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# FAST GEOMETRIC CORRECTION OF NOAA AVHRR

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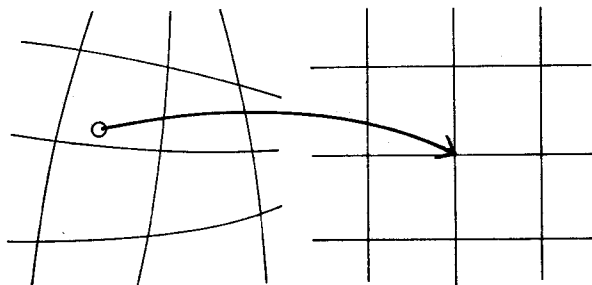
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Tokyo, Japan

## ABSTRACT

Fast and accurate geometric correction procedure for NOAA AVHRR data is described. The procedure to calculate the mapping function which gives the coordinate of original AVHRR data if the coordinate of the surface is given uses orbital data in stead of GCPs. The question of the earth rotation is solved by the geometry used in this paper and expansion of the equation of spherical trigonometry in the series of the term of the earth rotation. It is discussed how to make this procedure faster without increase of error. Error is estimated and showed that it is negligible in usual latitude.

## I. INTRODUCTION

AVHRR (Advanced Very High Resolution Radiometer) data of NOAA series satellites are very useful to Oceanography, Fishery and Meteorology. AVHRR provides us data twice a day. More than one NOAA series satellites are planned to be operational in the near future. NOAA satellites are polar orbital and resolution of AVHRR at nadir is 1.1 km. This resolution is much better than that of stationary satellites.



Original image

Output image

Fig. 1 A mapping of geometric correction

AVHRR data can be applied to observe dynamics of ocean. Application of AVHRR data is shown in [1]. In order to know a temporal change of ocean pattern such as a change of oceanic front or movement of eddies, it is very important to correct AVHRR data geometrically to superpose them. As AVHRR is operational has high resolution, we need fast and accurate geometric correction to take advantage of it.

## II. DIFFICULTIES

Geometric correction is a mapping from a coordinate of original data to that of output image (See Fig. 1). The question of geometric correction is to find a mapping function. In general most frequently used method to find the mapping function is to make use of GCPs (Ground Control Points). In this method we assume, first of all, the form of a mapping function, then determine coefficients of the function using GCPs. But this method is not good for AVHRR data. The reasons are as the follows.

- (1) If the application is oceanic, it is very difficult to find GCPs in the sea. A GCP is a clearly identified point whose location we know. GCPs are required to scatter all over the output image for better accuracy.
- (2) Covering area of AVHRR is so wide (the swath is up to 3000 km) that the form of a mapping function must be complex due to earth curvature.
- (3) We cannot ignore the rotation of the earth.
- (4) As the NOAA satellites are operational, intervention to find GCPs during the process of AVHRR data must be minimized.

The rest part of this paper describes how to determine the mapping function from the surface location to the position in AVHRR

data using orbital data. Orbital data is provided from NOAA by broadcasting. Unfortunately the mapping function is not derived straight forward. We solved the equation by approximation. But error is very small on normal latitudes.

Table 1 shows the characteristics of NOAA satellites and AVHRR which relate to geometric correction.

Table 1. Characteristics of NOAA satellites and AVHRR

Height	830 ~ 870 (km)
Scan angle	-55.4° ~ +55.4°
Swath	3000 (km)
Resolution	1.1 (km) at nadir
Scan speed	6 (lines/sec)
Period	100 (min)
Inclination angle	99°

### III. NOTATION

For the simplicity, we assume the coordinate system of the surface is not geographical system but spherical coordinate whose center is the center of the earth. We use the following notation.

H : Height of the satellite (830km)  
R : Radius of the earth (6370km)  
 $\Omega_e$  : Angular velocity of the earth ( $7.29246 \times 10^{-5}$  rad/sec)  
 $\Omega_s$  : Angular velocity of the satellite ( $1.04 \times 10^{-3}$  rad/sec)  
 $\Omega = \Omega_s / \Omega_e$  (14.2)  
 $\alpha$  : Scan angle of AVHRR  
 $\beta$  : Inclination angle (99°)  
t : Time passed after equator crossing  
N : North Pole  
P : Point observed by AVHRR  
Q : Crossing point of the orbit plane at latitude when the satellite is at S  
 $Q_0$  : Crossing point of the satellite at latitude  
S : Point of the satellite observing P  
d : Distance SQ (rad.)  
f : Distance between P and S (rad.)  
l : Distance PQ (rad.)  
 $l_0$  : Distance PQ (rad.)  
 $\delta$  : Angle NQP  
 $\delta_0$  : Angle N $Q_0$ P  
 $\epsilon$  : Angle SQP  
 $\epsilon_0 = \gamma \pm (\delta_0 - \pi/2)$   
 $\phi$  : Angle PN $Q_0$  ( $\phi = |\phi_P - \phi_0|$ )  
 $\rho$  : Angle QN $Q_0$  (due to earth rotation)  
 $\phi_C$  : Longitude at the equator crossing

