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ESTIMATION OF A REMOTE SENSING SYSTEM POINT-SPREAD FUNCTION FROM MEASURED IMAGERY

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ABSTRACT

Satellite-based multispectral imaging systems have been in operation since 1972 and the latest in the Landsat series of sensors was launched in July 1982. One system parameter of interest is resolution and this paper discusses experiments to determine the actual overall resolution after launch. Atmospheric effects and postprocessing effects add to the pre-launch optical resolution. Scene structures, such as roads and field edges, were used with numerical estimation procedures to predict resolution in Landsat-4 Thematic Mapper imagery. A nominal resolution of 39 meters was determined as compared to the predicted 30 m prelaunch value.*

I. INTRODUCTION

In order to verify that a satellite-borne remote sensing system is operating within specifications, it is necessary to estimate the system parameters by analysis of the measured data. One parameter of particular interest is the sensor point-spread function (PSF) which determines the resolution of the system.

It is possible to obtain useful estimates of the PSF by analyzing data resulting from scanning ground elements having identifiable geometric and radiometric structures. The data are processed in such a way as to estimate the coefficients in a basis function representation of the PSF or in some cases to directly provide the PSF itself.

The measured data can be expressed in the spatial domain as a convolution of the scene with an overall point-spread function:

$$g(x,y) = h(x,y) * f(x,y)$$

where $f(x,y)$ is the earth scene

$h(x,y)$ is the overall point-spread function of the sensor system

$g(x,y)$ is the resulting image

Given $g(x,y)$, we wish to determine $h(x,y)$. To do this, some deterministic element of the input $f(x,y)$ must be known or assumed. Although the theory can take into account the two-dimensional nature of the element, the initial experiments have been limited to the one-dimensional case. If the overall PSF is separable, i.e., if $h(x,y)$ can be written as a product $h(x)h(y)$, then this approach provides a direct estimate of the two components. Otherwise it generates cross sections through the two-dimensional PSF along the x and y axes.

Three scene elements that would be useful for this type of analysis are:

1. An impulse represented by a narrow-width discontinuity along a row or column of the data.

2. A step function represented by an abrupt change in gray level along a row or column of the data.

3. A rectangular pulse represented by a sequence of two steps in opposite directions along a row or column of the data.

Use of an impulse type scene element is the simplest since the sensor response is directly proportionate to the PSF.

However, it is difficult to identify a discontinuity that is narrow enough that it may be considered to approximate an impulse. Some initial work was done using roads for this purpose. It was found, however, that because their width is not negligible, they consistently led to computed impulse response estimates that were too wide.

A general approach applicable to any type of (known) scene element has been developed. It is illustrated using scene elements of the second type listed above and gives results that appear to be in keeping with what would be expected based on the system specifications.

II. DATA SELECTION

There are a number of ways in which subsets of data from a scene can be selected for use in estimating parameters of the sensor. The one which is simplest to employ is to find a sequence of rows or columns in which a repetitive scene element of known geometrical configuration occurs. One example of this is a road that is bordered by constant reflectance materials on each side. It is not necessary that the reflectances be the same on each side, only that they be more or less constant. Another example is the border between fields containing different crops.

Because of the orbital inclination of Landsat and the propensity of man to arrange linear features, such as roads and field boundaries, in the cardinal compass directions, it is generally found that there is a spatial displacement in the scene coordinates of the linear elements from one row or column to the next. This has the desirable effect of providing a fine grid of samples of the system response when values from adjacent rows or columns are combined after correction for the spatial shift of the scene element. The procedure for combining the data is quite straightforward and can be illustrated as follows for a north-south road.

