Reprinted from

Ninth International Symposium

Machine Processing of

Remotely Sensed Data

with special emphasis on

Natural Resources Evaluation

June 21-23, 1983

Proceedings

A MATHEMATICAL MODEL FOR CROP SPECTRAL-TEMPORAL TRAJECTORIES BASED ON A PLANT GROWTH MODEL

T.L. WOOLFORD

Lockheed Engineering and Management
Services Company, Inc.
Houston, Texas

I. ABSTRACT

The Kubelka-Munk radiative transfer model is combined with an approximation of Kauth-Thomas greenness and brightness transforms to derive approximate closed form expressions for crop greenness and brightness surrogates in terms of canopy biomass. The greenness relation derived resembles an existing empirical relation between leaf area index and greenness. A simple growth model based on interception and utilization of photosynthetically active radiation is developed and used to describe the time evolution of greenness and brightness. The model developed does not yet yield definitive profile calculations but suggests a conceptual framework which may be found useful for further profile analysis.

II. INTRODUCTION

Functions[1,2] approximating the Kauth-Thomas (K-T) greenness[3] vs. time trajectories of crops are useful in processing[5] Landsat Multispectral Scanner (MSS) data for crop identification and for simulation of agricultural scenes.[6]

Greenness vs. time trajectories have been extensively studied and a useful profile form has been developed by Badhwar[2] and by Crist and Malila.[7] This form is empirical and there is no comparable relation for brightness vs. time.

The goals of this research are: a) to show a link between the theory of radiative interactions and the time behavior of greenness and brightness measures; and b) to derive the temporal behavior of brightness over a crop's growing season.

A model or theory accomplishing these goals must contain three components: 1) a model of radiation interaction relating canopy material physical properties to canopy reflectances; 2) an analysis relating K-T greenness and brightness (or surrogates) to variables and parameters occurring in the radiation interaction model; and 3) a plant growth model relating the time evolution of variables in the radiation interaction model to parameters which characterize a plant, its environment, and its growth.

In this effort it is convenient to use biomass as the primary measure of vegetation quantity.

III. BACKGROUND

There is[1] a strong correlation between K-T greenness and vegetation quantity. Daughtry et al.[8] and Badhwar[9] have used a relation of the form

$$\mathcal{G} = \overline{\mathcal{G}}_{\infty}(1-e^{-k \cdot \text{LAI}}) \tag{1}$$

where $\overline{\mathcal{G}}_{\infty}$ and k are empirically determined to be at least approximately constant to relate greenness $(\mathcal{G})$ to leaf area index (LAI).

Badhwar[1,2] and others have used a closed form expression

$$\mathcal{G}(t) = \mathcal{G}_{o}(t/t_{o})^{b} e^{-c(t-t_{o})^{2}} \qquad t \geq t_{o} \tag{2}$$
$$= \mathcal{G}_{o} \qquad t < t_{o}$$

where t_{o} is emergence time and $\mathcal{G}_{o}$ is soil greenness to estimate emergence times[2,3] and growth stages for crops. The constants b and c are primarily determined by crop species. The inflection points and maximum correlate with growth stage. The time between inflection points is correlated with crop growing season length. These observations suggest that greenness profiles have a well defined crop specific form highly correlated with plant growth.

There have been many studies[12-15] of the spectral properties of canopies and leaves in both the visible and infrared (IR). These studies show that leaf and canopy reflectances are causally related to chlorophyll content and to the number and size of interfaces between materials of different indices of refraction in the plant tissue.

As plants mature changes occur in the spectral properties of the plant components which are related to changes in histology and chlorophyll content. The relation between canopy reflectance and leaf reflectance is determined by the radiative interactions among the canopy components and is affected by areal biomass density or LAI and soil background reflectance.

Chlorophyll content and biomass density are factors which affect both plant growth and canopy

spectral response. It is thus reasonable to be-lieve that the link between greenness profiles and plant phenology is not fortuitous but a reflection of conditions and processes which indicate onto-genetic stage, affect further plant growth, and are directly instrumental in determining spectral response.

IV. APPROACH

The work reported in this paper consists of three parts: 1) analysis of canopy reflectance us-ing simple (Kubelka-Munk) radiative transfer theory to relate spectral response to the quantity and type of vegetation present, to chlorophyll content, and to parameters which are related to histology and are indicative of a plant's age and health; 2) analysis of the K-T greenness and brightness expressions to obtain simpler two wave-length band reflectance surrogates; and 3) formu-lation of a plant growth model describing the time evolution of the biomass components corres-ponding to definite spectral components of the canopy. Calculation of the greenness and bright-ness profiles to be expected for a typical, albeit hypothetical, crop is then straightforward.

