

$$\mathrm{Var}\,(L) = 4\sigma^2 \left[\sum_{i=1}^{n} \frac{1}{\sum_{X \varepsilon r_0} P_{i2}^2(x_i)}\right] . \tag{4-7}$$

To test the hypothesis that the Laplacian is zero, we form the ratio

$$F = \frac{\left(\sum_{i=1}^{n} b_{ii}\right)^2 \Big/ \sum_{i=1}^{n} \frac{1}{\sum_{X \varepsilon r_0} P_{i2}^2(x_i)}}{\varepsilon^2 \left(\sum_X (1) - (N+1)\right)} \tag{4-8}$$

which has an F-distribution with $\left(1, \sum_X 1 - (N+1)\right)$ degrees of freedom and reject the hypothesis for large values of F. Equation (2-10) can be used to test whether the gradients b_i, $1 \leqslant i \leqslant n$ are zero. The square of the magnitude of the gradient is given by

$$gr = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i}\right)^2 \Bigg|_{X=0} = \sum_{i=1}^{n} b_i^2 \ . \tag{4-9}$$

From equations (2-6), (4-2), (4-4), and (4-9), the expected value of the gradient can be obtained as

$$E\,[gr] = \sum_{i=1}^{n} \alpha_i^2 + \sigma^2 \sum_{i=1}^{n} \frac{1}{\sum_{X \varepsilon r_0} x_i^2} \ . \tag{4-10}$$

Thus it is seen that the gradient is biased.

B. HYPERCUBIC SURFACE

In terms of one-dimensional discrete orthogonal polynomials, the n-dimensional hypercubic surface can be written as follows:

$$g(X) = a_0 + \sum_{i=1}^{n} a_i P_{i1}(x_i) + \sum_{i=1}^{n} a_{ii} P_{i2}(x_i) + \sum_{\substack{i,j=1 \\ i<j}}^{n} a_{ij} P_{i1}(x_i) P_{j1}(x_j) + \sum_{i=1}^{n} a_{iii} P_{i3}(x_i) + \sum_{\substack{i,j=1 \\ i \neq j}}^{n} a_{iij} P_{i2}(x_i) P_{j1}(x_j) + \sum_{\substack{i,j,k=1 \\ i<j<k}}^{n} a_{ijk} P_{i1}(x_i) P_{j1}(x_j) P_{k1}(x_k) \ . \tag{4-11}$$

Positioning the coordinate system at the center of the region, the estimates for the coefficients a's that minimize the sum of squares of errors between the observed and estimated greytone values are given by equation (2-4). In terms of the coordinates $(x_1, x_2, \ldots, x_n)$, the hypercubic surface can be written as follows:

$$g(X) = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum_{\substack{i,j=1 \\ i<j}}^{n} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{iii} x_i^3 + \sum_{\substack{i=1 \\ i \neq j}}^{n} b_{iij} x_i^2 x_j + \sum_{\substack{i,j,k=1 \\ i<j<k}}^{n} b_{ijk} x_i x_j x_k \ . \tag{4-12}$$

From equations (3-7), (4-11), and (4-12), the a's and b's can easily be related.[7]

From equations (3-7), (4-2), (4-4), and (4-12), it can easily be seen that the solution for the second derivatives of the fitted surface evaluated at the center of the region r_0 are the same either by fitting hyperquadric or hypercubic surfaces. However, it is noted that the constant term and gradients differ. Similar conclusions can be drawn for higher order surfaces.