The coordinates of the peaks in the row data corresponding to the road are determined for a sequence of N rows. These data are then fitted with a least-squares straightline, providing an analytical expression for the road coordinates. The x-coordinates in each row are then modified by subtracting from them the least-squares estimate of the road location in that row. This converts the data to a coordinate system in which zero is the road center. Because of the small angular difference between the sensor coordinate system and the road direction,

the change in road coordinates from row to row is only a fraction of the pixel spacing and so represents a sampled response to the scene element corresponding to a subpixel translation. By combining the data from a number of rows, a set of finely sampled data is found. These data can be graduated using splines or other smoothing functions to give an average response function from which to estimate the system point-spread function.

III. ESTIMATION PROCEDURE

A straightforward estimate of the system point-spread function can be obtained if it is represented in terms of a finite sum of basis functions. The simplest approach is to employ a sequence of rectangular pulses extending over the spatial extent of the PSF. This will give a staircase approximation to the PSF; but if narrow impulses are employed, the steps will be small and the fidelity will be good. A smooth curve can be passed through the approximation desired. In order for this procedure to be practical, it is necessary to know or estimate geometrical structure of the scene element producing the measured response. In the case of a road, it is desirable to know the width of the road, the intensity level on each side of the road, and the intensity level of the road itself. For a field boundary, all that is required is to know the intensities on each side of the boundary. Since the scene elements are selected on the basis of regions of uniform intensity on each side of a discontinuity, these levels can be found directly from the data. This is all that is required for a step type of discontinuity, such as a field boundary. In the case of a road, the peak value obtained when the sensor is centered on the road can be used. However, this value will be somewhat low and should be increased by a correction factor that can be computed from an initial analysis or from analysis of the step-response data.

Mathematically, the analysis procedure can be carried out as follows: Consider the case of a separable point-spread function, i.e., one in which the total PSF can be expressed as the product of a PSF in the x-direction (rows) and a PSF in the y-direction (columns). Only the x-direction analysis will be described; however, the y-direction analysis is exactly analagous. Let $f(x)$ be the scene intensity as a function of the x-coordinate, let $h(x)$ be the system PSF, and let $g(x)$ be the system output. The output can be represented as the convolution of the scene and the PSF, i.e.,

$$g(x) = \int_{-\infty}^{\infty} f(\lambda)\, h(x-\lambda)\, d\lambda$$

Now let $h(x)$ be represented using a finite set of nonoverlapping rectangular basis functions. $h(x)$ is assumed to extend over an interval of $\pm\, P/2$ and the rectangular basis functions are assumed to have unit heights and widths of T units. Thus

$$\hat{h}(x) = \sum_i c_i \, \text{rect}(\frac{x-iT}{T}) \quad -P/2T \le i \le P/2T$$

$$\hat{g}(x) = \int_{-\infty}^{\infty} f(\lambda) \sum_i c_i \, \text{rect}(\frac{x-\lambda-iT}{T})\, d\lambda$$

$$= \sum_i c_i \, \phi_i\,(x)$$

where:

$$\phi(x) = \int_{-\infty}^{\infty} f(\lambda)\, \text{rect}(\frac{x-\lambda}{T})\, d\lambda$$

$$\phi_i(x) = \phi(x-iT)$$

The procedure is now to choose the unknown coefficients c_i so as to give the best approximation to $\hat{g}(x)$. This can be done by minimizing the mean square error between the known (measured) $g(x)$ and the estimated value over the region (x_1, x_2). The integral squared error is:

$$E^2 = \int_{x_1}^{x_2} [g(x) - \hat{g}\,(x)]^2 \, dx$$

$$= \int_{x_1}^{x_2} [g(x) - \sum_i c_i \, \phi_i(x)]^2 \, dx$$

The coefficients c_i are found by solving the system of equations that results from setting the partial derivatives with respect to each coefficient equal to zero. The resulting expression for the c_is is:

$$\underline{c} = \underline{R}^{-1}\, \underline{b}$$

where $\underline{c}$ is a vector with elements c_i, $\underline{R}$ is a symmetric matrix with elements