$\phi_0$  : Longitude of the point  $Q_0$   
 $\phi_P$  : Longitude of the point P  
 $\Delta L$  : Difference of lines  
 $\Delta \phi$  : Difference of longitude  
a1 : Defined by (20)  
b1 : Defined by (21)  
c1 : Defined by (23)  
s : Size of a pixel of an output image  
(i, j) : (line, column) of the output image  
 $(\phi_1, \theta_1)$  : Location of corner of output

### IV. GUIDELINE OF THE PROCEDURE

The mapping function we have to find gives us the coordinate of the AVHRR data if the coordinate of output image (i, j) is given. The coordinate of the AVHRR is given by  $(\alpha, t+d/\Omega_s)$ . Therefore the question is to determine  $(\alpha, d)$  if (i, j) is given. Fig. 2 shows the diagram how to calculate  $(\alpha, d)$  from (i, j). The main objective of this paper is to show how to calculate  $\rho$  fast and accurately from given  $(\phi, \theta)$ . Other parts in Fig. 2 can be calculated straight forward.

### V. EQUATIONS FOR THE MAPPING FUNCTION

#### 1. MERCATOR PROJECTION

Coordinate system of the output image may be arbitrary. Given a coordinate system we get the function from (i, j) to  $(\phi, \theta)$ .

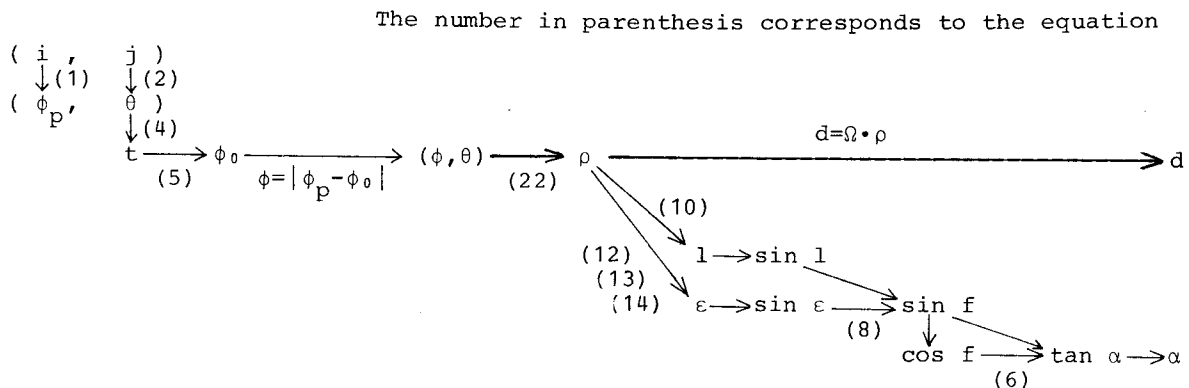


Fig. 2 Diagram to calculate  $(\alpha, d)$  from given (i, j)

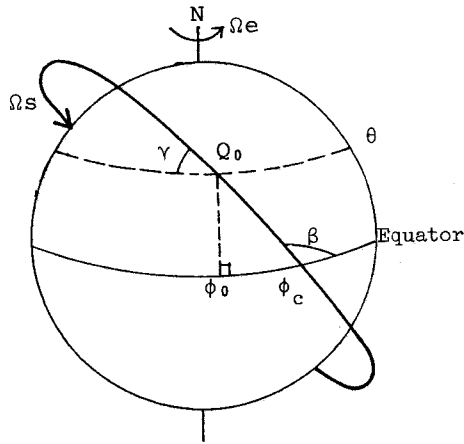


Fig. 3 Geometry of inclination angle

We chose the mercator system, so we use the following equations.

$$\phi_p - \phi_1 = \frac{i \cdot s}{R} \quad (1)$$

$$\tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = \exp\left(\theta_1 + \frac{j \cdot s}{R}\right) \quad (2)$$

where s is the size of a pixel of an output image.

