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A METHOD FOR DESCRIBING A THREE-DIMENSIONAL SHAPE

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I Abstract

This paper describes a new approach to both a two-dimensional shape and a three-dimensional shape. It is based on the notion that the two-dimensional shape is described by a chain of discrete points, and the three-dimensional shape is described by a latice-curved surface. This approach uses two special parameters which are the sum of distances and the sum of angles. There are three stages in this approach: 1) to describe a two-dimensional shape, 2) to describe a three-dimensional shape, and 3) to describe a shape within a digital image which contains more than two grey levels. This approach is useful because how to use the information of shape to process images becomes more and more important.

II Introduction

In digital image processing, the grey levels of images are often considered to the exclusion of other information about the shape of objects in the image. The shapes may, in fact, be the more important aspects of the image.

Of course, there are a lot of methods of shape description which have been previously discovered. But there are not yet any methods to describe a general three-dimensional shape.

The method described in this paper demonstrates that many kinds of descriptions can be used to describe every kind of shape in two-dimensional measurement & in three-dimensional measurement as well.

On the other hand, this method can be easily used for digital image processing, because it is based on the fact that the shape can be described by a chain of discrete points.

Although the descriptors are great in number, owing to the simple measurement of each descriptor, it is much easier to execute this method on the computer.

III Describing the Two-dimensional Shape

Since every image can be digitized and represented as a dot matrix which has the same distance between the line and line/column and column. The eight neighboring points of a given point which is located on the curve of the shape are those that occur directly above and below, directly to the right and left, and on the four diagonals.

The general approach is based on a calculation of the sum of the distances between a point in the image and its eight neighboring points as well as the sum of the angles between the abovementioned point and its eight neighboring points, where the distance and the angle are based on positional differerence in combination with grey level difference.

First, consider a binary image, one in which only two grey levels occur, "black" and "white" as dipicted in Figure 1. The goal of this research is to describe the "black" line in such a way that its shape is unambiguous.

An essential hypothesis must first be made: That the values of distances and angles are equal to zero, if the grey levels between two points are different.

Using this concept of distance and angle, three types of distances and nine types of angles may occur between any two points, as illustrated in Figure 1.

a=the diagonal distance between two points with the same grey level values.

b=the orthogonal distance between two points with same grey level values.

o=the distance between two points with different grey level values.

$\angle BAB=\alpha=$ the angle between a given point and its horizontal-right neighboring point with the same grey level.

$\angle BAC=\beta=$ the angle between a given point and its above-right neighboring point with the same grey level.

$\angle BAD=\gamma=$ the angle between a given point and its above neighboring point with the same grey level.

$\angle BAE=\delta=$ the angle between a given point and its above-left neighboring point with the same grey level.

$\angle BAF=\epsilon=$ the angle between a given point and its horizontal-left neighboring point with the same grey level.

$\angle BAG=\zeta=$ the angle between a given point and its below-left neighboring point with the same grey level.

$\angle BAH=\eta=$ the angle between a given point and its below neighboring point with the same grey level.

$\angle BAI=\kappa=$ the angle between a given point and its below-right neighboring point with the same grey level.

o = the angle between a given point and its neighboring point with a different grey level.

First, using this approach, it is possible to calculate the sum of the distances for point A (h_A) as the distance between point A and it eight neighboring points as follows:

$$h_A=\sum_{i=1}^{8} dis_i$$

This expression can also be written as follows:

$$H_A=\mu \frac{h_A}{4(a+b)}$$

where μ is a proportional constant.

This operation makes H independent of the scale of the image. This is the term that is used in the following discussion. Even when every possible combination of "white" and "black" points within a neighborhood is considered there are a total of only 25 different values that H may have. If none of the eight neighboring points has the same grey value, there is only one possible value of H for that point. If one of the eight neighboring points has a value different from the center point, then H may have two values, and so on, as shown in table 1.