The means and variances of the first and second derivatives of the hypercubic surface evaluated at the center of the region are obtained in the following. Since a's are unbiased, b_i's, and b_{ii}'s are unbiased. The variance of b_{ii}'s is given by the variance of a_{ii}'s of equation (2-7). The variance of b_i's can be obtained as

$$\mathrm{Var}(b_i) = \sigma^2 \left[\frac{1}{\Sigma P_{i2}^2(x_i)} + \frac{\mu_{i4}^2}{\mu_{i2}^2} \frac{1}{\Sigma P_{i3}^2(x_i)} + \sum_{\substack{j=1 \\ \neq i}}^{n} \frac{\mu_{j2}^2}{\mu_{j0}^2} \frac{1}{\Sigma P_{j2}^2(x_j)\, P_{i1}^2(x_i)} \right] . \quad 1 \leqslant i \leqslant n \qquad (4\text{-}13)$$

The Laplacian is unbiased and its variance is given by equation (4-7).

V. PARAMETERS OF THE HYPERSURFACES UNDER ROTATION OF THE COORDINATE SYSTEM

Very often, such as in the mapping of lineaments from remotely sensed data, it is required to obtain the directional edges and lines. It is the purpose of this section to derive expressions for the parameters of the fitted surfaces when the coordinate system is rotated and obtain variances for the derivatives of the fitted surfaces evaluated at the center of the region.

Let $X = (x_1, x_2, \ldots, x_n)^T$ be a point in the n-dimensional space referred to the original coordinate system, and let $Y = (y_1, y_2, \ldots, y_n)^T$ be the corresponding point referred to the rotated coordinate system. Let D be the orthonormal rotation matrix. Then we have

$$X = DY . \qquad (5\text{-}1)$$

Let g(X) be the estimated greytone surface expressed in terms of original coordinate system, and let it be g(Y) when expressed in terms of rotated coordinate system. Since D is an orthonormal matrix, we have the relationships

$$D^T = D^{-1} \text{ and } D^TD = DD^T = I . \qquad (5\text{-}2)$$

A. HYPERQUADRIC SURFACE

From Equation 5-1, the coordinates x_i can be expressed in terms of the coordinates y_i and the elements of matrix D as

$$x_i = \sum_{j=1}^{n} d_{ij} y_j , \quad 1 \leqslant i \leqslant n . \qquad (5\text{-}3)$$

The hyperquadric greytone surface, when expressed with respect to the rotated coordinate system, can be written as

$$g(Y) = C_0 + \sum_{i=1}^{n} C_i y_i + \sum_{i=1}^{n} C_{ii} y_i^2 + \sum_{\substack{i,j=1 \\ i<j}}^{n} C_{ij} y_i y_j . \qquad (5\text{-}4)$$

Using Equation 5-3 in Equation 4-3 and comparing the coefficients of the resulting expression with that of the ones in Equation 5-4 yield

$$C_0 = b_0 , \quad C_i = \sum_{\ell=1}^{n} b_\ell d_{\ell i} \qquad , \; 1 \leqslant i \leqslant n$$

$$C_{ii} = \sum_{\ell=1}^{n} b_{\ell\ell} d_{\ell i}^2 + \sum_{\substack{\ell,m=1 \\ \ell<m}}^{n} b_{\ell m} d_{\ell i} d_{mi} , \; 1 \leqslant i \leqslant n \qquad (5\text{-}5)$$

$$C_{ij} = 2 \sum_{\ell=1}^{n} b_{\ell\ell} d_{\ell i} d_{\ell j} + \sum_{\substack{\ell,m=1 \\ \ell<m}}^{n} b_{\ell m}(d_{\ell i} d_{mj} + d_{\ell j} d_{mi}). \qquad 1 \leqslant i,j \leqslant n, \; i<j$$

Since the coefficients b's are unbiased, the coefficients c's also are unbiased. The variances of C_i and C_{ii} are given by the following:

$$\mathrm{var}(C_i) = \sigma^2 \left[\sum_{\ell=1}^{n} d_{\ell i}^2 \cdot \frac{1}{\underset{X\varepsilon r_0}{(\Sigma\, x_\ell^2)}} \right] \quad (5\text{-}6)$$

$$\mathrm{var}(C_{ii}) = \sigma^2 \left[\sum_{\ell=1}^{n} d_{\ell i}^4 \frac{1}{\underset{X\varepsilon r_0}{\Sigma\, P_{\ell 2}^2}\, (x_\ell)} + \right.$$

$$\left. \sum_{\substack{\ell,m=1 \\ \ell < m}}^{n} d_{\ell i}^2 d_{mi}^2 \cdot \frac{1}{\underset{X\varepsilon r_0}{\Sigma\, (x_\ell x_m)^2}} \right] . \quad (5\text{-}7)$$