## 2. INCLINATION ANGLE AND LONGITUDE OF THE SATELLITE AT LATITUDE $\theta$

Relation between crossing angle  $\gamma$  at latitude  $\theta$  and inclination angle at equator is given by (3).

$$\cos \gamma = \frac{\cos \beta}{\cos \theta} \quad (3)$$

Latitude of the satellite  $\phi_0$  at the time t after equator crossing is given by (4).

$$\sin \theta = \sin(t \Omega_s) \sin \beta \quad (4)$$

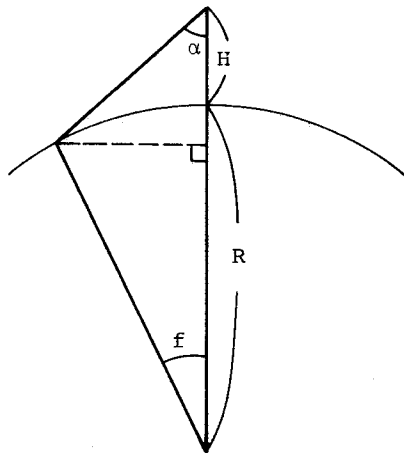


Fig. 4 Geometry of scan angle

Longitude of the satellite  $\phi_0$  at latitude  $\theta$  is given by (5), which contains the term to compensate the earth rotation during the time t.

$$\cos(\phi_c - \phi_0 - t \Omega_e) = \frac{\cos(t \Omega_s)}{\cos \theta} \quad (5)$$

Fig. 3 shows the geometry. The equations (3), (4) and (5) are derived from the spherical right triangle.

## 3. RELATION BETWEEN SCAN ANGLE $\alpha$ AND f

f is the distance between P and nadir of the satellite. f is related to scan angle  $\alpha$  of AVHRR (see Fig. 4). As we showed in Fig. 2, we need to calculate  $\alpha$  from given f. The function from f to  $\alpha$  is given by (6).

$$\tan \alpha = \frac{\sin f}{\frac{H+R}{R} - \cos f} \quad (6)$$

where H is the height of the satellite and R is the radius of the earth. The equation (6) is easy to derive if you put dotted line in Fig. 4.

## 4. GEOMETRY OF OBSERVED POINT P AND SATELLITE POSITION S

Let the satellite be at S. AVHRR with scan angle f (which is derived using (6)) observes the point P whose coordinate is  $(\phi_p, \theta)$ . Q is the crossing point between the plane of latitude  $\theta$  and the orbit plane of the satellite. As AVHRR is scanning at right angle to the orbit, angle PSQ is right angle. As shown in Fig. 5, let d be the distance between S and Q, l be the distance between P and Q, and  $\epsilon$  be the angle SQP. Because of the spherical right triangle PSQ, we get the following

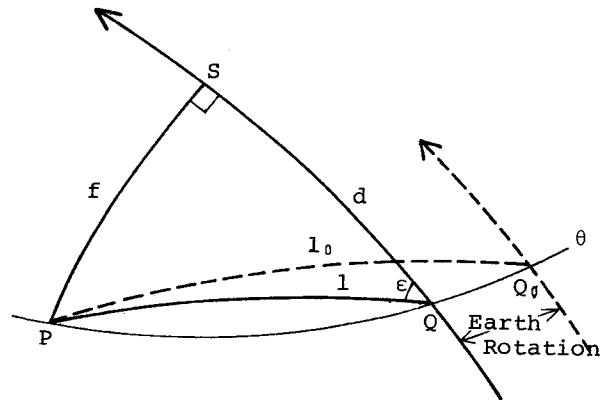


Fig. 5 Geometry of Observation

two important equations, (7) and (8).

$$\tan d = \cos \varepsilon \cdot \tan l \quad (7)$$

$$\sin f = \sin \varepsilon \cdot \sin l \quad (8)$$

Note that Q is not the real crossing point of the satellite at latitude  $\theta$ . The earth rotates while the satellite goes from  $Q_0$  to S. Difficulty comes from here.