Table 1. Total of all possible values of H

Number of neighboring points identical to central point	Number of ways H can be evaluated
0	1
1	2
2	3
3	4
4	5
5	4
6	3
7	2
8	1
	Total: 25

Therefore it can be seen that the values of H represents the similarities & differences that occur between the values of a point and all its neighboring points. The 25 different forms that H may take are shown in Table 2.

Table 2. All possible forms that H can take.

$H_{01}=\mu\frac{4a+4b}{4(a+b)}$

$H_{11}=\mu\frac{4a+3b}{4(a+b)}$

$H_{12}=\mu\frac{3a+4b}{4(a+b)}$

$H_{21}=\mu\frac{4a+2b}{4(a+b)}$

$H_{22}=\mu\frac{3a+3b}{4(a+b)}$

$H_{23}=\mu\frac{2a+4b}{4(a+b)}$

$H_{31}=\mu\frac{4a+b}{4(a+b)}$

$H_{32}=\mu\frac{3a+2b}{4(a+b)}$

$H_{33}=\mu\frac{2a+3b}{4(a+b)}$

$H_{34}=\mu\frac{a+4b}{4(a+b)}$

$H_{41}=\mu\frac{4a}{4(a+b)}$

$H_{42}=\mu\frac{3a+b}{4(a+b)}$

$H_{43}=\mu\frac{2a+2b}{4(a+b)}$

$H_{44}=\mu\frac{a+3b}{4(a+b)}$

$H_{45}=\mu\frac{4b}{4(a+b)}$

$H_{51}=\mu\frac{3a}{4(a+b)}$

$H_{52}=\mu\frac{2a+b}{4(a+b)}$

$H_{53}=\mu\frac{a+2b}{4(a+b)}$

$H_{54}=\mu\frac{3b}{4(a+b)}$

$H_{61}=\mu\frac{2a}{4(a+b)}$

$H_{62}=\mu\frac{a+b}{4(a+b)}$

$H_{63}=\mu\frac{2b}{4(a+b)}$

$H_{71}=\mu\frac{a}{4(a+b)}$

$H_{72}=\mu\frac{b}{4(a+b)}$

$H_{81}=\mu\frac{0}{4(a+b)}$

Second, using this approach, it is possible to calculate the sum of the angles for point $A(\theta'_A)$ as the angle between point A and its eight neighboring points as follows:

$$\theta'_A=\sum_{i=1}^{8} \text{Ang}_i$$

This expression can also be written as follows:

$$\theta=\nu\frac{\theta'_A}{\alpha+\beta+\gamma+\delta+\epsilon+\zeta+\eta+\kappa}$$

where ν is a proportional constant.

There are a total of 256 different values that θ may have. If none of the eight neighboring points has the same grey value. There is only one possible value of θ for that point, if one of the eight neighboring points has a value different from the center point, then θ may have eight values, and so on, as shown in table 3.

Table 3. Total of all possible values of θ

Number of neighboring points identical to central point	Number of ways θ can be evaluated
0	1
1	8
2	28
3	56
4	70
5	56
6	28
7	8
8	1
	Total:256

Using the parameter of sum of distances H, and the parameter of sum of angles θ, a new two-dimensional coordinate is built as show in figure2.

A two-dimensional shape fraction can be written as:

$$f(H,\theta)=\sqrt{H^2+\theta^2}$$

There are a total of 256 different values that $f(H,\theta)$ may have. The first 5 different forms that f may have are shown in table 4. In the notation used, the first digit of each subscript refers to the total number of neighboring points that differ from the center point in grey level value, and the second digit reflects the number of different forms that $f(H,\theta)$ may take depending on the position of each of the values.