Since sum of squares of errors of fitted surface is independent of rotation in the coordinate system, statistical tests similar to Equation 4-8 can be set up to test the hypothesis that the estimated coefficients C_i and C_{ii} are zero for directional edge and line detection.

For $j \neq k$, the covariance of C_j and C_k can be expressed as

$$\mathrm{cov}(C_j, C_k) =$$

$$\sigma^2 \left\{ \sum_{\ell=1}^{n} \sum_{m=1}^{n} d_{\ell j} d_{mk} \left[\frac{\Sigma\, x_\ell x_m}{(\Sigma\, x_\ell^2)\ (\Sigma\, x_m^2)} \right] \right\} . \quad (5\text{-}8)$$

When the size of the neighborhood is the same in x_ℓ and x_m coordinate axes, from Equation 5-8 it is seen that the coefficients C_j and C_k are independent when $j \neq k$.

B. HYPERCUBIC SURFACE

The hypercubic greytone surface, when expressed with respect to the rotated coordinate system, can be written as

$$g(Y) = C_0 + \sum_{i=1}^{n} C_i y_i + \sum_{i=1}^{n} C_{ii} y_i^2 +$$

$$\sum_{\substack{i,j=1 \\ i<j}}^{n} C_{ij} y_i y_j + \sum_{i=1}^{n} C_{iii} y_i^3 + \sum_{\substack{i,j=1 \\ i \neq j}}^{n} C_{iij} y_i^2 y_j +$$

$$\sum_{\substack{i,j,k=1 \\ i<j<k}}^{n} C_{ijk} y_i y_j y_k . \quad (5\text{-}9)$$

Using Equation 5-3 in Equation 4-13 and comparing the coefficients of the resulting expression with that of the ones in Equation 5-9, expressions for the c's can easily be obtained in terms of the b's and the elements of the rotation matrix D.[7]

Since the coefficients b's are unbiased, the coefficients c's are also unbiased. The variances of C_i and C_{ii}, the first and second derivatives of the hypercubic surface evaluated at the center of the region r_0, are given in the following:

$$\mathrm{var}(C_i) = \sum_{r=1}^{n} d_{ri}^2 \cdot \mathrm{var}(b_r) \quad (5\text{-}10)$$

where $\mathrm{var}(b_r)$ is given by Equation 4-15. The $\mathrm{var}(C_{ii})$ is given by Equation 5-7. Since sum of squares of errors of fitted surface is independent of rotation in the coordinate system, statistical tests similar to Equation 4-8 can be set up to test the hypothesis that the estimated coefficients C_i and C_{ii} are zero for directional edge and line detection.

VI. DIRECTION ISOTROPIC PROPERTIES OF THE DERIVATIVES

In this section, we show that some functions of the partial derivatives of the n-dimensional function are invariant under rotation of the domain of the n-dimensional function. Let H be the orthonormal rotation matrix, where

$$Y = HX . \quad (6\text{-}1)$$

From Equations 5-1 and 6-1, it is seen that $H = D^{-1}$. Let g(X) be the greytone surface expressed in terms of original coordinates x_i and g(Y) be the greytone surface expressed in terms of rotated coordinates y_i. We shall now have the following results:

A. THEOREM 2

The magnitude of the gradient of the n-dimensional greytone surface is invariant under rotation of the coordinate system.

Proof: See Reference 7.