### 5. EFFECT OF EARTH ROTATION

Our goal is to solve f when the point P is given. Because of the earth rotation we cannot derive f from coordinate of P straight forward. Let  $\rho$  be the angle of the earth rotation while the satellite goes along d (see Fig. 6). As the Fig. 6 shows there are four cases, A, B, C, and D.  $Q_0$  is the real crossing point of the satellite at latitude  $\theta$ , but Q moves from  $Q_0$  due to the earth rotation  $\rho$ . Let  $\phi$  be the angle  $PNQ_0$ ,  $\delta$  be the angle  $NQP$ ,  $\delta_0$  be the angle  $NQ_0P$ , and  $l_0$  be the distance  $PQ_0$ . Spherical triangles  $PNQ_0$  and  $PNQ$  are isosceles. These triangles suffice the

equations (9), (10), (11) and (12).

$$\sin l_0 = \sin^2 \theta + \cos^2 \theta \cos \phi \quad (9)$$

$$\cos l = \sin^2 \theta + \cos^2 \theta \cos(\phi - \rho) \quad (10)$$

$$\tan \delta_0 = \frac{1}{\sin \theta} \cot \frac{\phi}{2} \quad (11)$$

$$\tan \delta = \frac{1}{\sin \theta} \cot \frac{\phi - \rho}{2} \quad (12)$$

As shown in Fig. 6  $\varepsilon$  and  $\delta$  are related either by (13) in case A and D or (14) in case B and C.

$$\varepsilon = \gamma + \delta - \frac{\pi}{2} \quad (13)$$

$$\varepsilon = \gamma - \delta + \frac{\pi}{2} \quad (14)$$

We define  $\varepsilon_0$  by (15).

$$\varepsilon_0 = \gamma \pm (\delta_0 - \frac{\pi}{2}) \quad (15)$$

Equation (10) gives us relation between  $\rho$  and  $l$ , and (11), (13) and (14) gives us

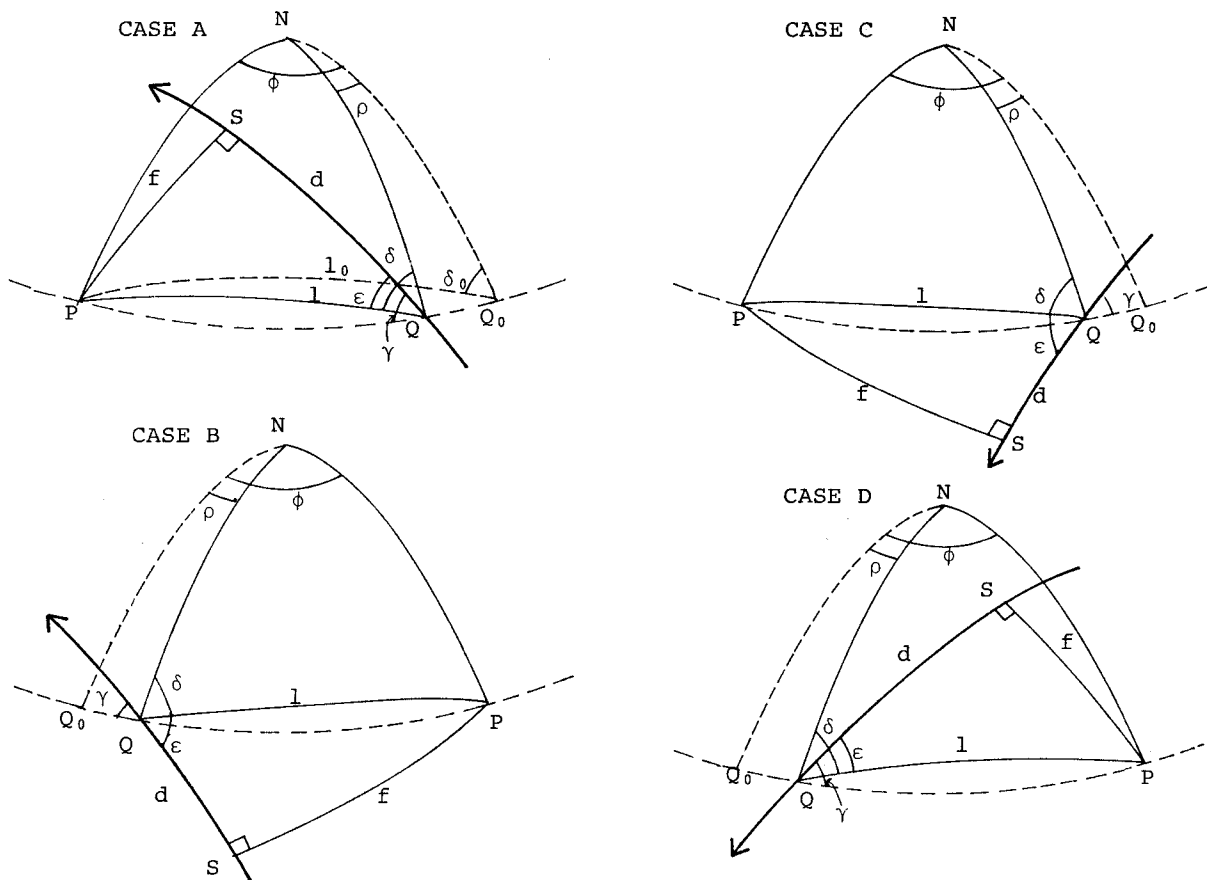


Fig. 6 Geometry of observation with the earth rotation

relation between  $\rho$  and  $\epsilon$ . This means the right hand side of the equation (7) is a function of  $\rho$ . On the other hand  $d$  is represented by  $\rho$  as (16).