Appropriate greenness and brightness surrogates are found to be a linear combination of reflect-ance of the red chlorophyll absorption spectral region and of a band in the IR. The reflectances in these expressions are evaluated using Kubelka-Munk (KM) theory and greenness and brightness are obtained as explicit functions of biomass, canopy infinite depth reflectances (IDRs), and extinction coefficients at the two wavelengths. The IDRs and extinction coefficients are related to the ab-sorption and scattering coefficients for green (chlorophyll containing) and nongreen material.

A model which consists of three differential equations is developed for the time evolution of live green, live nongreen, and senescent or dead biomass. The model is based on interception of photosynthetically active radiation (PAR) by the canopy, transformation of the radiation energy to chemical energy, and partitioning of the photo-synthate among the three biomass components. It includes the effect of plant respiration and senescence.

V. THE RADIATION INTERACTION MODEL

Kubelka-Munk (KM) theory is the simplest radi-ative transfer model relating canopy reflectance and transmission to biomass density, canopy compo-sition, and component optical parameters. It is a bulk scattering theory which assumes that the canopy can be adequately idealized as a laterally uniform collection of randomly oriented scattering and absorbing elements. The one layer model used here also assumes vertical uniformity. The model also assumes that incident radiation is diffuse. For strongly ordered canopies and direct illumina-tion these assumptions may be inadequate but it is

believed that this idealization is conceptually useful.

It is reasonable to believe that for a speci-fic plant species under fixed cultural and envir-onmental conditions green LAI and green biomass are proportional. It is also likely that at a given growth stage there will be a strong linear correlation between green biomass and total biomass. For further model development it is desirable to use biomass as the primary measure of plant compo-nent density.

The plant canopy is assumed to be a homogen-eous mixture of randomly oriented components having different scattering and absorption proper-ties. The total radiative scattering and absorp-tion coefficients are assumed to be linear func-tions of the amounts of different materials present in the canopy. The component absorption coeffic-ients are functions of pigmentation and the number of refractive index interfaces present.

Different interaction mechanisms are prominent in different spectral regions—chlorophyll absorp-tion in the visible region of the spectrum and scattering (because of the lack of absorption) in the near IR.

A. KUBELKA-MUNK THEORY

In this approximation the reflectance R_λ is

$$R_\lambda = R_{o,\lambda} + \frac{(R_{\infty,\lambda} - R_{o,\lambda})(1 - R_{o,\lambda} R_{\infty,\lambda})(1 - e^{-2k_\lambda m})}{(1 - R_{o,\lambda} R_{\infty,\lambda}) - (R_{\infty,\lambda} - R_{o,\lambda}) R_{\infty,\lambda} e^{-2k_\lambda m}} \quad (3)$$

where m is the biomass density, R_o soil background reflectance,

$$R_{\infty,\lambda} = 1 + \frac{\alpha_\lambda}{\sigma_\lambda} \left[1 - \sqrt{1 + \frac{2\sigma_\lambda}{\alpha_\lambda}} \right] \quad (4)$$

and

$$k_\lambda = \left[1 + \frac{\alpha_\lambda}{\sigma_\lambda} + \left[\frac{\alpha_\lambda}{\sigma_\lambda} \right]^2 \right]^{\frac{1}{2}} \quad (5)$$

The quantity $R_{\infty,\lambda}$ is the IDR of the canopy. The subscript λ designates a specific small wavelength interval $(\lambda, \lambda + \Delta\lambda)$ and α_λ and σ_λ are the mass absor-ption and scattering coefficients.

B. THE LINEARIZED KUBELKA-MUNK EQUATION

Simpler expressions for reflectance and trans-mission can be obtained since $0 < R_{\infty,\lambda} < 1$ and $0 < R_{o,\lambda} < 1$. Equation (3) is then approximately

$$R_\lambda = R_{o,\lambda} + (R_{\infty,\lambda} - R_{o,\lambda})(1 - e^{-2k_\lambda m}) \quad (6)$$

Park and Deering have found this approximation to agree with the KM equation to within 0.2% (red reflectivity) and LeMaster and Chance[21] have ob-tained the same equation as an approximation to the one layer Suits[22] model.