B. THEOREM 3

The Laplacian of the n-dimensional greytone surface is invariant under rotation of the coordinate system. That is,

$$\sum_{i=1}^{n} \frac{\partial^2 g}{\partial x_i^2} = \sum_{i=1}^{n} \frac{\partial^2 g}{\partial y_i^2} . \quad (6\text{-}2)$$

Proof: See Reference 7.

C. THEOREM 4

The sum of squares of second-order partial derivatives of n-dimensional greytone surface is invariant under rotation of the coordinate system. That is,

$$\sum_{i=1}^{n} \left(\frac{\partial^2 g}{\partial x_i^2}\right)^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)^2 =$$

$$\sum_{i=1}^{n} \left(\frac{\partial^2 g}{\partial y_i^2}\right) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{\partial^2 g}{\partial y_i \partial y_j}\right)^2 . \qquad (6\text{-}3)$$

Proof: See Reference 7.

D. THEOREM 5

The sum of squares of third-order partial derivatives of n-dimensional greytone surface is invariant under rotation of the coordinate system. That is,

$$\sum_{i,j,k=1}^{n} \left(\frac{\partial^3 g}{\partial x_i \partial x_j \partial x_k}\right)^2 = \sum_{i,j,k=1}^{n} \left(\frac{\partial^3 g}{\partial y_i \partial y_j \partial y_k}\right)^2 . \qquad (6\text{-}4)$$

Proof: See Reference 7.

Proceeding in a similar manner, it can be shown that the results similar to Equations 6-3 and 6-4 hold good for higher-order partial derivatives.

VII. RECURSIVE RELATIONS BETWEEN THE COEFFICIENTS OF FITTED SURFACES OF SUCCESSIVE NEIGHBORHOODS

In multidimensional edge and line detection, each image point is processed by fitting a hypersurface to the greytone surface in a hyperrectangular region having the image point at its center. Hyperrectangular regions of neighboring image points overlap. The computational efficiency can be considerably improved by relating the coefficients of fitted hypersurfaces of neighboring regions in terms of greytone function values in the nonoverlapped region.

Consider a three-dimensional hyperquadric surface. Let the image points be successively processed in the direction of coordinate axis x_2. Let R_1 and R_2 be the hyperrectangular regions of successive image points. From Equations 4-2, 4-3, and 4-4, it is seen that the denominators of the coefficients of the fitted surfaces are independent of the picture function and depend only on the sizes of the neighborhood regions. Consider only the numerators of the coefficients. Let them be represented in the region R_ℓ for ℓ = 1, 2 as

$$C_o(\ell) = \sum_{X \epsilon R_\ell} f(X) \ ,$$

$$C_{ij}(\ell) = \sum_{X \epsilon R_\ell} x_i x_j \, f(X) \qquad , 1 \quad \underset{i < j}{i,j} \quad 3$$

$$C_i(\ell) = \sum_{X \epsilon R_\ell} x_i \, f(X) \ , \ C_{ii}(\ell) =$$

$$\sum_{X \epsilon R_\ell} P_{i2}(x_i) \, f(X) \quad , 1 \leqslant i \leqslant 3 \ . \qquad (7\text{-}1)$$

Let N_2 be the size of the neighborhood in the x_2-coordinate direction, and let it be odd. Let $N_{22} = N_2/2$ and be truncated to an integer value. Let $f_\ell(x_1,x_2,x_3)$ be the greytone function in the region R_ℓ, ℓ = 1, 2.