$$r_{ij} = r_{ji} = \int_{x_1}^{x_2} \phi_i(x)\, \phi_j(x)\, dx$$

and $\underline{b}$ is a vector with elements

$$b_i = \int_{x_1}^{x_2} g(x)\, \phi_i(x)\, dx$$

For a typical problem, the PSF extent might be approximated by 20 or more coefficients extending over a spatial extent of 5-6 pixels. The solution would require computing the elements r_{ij} which are samples of the autocorrelation function of the input image convolved with the rectangular basis function, computing the b_i which are the projections of the output on the ith basis function, and then multiplying the inverse of the $\underline{R}$ matrix by the $\underline{b}$ vector. Because of the smoothing that was done in generating the original $g(x)$ function, the solution will be well-behaved.

IV. EXPERIMENTAL RESULTS

The procedure that has been described is suitable for use with a variety of scene events that can be identified in Landsat imagery. The simplest case and the one that will be considered here is that of a boundary between two different intensity levels that extends in a north-south direction. This corresponds to an underlying functional form of a step discontinuity. Scene structures corresponding to field boundaries were selected from a rural area in Landsat-4 Thematic Mapper imagery of Webster County, Iowa** for use in the analysis. Data from consecutive rows on each side of the boundary were examined to be sure that no anomalous behavior was occurring. The least-squares straightline for the boundary representation was computed and the system response for consecutive rows adjusted to correspond to a boundary position on the x-direction of zero. Data for two different boundaries are shown in Figures 1 and 2. In Figure 1, fourteen scan lines are employed and in Figure 2, twelve scan lines.

The data of Figures 1 and 2 were smoothed using a cubic spline subroutine with five knots. The resulting response functions are shown in Figures 3 and 4 along with the assumed intensity level of the underlying scene. Data from the smoothed response functions were used to solve for the coefficients in the basis function representation of the PSF, as described previously. For this analysis, the PSF was assumed to be limited in extent to 9 sampling intervals and a total of 21 coefficients was calculated. The resulting PSFs are shown in Figures 5 and 6.

** The data used were from Band 4 (.76-.90 μm) of the Landsat-4 Thematic Mapper from Scene 40049-16264 obtained on Sept. 3, 1982 from the fully corrected P tape.

In interpreting the results of estimating the PSF of a sensor, it is convenient to use parameters that describe some of the general properties of the PSF. Two such parameters are the equivalent width, W_{eq}, and the half-amplitude width, $W_{\frac{1}{2}}$, of the main lobe. These are defined as:

1. Half amplitude width $W_{\frac{1}{2}}$ = width at which the magnitude of $h(x)$ falls to one-half of its value at the origin.

2. Equivalent width $W_e = \dfrac{\int_{-\infty}^{\infty} h(x)\, dx}{h_{max}}$

This is the width of a rectangle having the amplitude of $h(x)$ at its maximum and a width such that it has the same area as $h(x)$. For the two scene elements analyzed, these parameters are as follows:

Table 1. Results of PSF Width Estimation.

Parameter	Data Set 1	Data Set 2
W_{eq}	1.53	1.38
$W_{\frac{1}{2}}$	1.44	1.28

The units of W_{eq} and $W_{\frac{1}{2}}$ are sampling intervals. The sampling spacing for the Thematic Mapper imaging is 28.5 meters.

Because of the special nature of the underlying scene element that produced the data used in this analysis, there is an alternative way to estimate the point-spread function. The scene element can be modeled as a step superimposed on a constant background, i.e.,

$$f(x) = A + B\, u(x)$$

where A is the background and B is the amplitude change across the boundary. The output is then given by

$$g(x) = [A + B\, u(x)] * h(x)$$

Taking the derivative of both sides of this equation gives

$$g'(x) = \frac{d}{dx}(A + B\, u(x)) * h(x)$$

$$= B\, \delta(x) * h(x)$$

$$= B\, h(x)$$

$$\therefore\ h(x) = \frac{1}{B} g'(x)$$

The PSF is therefore the derivative of the output (measured) function scaled by the amplitude of the step. This quantity can be computed directly from the smoothed representations given in Figures 3 and 4. The results of this computation are shown in Figures 7 and 8 along with the coefficients previously determined. It is seen that there is excellent agreement between these figures and the results of the more general solution given in Figures 5 and 6.