Table 4. The first 5 different forms that $f(H,\theta)$ may take

$$f(H_{01},\theta_{01})=\sqrt{\mu^2+\nu^2}$$

$$f(H_{11},\ \theta_{11}) = \sqrt{\mu^2 \frac{(\Sigma - b)^2}{\Sigma^2} + \nu^2 \frac{(\Omega - \alpha)^2}{\Omega^2}}$$

$$f(H_{12},\ \theta_{12}) = \sqrt{\mu^2 \frac{(\Sigma - a)^2}{\Sigma^2} + \nu^2 \frac{(\Omega - \beta)^2}{\Omega^2}}$$

$$f(H_{11},\ \theta_{13}) = \sqrt{\mu^2 \frac{(\Sigma - b)^2}{\Sigma^2} + \nu^2 \frac{(\Omega - \gamma)^2}{\Omega^2}}$$

$$f(H_{12},\ \theta_{14}) = \sqrt{\mu^2 \frac{(\Sigma - a)^2}{\Sigma^2} + \nu^2 \frac{(\Omega - \delta)^2}{\Omega^2}}$$

where $\Sigma = 4a + 4b$

$$\Omega = \alpha + \beta + \gamma + \delta + \epsilon + \zeta + \eta + \kappa$$

The 256 different decriptions are able to describe every kind of two-dimensional shape.

IV. Describing the Three-dimensional Shape

Every three-dimensional object can be represented as a cubic lattice and, every three-dimensional shape can be represented as a lattice-curve surface.

There are 26 neighboring points for each center point as shown in Figure 3.

Taking further steps to develop a two-dimensional shape describing the use of the third parameter of the sum of angles which are measured on a vertical plane perpendicular to the horizontal plane which contains angle θ'. The new parameter of the sum of angles can be written as the following:

$$\varphi' = \sum_{i=1}^{8} \mathrm{Ang}_i$$

The above-mentioned hypothesis that the value of angles are equal to zero, if the grey levels between two points are different, is used for measurement of values of φ .

This expression can also be written as the following:

$$\varphi = \rho \frac{\varphi'}{\alpha + \beta + \gamma + \delta + \epsilon + \zeta + \eta + \kappa}$$

as illustrated in Figure 3.

$\varphi'_1 = \angle BAB = \angle CAC = \angle DAD = \angle EAE = \angle FAF = \angle GAG = \angle HAH = \angle IAI = \alpha$

$\varphi'_2 = \angle BAB' = \angle CAC' = \angle DAD' = \angle EAE' = \angle FAF' = \angle GAG' = \angle HAH' = \angle IAI' = \beta$

$\varphi'_3 = \angle BAA' = \angle CAA' = \angle DAA' = \angle EAA' = \angle FAA' = \angle GAA' = \angle HAA' = \angle IAA' = \gamma$

$\varphi'_4 = \angle BAF' = \angle CAG' = \angle DAH' = \angle EAI' = \angle FAB' = \angle GAC' = \angle HAD' = \angle IAE' = \delta$

$\varphi'_5 = \angle BAF = \angle CAG = \angle DAH = \angle EAI = \angle FAB = \angle GAC = \angle HAD = \angle IAE = \epsilon$

$\varphi'_6 = \angle BAF'' = \angle CAG'' = \angle DAH'' = \angle EAI'' = \angle FAB'' = \angle GAC'' = \angle HAD'' = \angle IAE'' = \zeta$

$\varphi'_7 = \angle BAA'' = \angle CAA'' = \angle DAA'' = \angle EAA'' = \angle FAA'' = \angle GAA'' = \angle HAA'' = \angle IAA'' = \eta$

$\varphi'_8 = \angle BAB'' = \angle CAC'' = \angle DAD'' = \angle EAE'' = \angle FAF'' = \angle GAG'' = \angle HAH'' = \angle IAI'' = \kappa$

ρ is a proportional constant. There are a total of 256 different values that φ may have.

In Figure 3, a new angle is needed for θ at point A' and A". It is determined as τ .

The parameter θ' which is measured on a horizontal plane can be written as the following:

$$\theta' = \sum_{i=1}^{9} \mathrm{Ang}_i$$

This expression can also be written as the following:

$$\theta = \nu \frac{\theta'}{\alpha + \beta + \gamma + \delta + \epsilon + \zeta + \eta + \kappa + \tau}$$

There are a total of 512 different values that may have as shown in table 5.