In computing the coefficients of the fitted hypersurfaces in each neighborhood, the coordinate system is positioned at the center of the neighborhood. The values that the variable x_2 takes in each neighborhood are $(-N_{22}, -N_{22} + 1, \ldots, 0, \ldots, N_{22} - 1, N_{22})$. The greytone functions $f_1(X)$ and $f_2(X)$ of successive neighborhoods are related as follows:

$$f_1(x_1, -N_{22} + i, x_3) =$$

$$f_2(x_1, -N_{22} + i - 1, x_3) \ . \qquad (7\text{-}2)$$

$$i = 1, 2, \ldots, N_2 - 1$$

Let r_{13} be the domain of x_1, x_3 over the hyperrectangular region. Let us define the following variables over the domain of x_1, x_3:

$$\nu_1 = \sum_{r_{13}} f_1(x_1,-N_{22},x_3)$$

$$\nu_2 = \sum_{r_{13}} f_2(x_1,N_{22},x_3)$$

$$\delta_1 = \sum_{r_{13}} x_1 f_1(x_1,-N_{22},x_3)$$

$$\delta_2 = \sum_{r_{13}} x_1 f_2(x_1,N_{22},x_3)$$

$$\Gamma_1 = \sum_{r_{13}} x_3 f_1(x_1,-N_{22},x_3)$$

$$\Gamma_2 = \sum_{r_{13}} x_3 f_2(x_1,N_{22},x_3) \qquad (7\text{-}3)$$

$$\lambda_1 = \sum_{r_{13}} x_1^2 f_1(x_1,-N_{22},x_3)$$

$$\lambda_2 = \sum_{r_{13}} x_1^2 f_2(x_1,N_{22},x_3)$$

$$\xi_1 = \sum_{r_{13}} x_3^2 f_1(x_1,-N_{22},x_3)$$

$$\xi_2 = \sum_{r_{13}} x_3^2 f_2(x_1,N_{22},x_3)$$

$$\eta_1 = \sum_{r_{13}} x_1 x_3 f_1(x_1,-N_{22},x_3)$$

$$\eta_2 = \sum_{r_{13}} x_1 x_3 f_2(x_1,N_{22},x_3) \, .$$

In terms of the variables of Equation 7-3, the coefficients c's of the hyperquadric fitted surfaces in regions R_1 and R_2 are related as follows:

$$c_0(2) = c_0(1) - \nu_1 + \nu_2$$

$$c_1(2) = c_1(1) - \delta_1 + \delta_2$$

$$c_2(2) = c_2(1) - c_0(1) + (N_{22} + 1)\ \nu_1 + N_{22}\ \nu_2$$

$$c_3(2) = c_3(1) - \Gamma_1 + \Gamma_2$$

$$c_{11}(2) = c_{11}(1) - \lambda_1 + \lambda_2 - \frac{\mu_{12}}{\mu_{10}} (-\nu_1 + \nu_2)$$

$$c_{22}(2) = c_{22}(1) - 2\ c_2(1) + c_0(1) +$$

$$\nu_1 \left(\frac{\mu_{22}}{\mu_{20}} - N_{22}^2 - 2\ N_{22} - 1\right) + \nu_2 \left(N_{22}^2 - \frac{\mu_{22}}{\mu_{20}}\right)$$

$$c_{33}(2) = c_{33}(1) - \xi_1 + \xi_2 - \frac{\mu_{32}}{\mu_{30}} (-\nu_1 + \nu_2)$$

$$c_{12}(2) = c_{12}(1) - c_1(1) + (N_{22} + 1)\ \delta_1 + N_{22}\ \delta_2$$

$$c_{13}(2) = c_{13}(1) - \eta_1 + \eta_2$$

$$c_{23}(2) = c_{23}(1) - c_3(1) + (N_{22} + 1)\ \Gamma_1 + N_{22}\ \Gamma_2 \, . \qquad (7\text{-}4)$$

Using Equation 7-4, the coefficients of the fitted hyperquadric surface of region R_2 can be computed in terms of the coefficients of the fitted hyperquadric surface of region R_1 and the greytone function values over the domain of x_1, x_3 when $x_2 = -N_{22}$ and N_{22}. In processing the image points along coordinate axis x_2, the variables of Equation 7-4 are computed for each x_2. To process the neighboring plane of data by moving along coordinate axis x_1, recursive relations for the variables of Equation 7-3 can easily be obtained.[7]