The approximation of transmittance is

$$T = e^{-k_\lambda m} \quad (7)$$

which shows that k_λ is the extinction coefficient for the canopy.

Reflectance at any wavelength depends on the canopy composition through its effect on k_λ and $R_{\infty,\lambda}$ or equivalently α_λ and σ_λ.

C. THE CANOPY AS A MIXTURE OF MATERIALS

For a homogeneous mixture of materials with absorption coefficients α_{λ_i} and scattering coefficients σ_{λ_i} the total absorption and scattering coefficients are

$$\alpha_\lambda = \frac{1}{m_T} \sum_{i=1}^{n} \alpha_{\lambda_i} m_i \qquad (8a)$$

$$= \frac{1}{m_T} \sum_{i=1}^{n} \alpha_{\lambda_i} m_i \qquad (8b)$$

$$m_T = \sum_{i=1}^{n} m_i \qquad (8c)$$

where n is the number of components, m_i is the mass density of the ith component and m_T the total mass density of the composite material.

In a two component system where m_1 is the green biomass and m_2 is the non-chlorophyll containing biomass we get from (8)

$$\alpha_\lambda = (1 + \mu_\lambda \chi)\alpha_{\lambda_1}/(1+\chi) \qquad (9a)$$

$$\sigma_\lambda = (1 + \xi_\lambda \chi)\sigma_{\lambda_1}/(1+\chi) \qquad (9b)$$

$$m_T = m_1 + m_2 \qquad (9c)$$

where

$$\mu_\lambda = \frac{\alpha_{\lambda_2}}{\alpha_{\lambda_1}}; \qquad \xi_\lambda = \frac{\sigma_{\lambda_2}}{\sigma_{\lambda_1}}; \qquad \chi = \frac{m_1}{m_2}$$

and equations (4) and (5) become

$$R_{\infty,\lambda} = 1 - \frac{(1+\chi\mu)}{(1+\chi\xi)}(\alpha_{\lambda_1}/\sigma_{\lambda_1}) \left[\sqrt{1 + \frac{2(1+\chi\xi)\ \sigma_{\lambda_1}}{(1+\chi\mu)\ \alpha_{\lambda_1}}} - 1 \right] \qquad (10)$$

$$k_\lambda = \left[1 + \frac{(1+\chi\mu)}{(1+\chi\xi)}(\alpha_{\lambda_1}/\sigma_{\lambda_1}) + \frac{(1+\chi\mu)^2}{(1+\chi\xi)^2}\left(\frac{\alpha_{\lambda_1}}{\sigma_{\lambda_1}}\right)^2 \right]^{\frac{1}{2}}$$

$$\times \frac{(1+\xi\chi)}{(1+\chi)}\sigma_{\lambda_1} \qquad (11)$$

Equations (10) and (11) can be expressed in terms of the IDRs and extinction coefficients for the pure components. The resulting form is more convenient for some calculations. Define $_1R_{\infty,\lambda}$ to be the IDR for the green biomass component; $_2R_{\infty,\lambda}$ the IDR for nongreen biomass, $_1k_{\infty,\lambda}$ the extinction coefficient for green biomass, and $_2k_{\infty,\lambda}$ the extinction coefficent of the nongreen biomass. The the IDR for the composite canopy is

$$R_{\infty,\lambda} = (1+F) - \sqrt{(1+F)^2 - 1} \qquad (12)$$

where F is the so-called Kubelka-Munk function[16]:

$$F = \frac{1}{(1+\chi\xi_\lambda)}\left[\frac{(1-_1R_{\infty,\lambda})^2}{2_1R_{\infty,\lambda}} + \chi\xi_\lambda \frac{(1-_2R_{\infty,\lambda})^2}{2_2R_{\infty,\lambda}} \right]$$

The composite extinction coefficient is

$$K_\lambda = \frac{1}{(1+\chi)}\sqrt{(1+\theta)(_1k^2_{\infty,\lambda}) + \chi^2(_2k^2_{\infty,\lambda})} \qquad (13)$$

where

$$\theta_\lambda = \chi\left[2\xi_\lambda + (\xi_\lambda + \mu_\lambda)\frac{\alpha_{\lambda_1}}{\sigma_{\lambda_1}} + 2\mu_\lambda \frac{\alpha^2_{\lambda_1}}{\sigma^2_{\lambda_1}}\right]/\left[1 + \frac{\alpha_{\lambda_1}}{\sigma_{\lambda_1}} + \frac{\alpha^2_{\lambda_1}}{\sigma^2_{\lambda_1}}\right]$$

VI. GREENESS AND BRIGHTNESS SURROGATES

Kauth-Thomas greeness is a linear combination of radiances in which the coefficients of two visible channel radiances are negative while the coefficients of two IR channels are positive. Brightness is a linear combination of channel radiances in which all coefficients are positive. The behavior of MSS K-T greeness and brightness expressions are similar to combinations of one red and one IR reflectance.