Table 5. Total of all possible values of

Number of neighboring points identical to central point	Number of ways can be evaluated
0	1
1	9
2	36
3	84
4	126
5	126

6	84
7	36
8	9
9	1
	Total: 2

Using the parameter of the sum of distance H and the two parameters of the sum of angles θ and φ a new three-dimensional coordinate is built as shown in figure 4.

A three-dimensional shape descriptor can be written as:

$$f(H, \theta, \varphi) = \sqrt{H^2 + \theta^2 + \varphi^2}$$

Using a formula as follows:

$$C_m^{m-n} = \frac{m!}{n!(m-n)!}$$

where m is the number of neighboring points as 26

n is the number of neighboring points identical to the central point.

There are a total of 67,108,864 different values that $f(H, \theta, \varphi)$ may have as shown in Table 6.

Table 6. Total of all possible values of $f(H, \theta, \varphi)$

Number of neighboring points identical to central point	Number of ways $f(H, \theta, \varphi)$ can be evaluated
0	1
1	26
2	325
3	2600
4	14950
5	65780
6	230230
7	657800
8	1562275
9	3124550
10	5311735
11	7726160
12	9657700
13	10400600
14	9657700
15	7726160
16	5311735
17	3124550
18	1562275
19	657800
20	230230
21	65780
22	14950
23	2600
24	325
25	26
26	1
	Total=2^{26}

The first 5 different forms that $f(H, \theta, \varphi)$ may have are shown in Table 7.

Table 7. The first 5 different forms that $f(H, \theta, \varphi)$ may have.

$$f(H_{01}, \theta_{01}, \varphi_{01}) = \sqrt{\mu^2 + \nu^2 + \rho^2}$$

$$f(H_{11}, \theta_{11}, \varphi_{11}) = \sqrt{\mu^2 \frac{(\Sigma-b)^2}{\Sigma^2} + \nu^2 \frac{(\Omega-\alpha)^2}{\Omega^2} + \rho^2 \frac{(\psi-\alpha)^2}{\psi^2}}$$

$$f(H_{11}, \theta_{11}, \varphi_{12}) = \sqrt{\mu^2 \frac{(\Sigma-b)^2}{\Sigma^2} + \nu^2 \frac{(\Omega-\alpha)^2}{\Omega^2} + \rho^2 \frac{(\psi-\beta)^2}{\psi^2}}$$

$$f(H_{11}, \theta_{11}, \varphi_{13}) = \sqrt{\mu^2 \frac{(\Sigma-b)^2}{\Sigma^2} + \nu^2 \frac{(\Omega-\alpha)^2}{\Omega^2} + \rho^2 \frac{(\psi-\kappa)^2}{\psi^2}}$$

$$f(H_{12}, \theta_{12}, \varphi_{14}) = \sqrt{\mu^2 \frac{(\Sigma-b)^2}{\Sigma^2} + \nu^2 \frac{(\Omega-\beta)^2}{\Omega^2} + \rho^2 \frac{(\psi-\alpha)^2}{\psi^2}}$$

where Σ, Ω, ψ, can be written as the following:

$$\Sigma = 4a + 4b$$

$$\Omega = \alpha + \beta + \gamma + \delta + \epsilon + \zeta + \eta + \kappa + r$$

$$\psi = \alpha + \beta + \gamma + \delta + \epsilon + \zeta + \eta + \kappa$$

The 67,108,864 different descriptors are able to describe every kind of three-dimensional shape.

V Describing a Shape Within a Digital Image Which Contains More Than Two Grey Levels

The following approach will be extended to images which contain more than two grey levels. These images will be treated as "three-dimensional" images with the grey values as the third dimension.

Figure 5 is an example of an image, where object M and object N have different grey levels. The black lines represent the boundary lines of the two objects, and points A and B are two arbitrary points in region M, the former located within the region and the latter on the boundary.