All of the estimates suggest the existence of sidelobes on the PSF. This is evidenced by the overshoot seen in Figures 3 and 4 and in the resulting PSF estimates in Figures 5-8. More extensive analysis using different scene structures will be required to accurately quantify the nature and extent of the sidelobe structure.

V. SUMMARY

The problem of estimating the overall point-spread function of multispectreal scanner systems was studied using real scene data and known geometric structures in the scene. A direct solution to an approximate form of the PSF was made along with a method using the derivative of an estimated edge response. Both results agreed closely. The TM scanner system specifications are given in line-spread function width and these values are listed with the experimental results in terms of meters in Table 2. The estimated values are very reasonable, considering the number of factors which could be influencing the result. The atmosphere will have a blurring effect on the overall PSF as well as on cubic convolution resampling effects and possible electronic effects not accounted for in the specification. Also,

Table 2. Comparison of PSF Width Estimation Results on Thematic Mapper Specifications[2] Based on Altitude of 705.3 km and pixel spacing of 28.5 M.

Av. PSF Est. Width (Pixels)		Av. PSF Est. Width (Meters)		LSF Width μ Radians	LSF Width Meters
$W_{\frac{1}{2}}$	W_{eq}	$W_{\frac{1}{2}}$	W_{eq}		
1.36	1.45	38.8	41.3	43.2	30.5

the specific definition of the LSF speci-
fication is not known nor is the actual
altitude at the instant the data were
acquired. Thus the nominal overall PSF
half-amplitude width of 39 m is reason-
able; however, a greater sample of scene
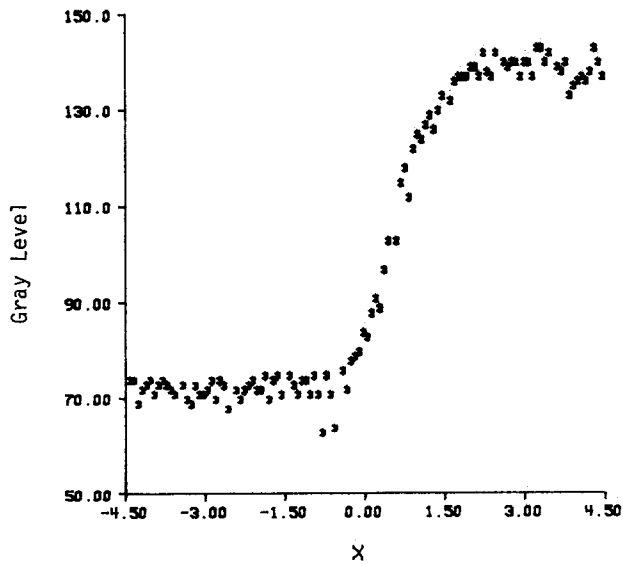objects should be evaluated to further
verify this result.
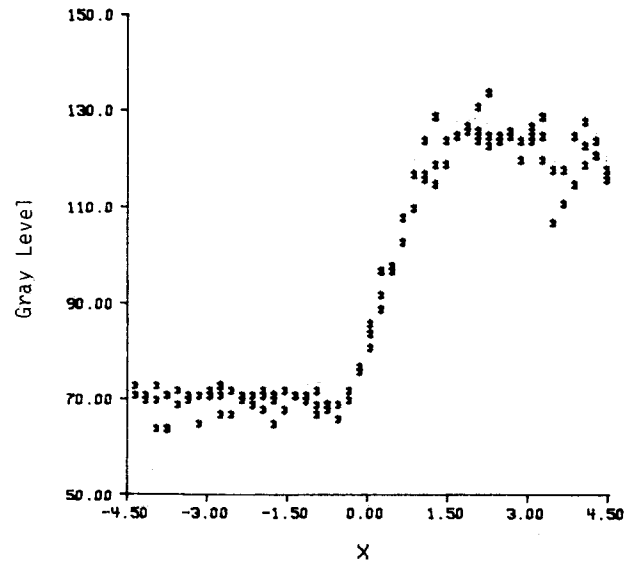