Expressions for the greeness and brightness surrogates are obtained by combining their definition with the KM solutions. The expression for greeness in terms of biomass that is obtained resembles the empirically determined greeness LAI relation (1) but is more complicated. The most important aspect of this derivation is that a relation for brightness is obtained from a parallel derivation.

A. SURROGATE DEFINITION

The contribution of the two visible bands can be approximated by a single reflectance R_1. The Landsat MSS Ch 2 reflectance is chosen for this purpose. Contributions of the two IR channels can be approximated by a single reflectance R_2. Greeness and brightness surrogates are then defined by

$$G = -g_1 R_1 + g_2 R_2; \qquad g_1, g_2 > 0 \qquad (14a)$$

$$B = b_1 R_1 + b_2 R_2; \qquad b_1, b_2 > 0 \qquad (14b)$$

B. SURROGATE DEPENDENCE ON BIOMASS

The greeness and brightness surrogates are related to soil reflectance, canopy extinction coefficient, and canopy IDR by the KM reflectance relations. Combining equations (14) with the linearized KM relations yields

$$G - G_0 = (\tilde{G}_\infty - G_0)(1 - e^{-2k_1 m}) \qquad (15a)$$

$$B - B_0 = (\tilde{B}_\infty - B_0)(1 - e^{-2k_1 m}) \qquad (15b)$$

where $G_0 = -g_1 R_{o,1} + g_2 R_{o,2} \qquad (15c)$

$$B_0 = b_1 R_{o,1} + b_2 R_{o,2} \qquad (15d)$$

$$\tilde{G}_\infty = G_\infty - g_2(R_{\infty,2} - R_{o,2})\left(\frac{e^{-2k_2 m} - e^{-2k_1 m}}{1 - e^{-2k_1 m}}\right) \qquad (15e)$$

$$\tilde{B}_\infty = B_\infty - b_2(R_{\infty,2} - R_{o,2})\left(\frac{e^{-2k_2 m} - e^{-2k_1 m}}{1 - e^{-2k_1 m}}\right) \qquad (15f)$$

$$G_\infty = -g_1 R_{\infty,1} + g_2 R_{\infty,2} \qquad (15g)$$

$$B_\infty = b_1 R_{\infty,1} + b_2 R_{\infty,2} \qquad (15h)$$

and $R_{\infty,1}$, $R_{\infty,2}$, $R_{o,1}$ and $R_{o,2}$ are the red and IR

canopy IDRs and red and IR soil reflectances while k_1 and k_2 are the red and IR extinction coefficients. Although the greeness and brightness surrogates have the same form their behavior may be significantly different because the second term of equations (15e) and (15f) will be a larger fraction of the total infinite canopy depth attribute in the case of brightness. Brightness will therefore be much more sensitive than greeness to changes in k_1, k_2 and biomass. The difference between canopy infinite depth IR reflectance and soil IR reflectance will also have a greater effect on brightness profiles.

If we choose g_1 and g_2 so that the soil greeness is zero, (15a) can be written

$$G = \tilde{G}_\infty(1-e^{-2k_1 m})$$

If LAI is proportional to biomass density (LAI $= \ell \cdot m$) $\quad G = \tilde{G}_\infty(1-e^{-\frac{2k_1}{\ell} LAI})$

which is equivalent to the empirical equation for greeness (1) after proper identification of $\tilde{G}_\infty$ and $\frac{-2k_1}{\ell}$.

The derivation leading to this equation for greeness has also led to an equation for brightness.

VII. THE GROWTH MODEL

Two links between the production of biomass and spectral interactions are exploited to provide a growth model: 1) the role of chlorophyll absorption both as a driver of plant growth and as the determinant of visible reflectance and 2) the relation between interception of radiation and reflectance. The absorption in the red chlorophyll band is taken to represent PAR absorption since both red and blue absorption depend primarily on chlorophyll concentration and will be approximately proportional.