Applying the same hypothesis about distance as was used before, the distance between A and its eight neighboring points can have only two forms, a or b; in the case of point B, however, there exist four different expressions of distance: a, b, c, and d, where a and b are constants, and c and d are variables that change in relation to the grey levels present.

In order to describe these four kinds of distances in the grey-scale image, it is best to do so using the concept of three-dimensional space as shown in Figure 6. In this figure, axes x and y are the horizontal and vertical image axes, and z is a grey-level axis, creating, therefore, a kind of three-dimensional representation of the image.

Consider Figure 6. When measured along the z axis, point B has a grey value of M; similarly points I and E, which are two of the eight neighboring points of B, have a grey value of N. The distances BI and BE can be represented, then, as

$$BI=\sqrt{a^2 + (M-N)^2}$$

$$BE=\sqrt{b^2 + (M-N)^2}$$

where a and b have the same meaning as they did in the earlier discussion of the binary image.

Supposing

$$h=\sqrt{a^2 + (M-N_i)^2}$$

$$r=\sqrt{b^2 + (M-N_j)^2}$$

the sum of the distances between an arbitrary point and its eight neighboring points can be written as:

$$l'(h,r)= \sum_{i=1}^{4} h_i + \sum_{j=1}^{4} r_j$$

given that:

$$l(h,r)= \mu \left(1- \frac{4(a+b)}{l'(h,r)}\right)$$

using the parameter as the sum of angles as described in part III, it is written as the following.

$$\theta'= \sum_{i=1}^{8} Ang_i$$

given that:

$$\theta = \nu \left(1- \frac{\theta'}{\Omega}\right)$$

where $\theta'= \alpha +\beta +\gamma + \delta + \varepsilon + \zeta + \eta + \kappa$

the definitions from α to κ are the same as described in part III.

A new function of the descriptor can be written as the following:

$$f(l,\theta)=\sqrt{l^2 + \theta^2}$$

As a result, the value $f(l,\theta)$ of a point which is located within the object having a grey level of M will be evaluated as zero; the value of a point that is included in the boundary is evaluated as a meaning of value, less than $\sqrt{\mu^2+ \nu^2}$.

The advantage of this method is that all of the information contained within the image is concentrated into several broundaries that describe the objects in the image. This fact can save computer time as well as finding the boundaries of objects in subsequent processing.

The data for the center of the objects can be simplified to a single function $f(l,\theta)$ using the two-dimensional Fourier Transform:

$$F(u,v)= \iint_{-\infty}^{\infty} f(l,\theta)\exp(-i2\pi(ul+v\theta))dld\theta$$

where u,v is the frequency of the change of the shape, including grey values. In its discrete form it appears as the following:

$$F(u,v)=\frac{1}{N}\sum_{1=0}^{N-1}\sum_{=0}^{N-1} f(1,\theta)\exp(-i2\pi(u1+v\theta)/N)$$

The inverse form is

$$f(1,\theta)=\frac{1}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} F(u,v)\exp(i2\pi(u1+v\theta)/N)$$

In digital image processing, the process of finding the boundary of an object or detecting it from within a whole image is the most critical task. The above-mentioned method can be used to detect various kinds of curves based on grey levels. Further research must be devoted to determining whether it is possible to combine curves with similar spectra, thereby creating a spectrum of a group which can be compared to a standard pattern spectrum for classification and recognition.

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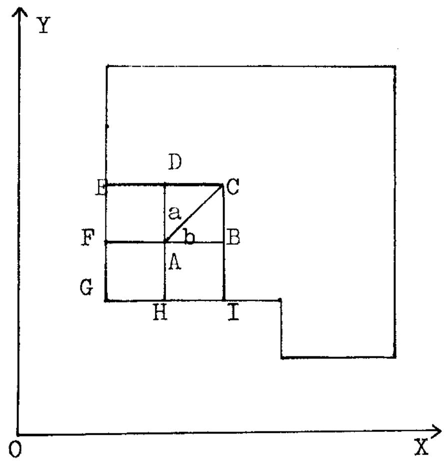