$$d = \Omega \rho \quad (16)$$

where  $\Omega$  is ratio of angular velocity of the satellite and the earth. As the both sides of the equation (7) can be represented by  $\rho$ , the question becomes to find  $\rho$  which suffices the equation (7). We solve the equation by approximation. That is, as  $\rho$  is small we expand both sides of (7) in a series of  $\rho$ . Then we ignore higher order of  $\rho$ . Each term of (7) is approximated as per the following. We ignore more than one order of  $\rho$ . The details of this expansion are shown in the Appendix I.

$$\tan d = d \quad (17)$$

$$\tan l = \tan l_0 (1 - a_1 \cdot \rho) \quad (18)$$

$$\cos \epsilon = \cos \epsilon_0 (1 - b_1 \cdot \rho) \quad (19)$$

where

$$a_1 = \pm \frac{\cos \theta \sin \phi}{\cos l_0 \sin^2 l_0} \quad (20)$$

sign is + in case A and C and - in case B and D

$$b_1 = \pm \frac{\tan \epsilon_0 \sin \theta}{2(1 - \cos^2 \theta \sin^2 \frac{\phi}{2})} \quad (21)$$

sign is + in case A and D and - in case B and C.

$l_0$  and  $\epsilon_0$  can be calculated using (9), (11) and (15) if  $\phi$  and  $\theta$  are given.

Assigning (17), (18) and (19) into (7) and ignoring of more than one order of  $\rho$  makes (7) the simple equation of  $\rho$ . The solution of the simple equation is given by (22).

$$\rho = \frac{1}{a_1 + b_1 + c_1} \quad (\text{radian}) \quad (22)$$

where

$$c_1 = \frac{\Omega}{\cos \epsilon_0 \tan l_0} \quad (23)$$

Note that if  $\phi$  is zero  $a_1$  and  $c_1$  become infinite and  $\rho$  is zero. It takes  $\rho/\Omega_e$  seconds and the satellite goes  $d = \Omega \rho$  while the earth rotate  $\rho$ . AVHRR scans 6 lines in one second, so the expression (24) gives the difference of the lines in which the original data that is to be picked up is.

$$\Delta L = \pm \frac{\rho}{\Omega_e} \times 6 \quad (24)$$

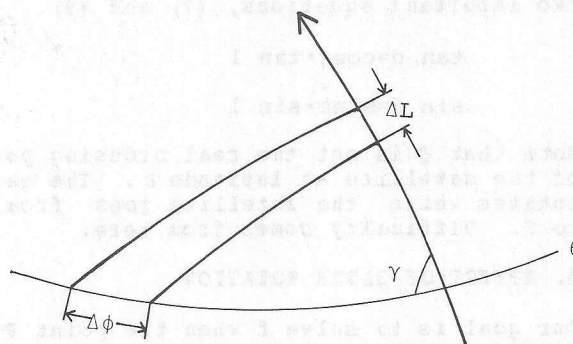


Fig. 7 Relation between  $\Delta\phi$  and  $\Delta L$

where sign is + in case A and C and - in case B and D.

Now we get  $\rho$ , then we can calculate  $\alpha$  according to the diagram of Fig. 2.

## VI. FAST CALCULATION

As the NOAA satellites are polar orbit, much difference of the longitude  $\Delta\phi$  cause a little difference of lines  $\Delta L$  of original AVHRR data (see Fig. 7). The grid in Fig. 8 shows this situation. In order to calculate fast we need to avoid unnecessary redundant calculation. Relation between  $\Delta L$  and  $\Delta\phi$  is given (25).

$$\Delta L = \frac{6}{\Omega_s} \cdot \frac{d\rho}{d\phi} \Delta\phi \quad (25)$$

where

$$\frac{d\rho}{d\phi} = \rho^2 \times \left\{ c_1(a_1 + b_1) - (3\cos^2 l_0 - 1)a_1^2 - \cot \phi \cdot a_1 - \frac{1}{\sin^2 \epsilon_0} b_1^2 - \frac{\cos^2 \theta \sin \phi}{2} b_1 \right\} \quad (26)$$