The fraction of visible (PAR) radiation not transmitted through the canopy is $1-e^{-k_1 M_1}$, where $M_1 = m_1$, the dry biomass of chlorophyll containing material. For canopies composed primarily of green leaves and stems, nearly all the intercepted PAR radiation will be absorbed by chlorophyll containing material and hence available to drive growth.

In this section an integral equation is proposed for total biomass growth under these conditions. It is a generalization of the equation used by Gallo, et al.[17] but differs from it by using green biomass rather than total biomass and by taking metabolic use of intercepted solar energy (respiration) into account.

The spectral profile equations require chlorophyll and nonchlorophyll containing component biomasses as inputs. To obtain these quantities the total biomass growth is assumed to result from the growth of live chlorophyll containing biomass

M_1, live non-chlorophyll containing biomass M_2, and senescent, chlorotic, or dead plant material M_3.

Equations for the three biomass components are formulated from the assumption that the PAR energy intercepted by green biomass and converted to chemical form by photosynthesis first provides for the respiratory requirement of the green biomass and that the residual photosynthate is distributed among green biomass production, and live nongreen biomass production and respiration.

Total live biomass accumulation is decreased by death and senescence processes which are assumed to be time dependent because of the effect of plant aging.

Live biomass which is not above ground (i.e., roots) is not explicitly considered in this model but may be taken into account by assuming that only a fraction of the live nongreen biomass affects reflectance.

A. MATHEMATICAL EXPRESSION OF GROWTH

The proposed equation for biomass production is

$$M_T = \int_{t_i}^{t_f} (\eta S P - \beta_1 M_1 - \beta_2 M_2)\, dt \qquad (16)$$

where M_T is the total biomass produced over the time interval (t_i, t_f), S is PAR insolation at the top of the plant canopy, η is the efficiency of conversion of energy into dry matter, P is the proportion of incident radiation intercepted by green biomass, β_1 is the respiration rate of green biomass, and β_2 is the respiration rate of live nongreen biomass.

In analogy with Gallo et al. P is

$$P = 1-e^{-k_1 M_1} \qquad (17)$$

Combining the two equations we obtain

$$M_T = \int [\eta S(1-e^{-k_1 M_1})-\beta_1 M_1-\beta_2 M_2]\, dt \qquad (18)$$

and the equivalent differential equation

$$\frac{dM_T}{dt} = \eta S(1-e^{-k_1 M_1})-\beta_1 M_1 -\beta_2 M_2 \qquad (19)$$

This equation is now split into three equations:

$$\frac{dM_1}{dt} = \Phi[\eta S(1-e^{-k_1 M_1})-\beta_1 M_1]-\gamma_1 M_1 \qquad (20)$$

$$\frac{dM_2}{dt} = (1-\Phi)[\eta S(1-e^{-k_1 M_1})-\beta_1 M_1] -\beta_2 M_2 -\gamma_2 M_2 \qquad (21)$$

$$\frac{dM_3}{dt} = \gamma_1 M_1 + \gamma_2 M_2 \qquad (22)$$

where Φ is the fraction of photosynthate used for the production of new green biomass, γ_1 is the death rate of green biomass components, and γ_2 is the death rate of nongreen biomass. Note that m_2 is

$$m_2 = M_2 + M_3 \qquad (23)$$

In general, Φ, η, S, and perhaps β_1 and β_2 as well as γ_1 and γ_2 will be functions of time because of environmental changes or plant physiological changes related to ontogenetic stage.

If the differential equations are solve numerically the time variation of these quantities can be taken into account, but in this paper calculations are made with all the quantities except γ_1 and γ_2 assumed constant.

VIII. NUMERICAL CALCULATIONS

In this section a greeness and brightness profile will be calculated for a hypothetical green plant canopy.

In this example the plant "death" rate in equations (20-22) is taken to be of the form

$$\gamma_1 = \gamma_{11} + \gamma_{12} t$$
$$\gamma_2 = \gamma_{21} + \gamma_{22} t$$

where γ_{11}, γ_{12}, γ_{21}, and γ_{22} are constants for a substantial portion of the plant growth period. This form is the simplest which embodies the idea of a plant component death rate which is proportional to the age of the live components.

The basic definition of "dead" biomass is biomass which is not metabolically active. In this example the carbohydrate stored in grain is considered to be "dead" as is the truly dead biomass derived from senescent leaves and stems.