FIGURE 1

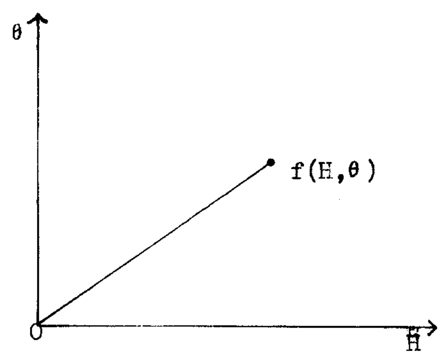

FIGURE 2

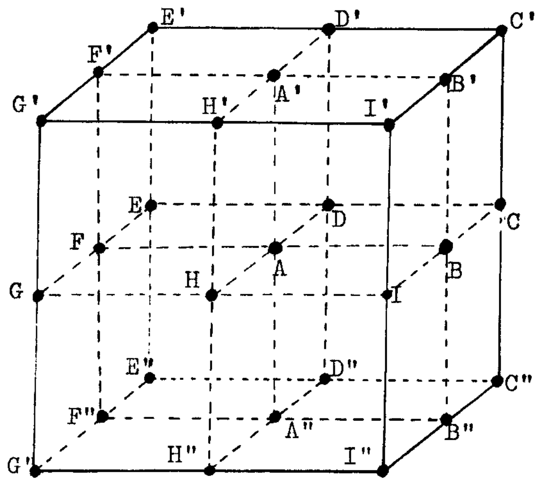

FIGURE 3

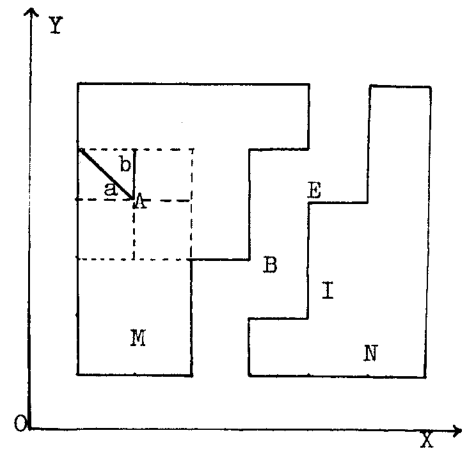

FIGURE 5

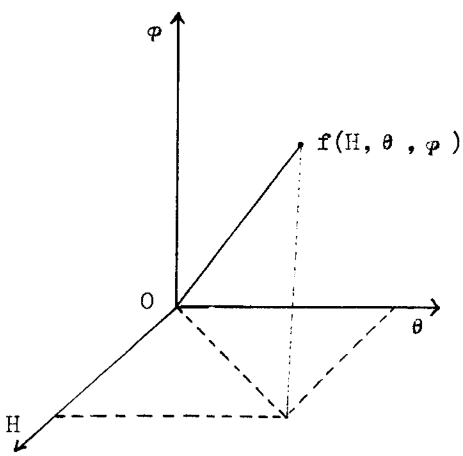

FIGURE 4

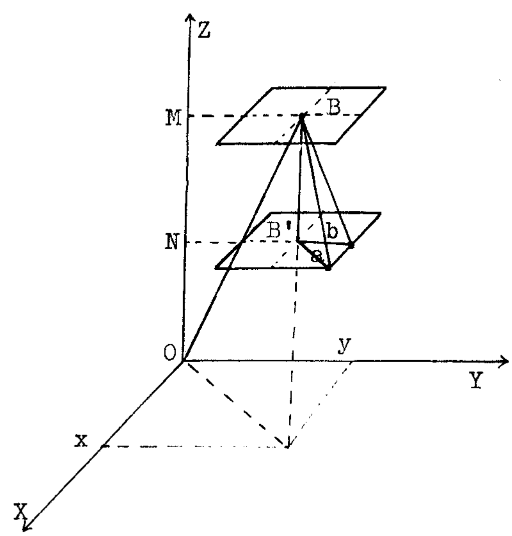

FIGURE 6