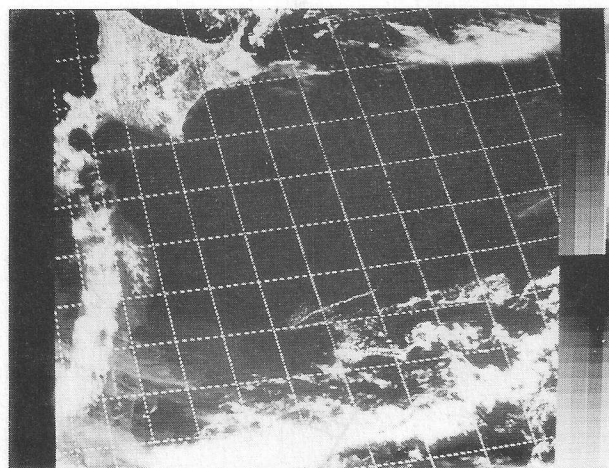


Fig. 8 Grid over the original AVHRR

Table 2 Error of  $\rho$  (the unit is line)

Longitude	-25°	-20°	-15°	-10°	-5°	0°	5°	10°	15°	20°	25°
Latitude											
10°	1.310	0.553	0.195	0.048	0.005	0.000	0.004	0.030	0.088	0.182	0.313
20°	2.020	0.797	0.258	0.058	0.005	0.000	0.004	0.024	0.056	0.092	0.117
30°	2.621	1.003	0.311	0.065	0.005	0.000	0.004	0.019	0.038	0.048	0.042
40°	2.399	1.105	0.339	0.070	0.005	0.000	0.004	0.017	0.030	0.032	0.020
50°	2.751	1.068	0.332	0.069	0.005	0.000	0.004	0.017	0.030	0.032	0.020
60°	2.239	0.905	0.293	0.064	0.005	0.000	0.004	0.020	0.038	0.048	0.042
70°	1.536	0.663	0.232	0.055	0.005	0.000	0.004	0.024	0.058	0.093	0.117
80°	0.764	0.376	0.151	0.042	0.005	0.000	0.005	0.033	0.102	0.216	0.376

Derivation of (26) is shown in the Appendix II. Note that the expression (26) converges (27) if  $\phi$  becomes zero.

$$\frac{d\rho}{d\phi} = \frac{\cos^2\gamma}{(\cos\theta\cos\gamma + \Omega)^2} \left( \frac{\Omega\cos\theta}{\cos\gamma} - 3\cos^2\theta \right) \quad (27)$$

This is derived from the equations (28), (29), (30) and (31) which hold when  $\phi$  is small.

$$\sin\phi = \phi \quad (28)$$

$$\sin l = l \quad (29)$$

$$l = \cos\theta \cdot \phi \quad (30)$$

$$\varepsilon = \gamma \quad (31)$$

$\Delta L$  in the highest resolution is 1 which corresponds to 1.1 km. We need to calculate (22) once in  $\Delta\phi$  given by (32).

$$\Delta\phi = \frac{\Omega e}{6} \cdot \frac{1}{\left(\frac{d\rho}{d\phi}\right)} \quad (32)$$

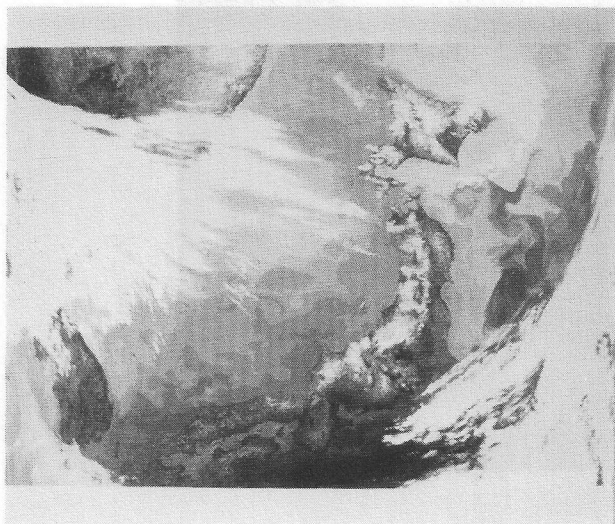


Fig. 9 Original AVHRR data

## VII. ERROR

An error of this method comes from approximation of (17), (18) and (19) and ignoring of higher order of  $\rho$ . So the error is contained only in (22). This error tends to be large if  $\phi$  is big or latitude is high. But the maximum of  $\phi$  is limited by the maximum scan angle. Table 2 shows the error of  $\rho$  on the unit of line. We assume  $\beta$  is 98.7°, the period is 101 min. and upward going. Note that if latitude is high reduced scale become small, that is the distance between the same longitude is small. For example, the maximum of  $f$  is 13° at the latitude 45°. (See coverage area of Fig. 10)