The quantity γ_{12} was not held constant in this example but varied to simulate the effect of changes in physiology with ontogenetic stage. Similarly the parameter γ_{22} was given two different values corresponding to vegetative growth and storage of carbohydrate in grain. The value of Φ was a constant ($\Phi = .4$) and was not varied from one growth stage to another though appropriate variation would probably lead to more realistic biomass vs. time curves.

Equations (20-22) were put into nondimensional form before integration to remove the need for absolute values of parameters and initial biomass. Biomass is nondimensionalized by dividing by $M^* = \xi S \cdot (1 \text{ day})$ so

$$x_1 = M_1/M^* \; ; \quad x_2 = M_2/M^* \; ; \quad x_3 = M_3/M^*,$$

while nondimensional time is $\tilde{t} = \beta_1 t$.

Green material biomass and the PAR extinction are related through light interception. Typically light is reduced to about 95% of its incident value at a LAI of about 5.0 with normal crop planting densities. (This is only meant to be a very rough estimate.) A value for k_1 of .599 where the units of k_1 must be $(LAI)^{-1}$ is compatible with this assumption. The "units" of mass must then be the quantity of green leaves required to produce an LAI of 1 for a one meter^2 area of soil. The object of this section is to display relative changes in biomass, reflectance, greeness and

brightness rather than magnitudes and since the quantities entering into reflectance calculations depend either on ratios of biomass or on the product of biomass and extinction coefficient, no difficulty is encountered with these "units". The relation between LAI, green biomass, and PAR attenuation establishes a value of σ_{λ_1} corresponding to these units if it is assumed that in the PAR and red absorption region $\alpha_{11}/\sigma_{11} \approx 4.0$ and $\xi_1 \approx 1.0$ where the subscript value $\lambda = 1$ corresponds to the red band or PAR.

A rough estimate of β_1 was made by comparing the slope of the total biomass equations with the slope of a logistic curve which would saturate at a biomass value of 5×10^3 times the biomass assumed present at t=1. Time from t=1 to inflection was 50 days. The value obtained was $\beta = .0735$ day^{-1}. This forces the units of ξS. In this example a value of $\xi S/\beta_1 = 5.965$ was chosen to be compatible with attainment of a maximum LAI of 5.0 if death rate terms in the growth equations were negligible. In addition to these quantities, β_2, γ_{11}, γ_{12}, γ_{21}, and γ_{22} must be specified. These are specified by their ratios to β_1, the green biomass respiration rate. The values used are:

$$\beta_2/\beta_1 = 1 \; ; \quad \gamma_{11}/\beta_1 = .00012; \quad \gamma_{21}/\beta_1 = .000012.$$

A plot of the nondimensional green, nongreen, and dead biomass is given in Figure 1. In this example the initial values

$$\gamma_{12}/\beta_1 = .004 \; ; \quad \gamma_{22}/\beta_1 = .000033$$

were used. γ_{12}/β_1 was changed to .004 with $\gamma_{22}/\beta_1 = .0031$ at $\tilde{t} = 38.7$. This represents a change from the vegetative growth to grain production phases of the growth cycle. At $\tilde{t} = 52.15$, γ_{12}/β_1 was set to .016. This corresponds to the rapid decrease of green material and LAI seen at the end of a growth season. It should be noted that neither the time scale nor the absolute value of the masses in Figure 1 are meaningful in themselves. It is the shape of the curve and the relative masses that are important. However, if we multiply all the biomasses by 2.55 to bring it to our originally estimated LAI of 5, we change the ratio $\xi S/\beta_1$ which will affect the shape of the curve. But if we also multiply β_1 by 2.55 and preserve the ratio we will change the value of k_1/β_1 which will change the shape of the curves unless we correspondingly modify k_1--the extinction coefficient. Further in our example the computed length of the growing season is much longer (by a factor of perhaps 6) than we initially assumed. Thus to adjust our curve to the assumed data we should have to increase β_1 and $\xi S/\beta_1$ while keeping the original value for k_1 and iterate on the other parameters until we again find acceptable biomass curves. If we continued this iterative procedure until a good fit for all observable quantities was obtained for an actual crop we would obtain new estimates of all the parameters of the differential equations. Experience indicates that it would also be necessary to develop a time varying value of Φ to achieve good results. Carrying out the parameter estimation procedure outlined above is beyond the scope of this paper but is mentioned to

indicate the steps which must be taken to apply the growth model and profile generation technique to a real crop. For the example of this section it should merely be noted that green LAI of only about 2 is reached by the growth model and the greeness of the hypothetical crop is therefore expected to be low. The build up of nongreen biomass in the simulated canopy is very large leading to negative greeness at the end of the growth period.