VIII. EXPERIMENTAL RESULTS

Some results are presented in this section applying the theory developed in the paper for the processing of a four-channel image of an Australian scene acquired by multispectral scanner aboard the Landsat. The size of the image is 512 x 512. Figures 8.1a and 8.1b are the Landsat Bands 4 and 7, respectively. It is seen that the details in the image vary from band to band, and different bands in general emphasize different features.[8]

The gradient at every pixel is computed by fitting a hyperquadric surface over a neighborhood of size 7 x 7 x 4 and using information from all the four bands. In using the F-test for the detection of significant edges, 95 percent confidence level is employed. Figure 8.2a is an image that shows the significant edges. Figure 8.2b is an image that shows the significant directional edges at an angle of 45 degrees to the direction of x_2 axis in counterclockwise direction.

In general, as the neighborhood moves along the direction of the gradient, the edge magnitude starts at a low value, reaches its maximum, and then drops off to a low value. It is desired to locate the peak of the edge profile. The following algorithm is employed for thinning the detected significant edges. It uses the thresholded gradient magnitude and gradient direction of the fitted surface at the center of the neighborhood. The measure of gradient magnitude is taken to be the computed F-value if the computed F-value exceeded the critical F-value. Otherwise, it is set to be zero.

The neighboring pixels along a direction normal to the direction of the edge are disqualified from being candidates for the edges if the

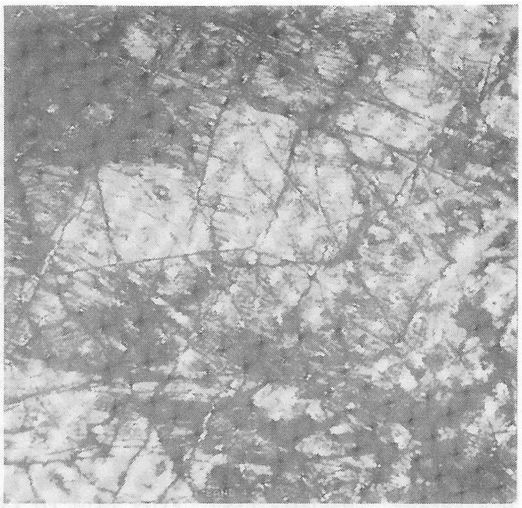

Figure 8-1a. An Australian Scene in Landsat Band 4 (512 x 512 Pixels)