Fig. 1. Fourteen row responses from Reg. 1

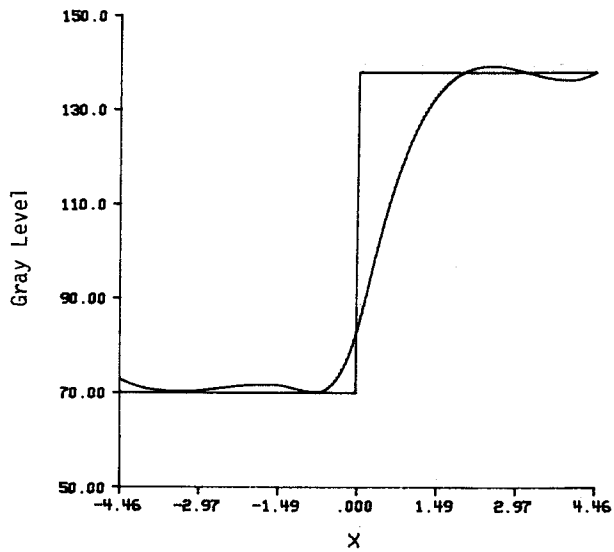Fig. 2. Twelve row responses from Reg. 2.

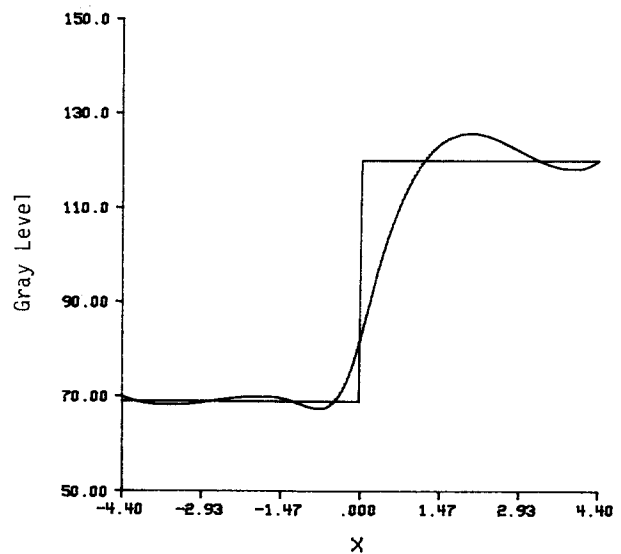Fig. 3. Smoothed estimate of the row re-
sponse from Region 1 and the underlying
scene intensity.

Fig. 4. Smoothed estimate of the row re-
sponse from Region 2 and the underlying
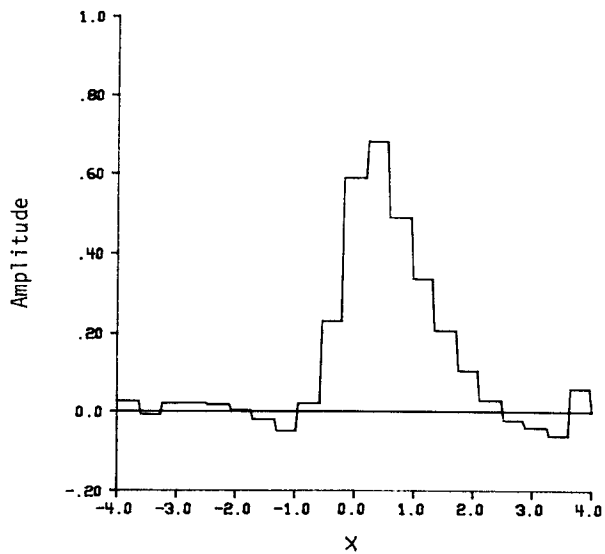scene intensity.