## VIII. RESULTS AND CONCLUSION

Fig. 11 shows the results of geometric correction of NOAA AVHRR. Coastal zone data are overlaid to prove accuracy of this procedure. Original data is shown in Fig. 9. This is north part of Japan. Fig. 10 shows the coverage area of Fig. 9, which is calculated only from orbital data. But orbital data which is parameter of this procedure is not so accurate. We used one GCP to transfer and adjust the image for Fig. 11.

The procedure discussed here is fast and accurate. As this procedure does not require human intervention, so this is suitable to process operational AVHRR data.

## ACKNOWLEDGEMENT

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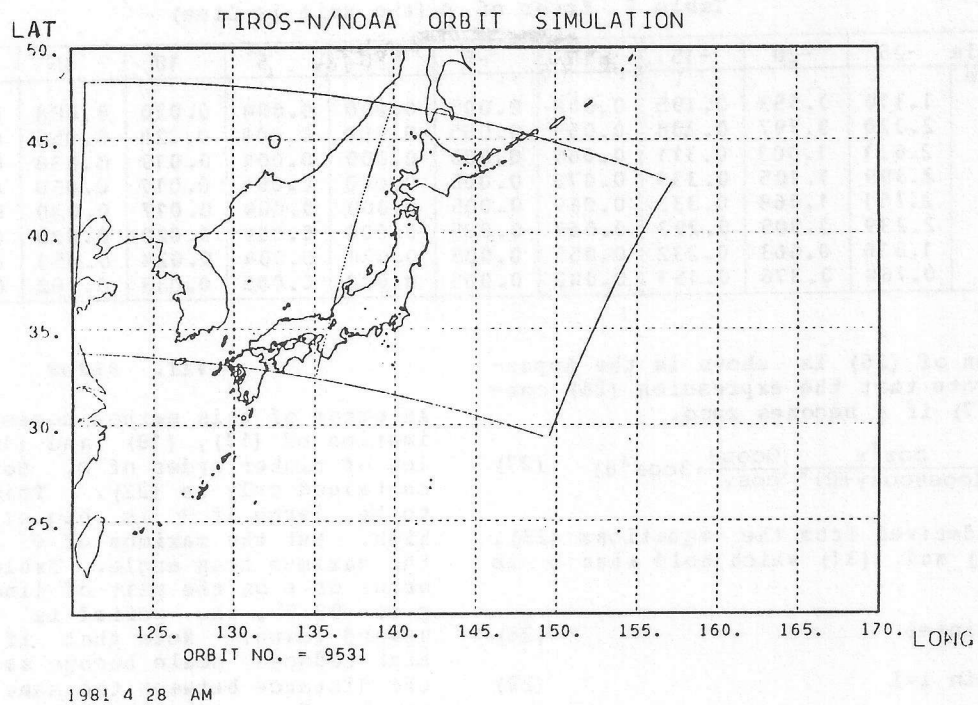