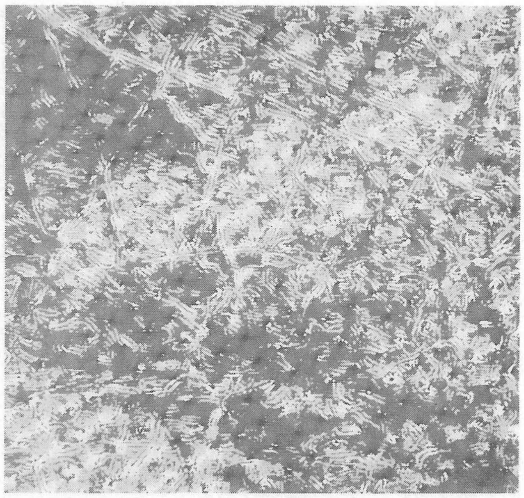

Figure 8-4. Detected Significant Lines at 95 Percent Confidence Level

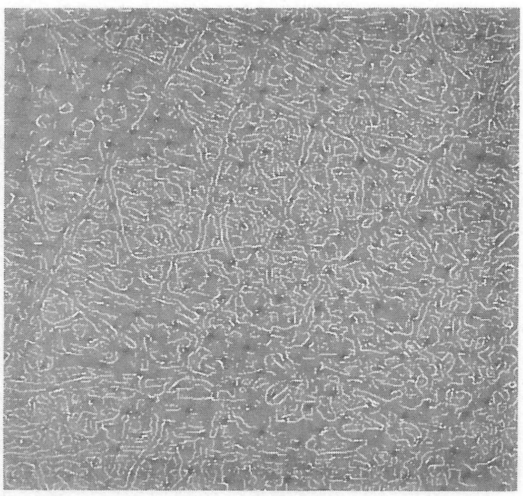

Figure 8-3. Thinned and Thresholded Gradient Magnitude Image

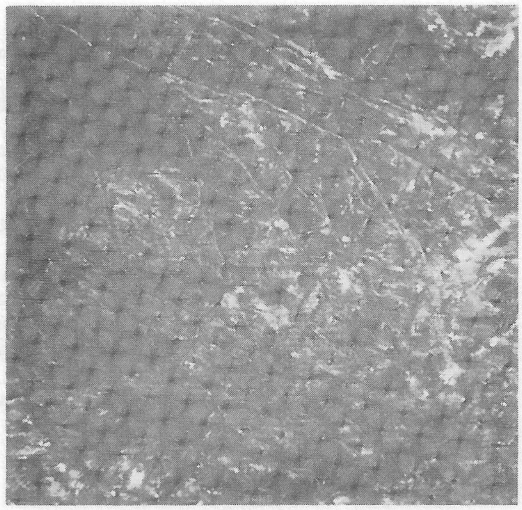

Figure 8-1b. An Australian Scene in Landsat Band 7 (512 x 512 Pixels)

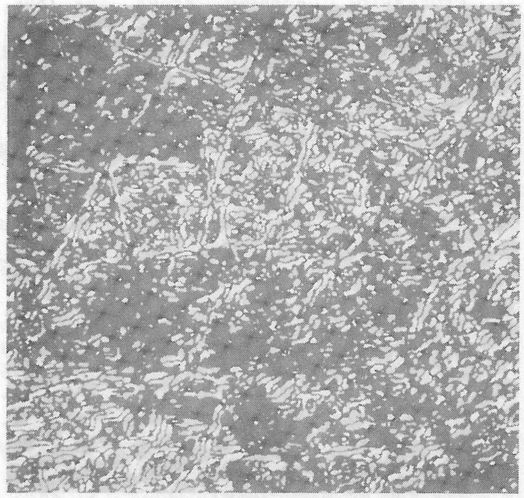

Figure 8-2b. Directional Gradient Magnitude Image Thresholded at 95 Percent Confidence Level

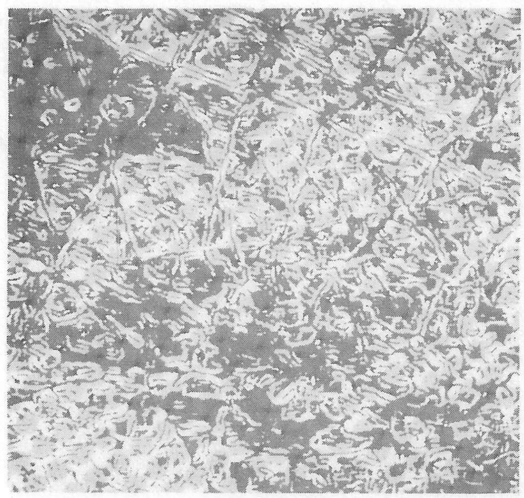

Figure 8-2a. Gradient Magnitude Image Thresholded at 95 Percent Confidence Level

following conditions are satisfied: the magnitude of the gradient at a pixel is larger than the magnitude of the gradients of its neighbors, and the direction of the gradients of the neighbors is within some allowable range from the direction of the gradient at the pixel under consideration. Figure 8.3 is the thinned magnitude gradient image using the above criteria.

Figure 8.4a is an image obtained by using second partial derivatives of hyperquadric surface over a neighborhood of size 7 x 7 x 4. The confidence level employed in the F-test is 95 percent. From Figures 8.1 and 8.4, it is seen that the significant lines in the image are extracted.

IX. REFERENCES

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