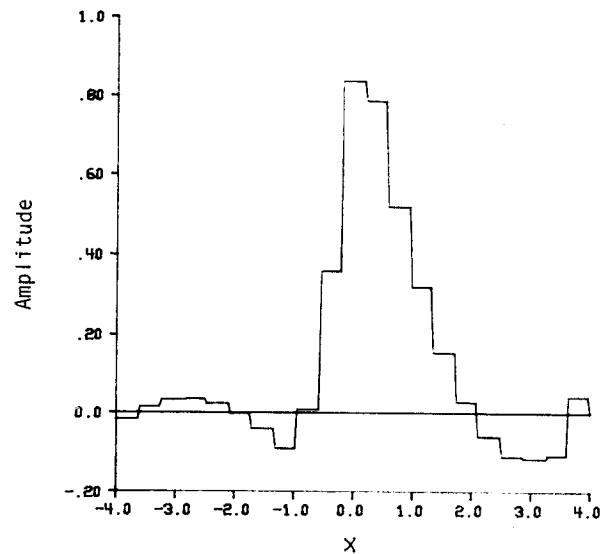Fig. 5. PSF estimate from smoothed data from Region 1.

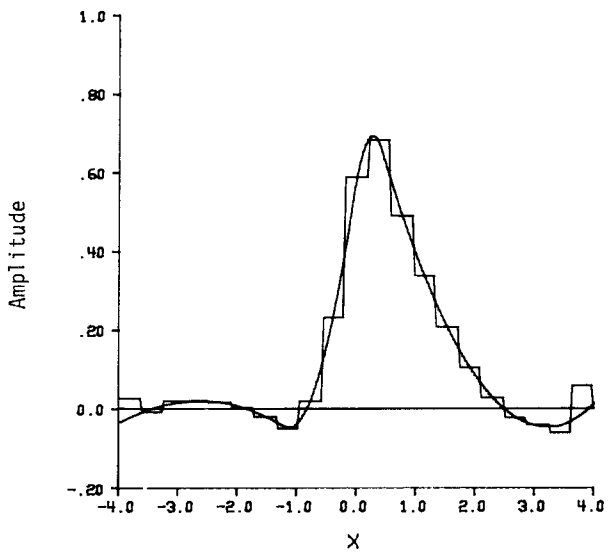Fig. 6. PSF estimate from smoothed data from Region 2.

Fig. 7. PSF estimate as derivative of step response based on data from Reg. 1 superimposed on coefficient estimates given in Figure 5.
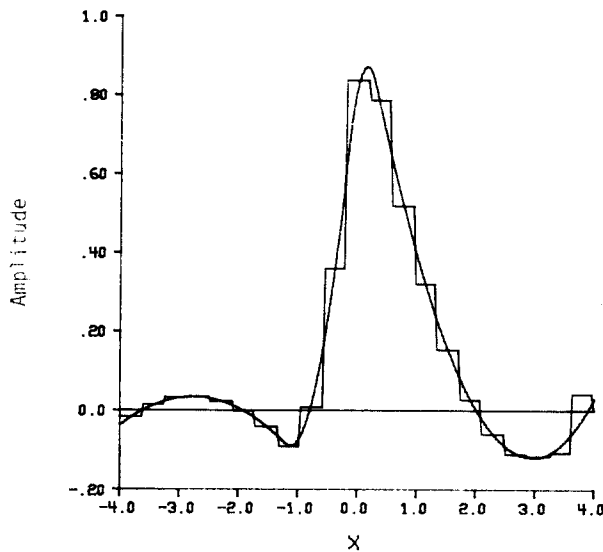
Fig. 8. PSF estimate as derivative of step response based on data from Reg. 2 superimposed on coefficient estimates given in Figure 6.

VI. REFERENCES

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