Equations (15) were used to generate the greeness and brightness surrogate vs. time curves in Figure 2. The values

$$g_1 = 1. \quad ; \quad g_2 = 1. \; ; \quad b_1 = 1. \; ; \quad b_2 = 1.$$

$$R_{o,1} = .20, \; ; \quad R_{o,2} = .40$$

were used and equations (12) and (13) were used to calculate the required IDRs and extinction coefficients. It is noted that despite the formal similarity of the greeness and brightness equations (15a and b) the curves resulting from numerical evaluation are quite different for greeness than for brightness. This difference in behavior results from the different effects of changing nongreen to green biomass ratios on $\tilde{G}_\infty$, $\tilde{B}_\infty$, G_∞, B_∞, k_1 and k_2. In Figures 1 and 2 it is noted that the greeness curve closely follows the green biomass curve whereas the brightness curve follows the sum of live and dead nongreen biomass.

The values of $\mu = (x_2 + x_3)/x_1$ and $m = x_1 + x_2 + x_3$ were obtained from the plant growth calculations previously discussed. The values of other parameters in (12) and (13) were obtained as described below.

The assumed value of α_{11}/σ_{11} corresponds to $_1R_{\infty,1} = .101$. It is assumed that $\xi_2 \approx 1$ and that the scattering coefficient in the visible is essentially the same as for the IR ($\sigma_{12} \approx \sigma_{22}$) so that $\sigma_{11} \approx \sigma_{22}$ (all scattering coefficients have approximately the same value). It is also assumed that $\alpha_{12} \approx 1.2\alpha_{22}$, that is, the visible absorption coefficient for nongreen material is slightly larger than the IR absorption for nongreen material. Also $\alpha_{21} \approx \alpha_{22}$ so the infrared absorption coefficient for green and nongreen material is about the same. The IR composite IDR for green and nongreen material is about .20 which leads to $\alpha_{21}/\sigma_{21} \approx \alpha_{22}/\sigma_{22} \approx .06$ and $\alpha_{12}/\sigma_{12} \approx .072$ and $\mu_2 = \alpha_{22}/\alpha_{21} \approx .06$ with $_1R_{\infty,2} = _2R_{\infty,2} = .70$. Then from equation (10) $_2R_{\infty,1} = .6857$. Using the assumed composite extinction coefficient $k_1 = .599$, the ratio $\chi = (x_2 + x_3)/x_1$ and equations (11) and (13), $_1k_{\infty,1}$ is found to be 1.1129 while $_2k_{\infty,1} = .25205$. Then $_1k_{\infty,2} = .25045 = _2k_{\infty,2}$.

IX. CONCLUSION

The main conclusion of this paper is that spectral temporal profiles can be understood in terms of a causal chain extending from the microscopic histological and biochemical level to large scale field spectral measurements. Using simple radiation interaction theory it is possible to relate greeness and brightness changes to changes in canopy biomass density and composition resulting from growth. Brightness shows a more complex behavior than greeness because of the effect of soil reflectance but is conceptually closely connected. The derived greeness surrogate expression is closely related to the empirical greeness function (1) whenever LAI is proportional to biomass. The quantity corresponding to $\tilde{G}_\infty$ is a weak function of time because of its dependence on biomass. This complication is an unavoidable consequence of the fact that greeness is a combination of the radiances at two wavelengts.

The ability to relate spectral temporal behavior to biophysical processes should facilitate application of multispectral analysis techniques to new situations and research problems. The work presented in this paper is not meant to be a substitute for proven profile techniques, but a conceptual tool to assist in understanding certain aspects of the physical and physiological basis of multitemporal behavior.

X. REFERENCES

1. Badhwar, G.D., 1982. Profile modeling for crop discrimination, Proc. Symp. Machine Processing of Remotely Sensed Data, Purdue University, West Lafayette, IN, 454-460.

2. Badhwar, G.D., 1980. Crop emergence date determination from spectral data. Photogram. Eng. Remote Sens. 46:369-377.

3. Badhwar, G.D. and K.E. Henderson, 1981. Estimating development stages of corn from spectral data--an initial model. Agron J. 73:748-755.