Fig. 10 Coverage Area of Fig. 9

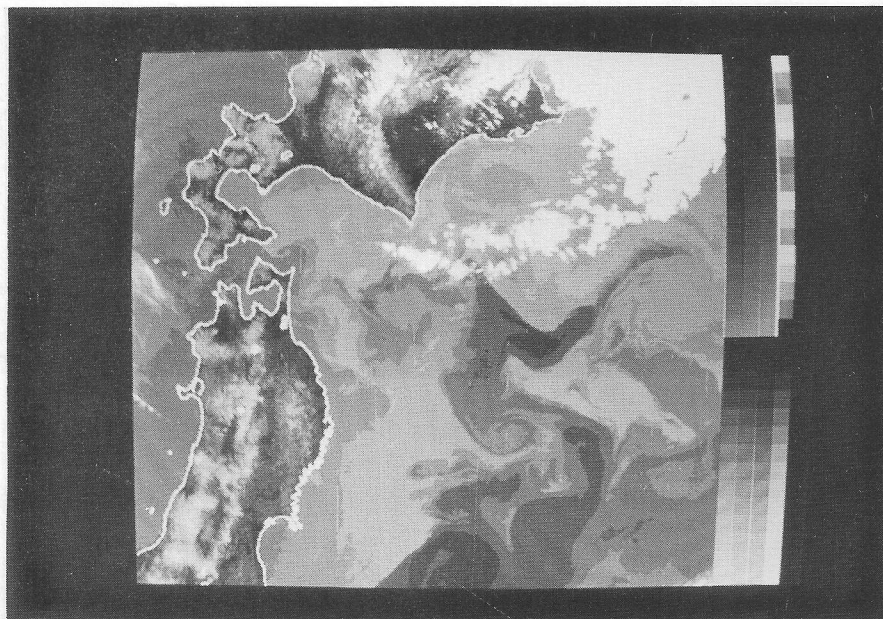


Fig. 11 Geometrically corrected image overlaid with the coastal zone



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APPENDIX I. Details of expansion

Relation between  $l$  and  $\rho$  is given by (10). Differentiation of both sides of the equation (10) by  $\rho$  leads to (A.1).

$$\frac{dl}{d\rho} = \frac{\cos^2 \theta \sin(\phi - \rho)}{\sin l} \quad (A.1)$$

The first two terms of Taylor expansion of  $\tan l$  in a series of  $\rho$  gives (A.2)

$$\begin{aligned} \tan l &= \tan l_0 + \frac{1}{\cos^2 l_0} \left( \frac{dl}{d\rho} \right)_{\rho=0} \cdot \rho \\ &= \cos l_0 (1 - b_1 \cdot \rho) \end{aligned} \quad (A.2)$$

Relation between  $\delta$  and  $\rho$  is given by (12). Differentiation of both sides of the equation (12) by  $\rho$  leads to (A.3).

$$\frac{1}{\cos^2 \delta} \cdot \frac{d\delta}{d\rho} = \frac{1}{2 \sin \theta \sin^2 \frac{\phi - \rho}{2}} \quad (A.3)$$

The expression (A.4) is derived from (12).

$$\begin{aligned} \frac{1}{\cos^2 \delta} &= 1 + \tan^2 \delta \\ &= \frac{\sin^2 \theta \sin^2 \frac{\phi - \rho}{2} + \cos^2 \frac{\phi - \rho}{2}}{\sin^2 \theta \sin^2 \frac{\phi - \rho}{2}} \end{aligned} \quad (A.4)$$

Assigning (A.4) into (A.3) leads to (A.5)

$$\begin{aligned} \frac{d\delta}{d\rho} &= \frac{\sin \theta}{2 (\sin^2 \theta \sin^2 \frac{\phi - \rho}{2} + \cos^2 \frac{\phi - \rho}{2})} \\ &= \frac{\sin \theta}{2 (1 - \cos^2 \theta \sin^2 \frac{\phi - \rho}{2})} \end{aligned} \quad (A.5)$$

The first two terms of Taylor expansion of  $\cos \varepsilon$  in a series of  $\rho$  gives (A.6).

$$\cos \varepsilon = \cos \varepsilon_0 - \sin \varepsilon_0 \left( \frac{d\varepsilon}{d\rho} \right)_{\rho=0} \cdot \rho \quad (A.6)$$

Equation (A.7) holds because of (13) and (14).

$$\frac{d\varepsilon}{d\rho} = \pm \frac{d\delta}{d\rho} \quad (A.7)$$

Assigning (A.7) and (A.5) into (A.6) leads to (19).

APPENDIX II. Derivation of (26)

We get (A.8) from (22).

$$\frac{1}{\rho} = a + b_1 + c_1 \quad (A.8)$$

Differentiation of both sides of (A.8) leads to (A.9)

$$-\frac{1}{\rho^2} \frac{d\rho}{d\phi} = \frac{da}{d\phi} + \frac{db_1}{d\phi} + \frac{dc_1}{d\phi} \quad (A.9)$$

The followings are derived from definition.

$$\frac{da}{d\phi} = \cot \phi \cdot a + (3 \cos^2 l - 1) a^2 \quad (A.10)$$

$$\frac{db_1}{d\phi} = \frac{\cos^2 \theta \sin \phi}{2} b_1 + \frac{1}{\sin} b_1^2 \quad (A.11)$$

$$\frac{dc_1}{d\phi} = -(a + b_1) c_1 \quad (A.12)$$

Assigning (A.10), (A.11) and (A.12) into (A.9) results in (26).

Mr. Yoshio Tozawa received the degree of Bachelor and Master of Science from the University of Tokyo in 1974 and 1976 respectively. He joined IBM Japan as a researcher at Tokyo Scientific Center. His background and interest is computer science. He is now involved in a project of Remote Sensing and Image Processing.