4. Kauth, R.J. and G.S. Thomas, 1976. The tasselled cap, a graphic description of the spectral temporal development of agricultural crops as seen by Landsat, Proc. Symp. Machine Processing of Remotely Sensed Data, Purdue University, West Lafayette, In. 4B-42.

5. Badhwar, G.D., J.G. Carnes, and W.W. Austin, 1982. Use of Landsat derived temporal profiles for corn-soybean feature extraction and classification. Remote Sensing of Environment, 12:57-59.

6. Badhwar, G.D., K.E. Henderson, D.E. Pitts, W.R. Johnson, M.L.Sestak, T.L. Woolford, and J.G. Carnes 1982. A comparison of simulated thematic mapper data and multispectral scanner data for Kingsbury County, South Dakota, Proc. Symp. Machine Processing of Remotely Sensed Data, Purdue University, West Lafayette, IN, 140-143.

7. Crist, E.P. and W.A. Malila, 1980. Temporal-spectral analysis techniques for vegetation applications of Landsat, Proc. 14th International Symp. Remote Sensing of the Environment, San Jose, Costa Rica. April 23-30, 1980. 2:1031-1040.

8. Daughtry, C.S.T., D.J. Knowlton, V.C. Vanderbilt, and M.E. Bauer, 1983. Spectral estimation of leaf area index, unpublished presentation at the JSC Quarterly Interchange Meeting, March 1-3, 1983, NASA Johnson Space Center, Houston, TX.

9. Badhwar, G.D., 1982. Unpublished results presented at the June 16, 1982 meeting of Fundamental Research Group at Goddard Space Flight Center, Greenbelt, Md.

10. Henderson, K.E. and G.D. Badhwar, 1982. An initial model for estimating soybean development stages from spectral data. Proc. Symp. on Machine Processing of Remotely Sensed Data, Purdue University, Lafayette, In. 199-204.

11. Badhwar, G.D., 1982. Extraction of temporal-spectral features for crop identification. JSC QTI, Jan. 19, 1982.

12. Bauer, M.E. and C.S.T. Daughtry, 1982. Spectral-agronomic relationships of corn, soybean, and what canopies, NASA JSC Quarterly Interchange, June 1982, Houston, Tx.

13. Gates, D.M., 1980. Biophysical Ecology, Springer-Verlag, New York, N.Y.

14. Knipling, E.G., 1970. Physical and physiological basis for the reflectance of visible and near-infrared radiation from vegetation. Remote Sens. Environ. 1:155-159.

15. Wiegand, C.L., H.W. Gausman, and W.A. Allen, 1972. Physiological factors and optical parameters as bases of vegetation discrimination and stress analysis, Proc. Operational Remote Sensing Seminar, Amer. Soc. of Photogram.

16. Kortüm, G., 1969. Reflectance Spectroscopy, Springer-Verlag, New York, N.Y.

17. Gallo, K.P., C.C. Brooks, C.S.T. Daughtry, M.E. Bauer, and V.C. Vanderbilt, 1982. Spectral estimates of intercepted solar radiation by corn and soybean canopies. Proc. of the 8th International Processing of Remotely Sensed Data Symposium, Purdue, IN.

18. LeMaster, E.W., J.E. Chance, and C.L. Wiegand, 1980. A seasonal verification of the Suits spectral reflectance model for wheat. Photogram. Eng. and Remote Sensing, V.46,#1, 107-114.

19. Tucker, C.J., 1980. A spectral method for determining the percentage of green herbage material in clipped samples. Remote Sensing of Environment, 9:175-181.

20. Park, J.K. and D.W. Deering, 1981. Relationships between diffuse reflectance and vegetation canopy variables based on the radiative transfer theory. NASA TM 82067.

21. LeMaster, E.W. and J.E. Chance, 1977, in Proc. 11th Int. Symp. Remote Sensing of the Environment, University of Michigan, Ann Arbor, Michigan, 1:703.

22. Suits, G.H., 1972. The calculation of the bidirectional reflectance of a vegetative canopy. Remote Sensing of Environment, 2:117-125.

Thomas L. Woolford received the B.S. degree in Chemistry from New Mexico Highlands University in 1958, and the M.S. degree in Physics from Arizona State University in 1969. He has had experience in the simulation and modeling of physical, chemical, engineering, and biological systems. He is currently employed by Lockheed Engineering and Management Services Company in Houston, Texas, where he is involved in the modeling and simulation of canopy reflectance using agronomic characteristics.
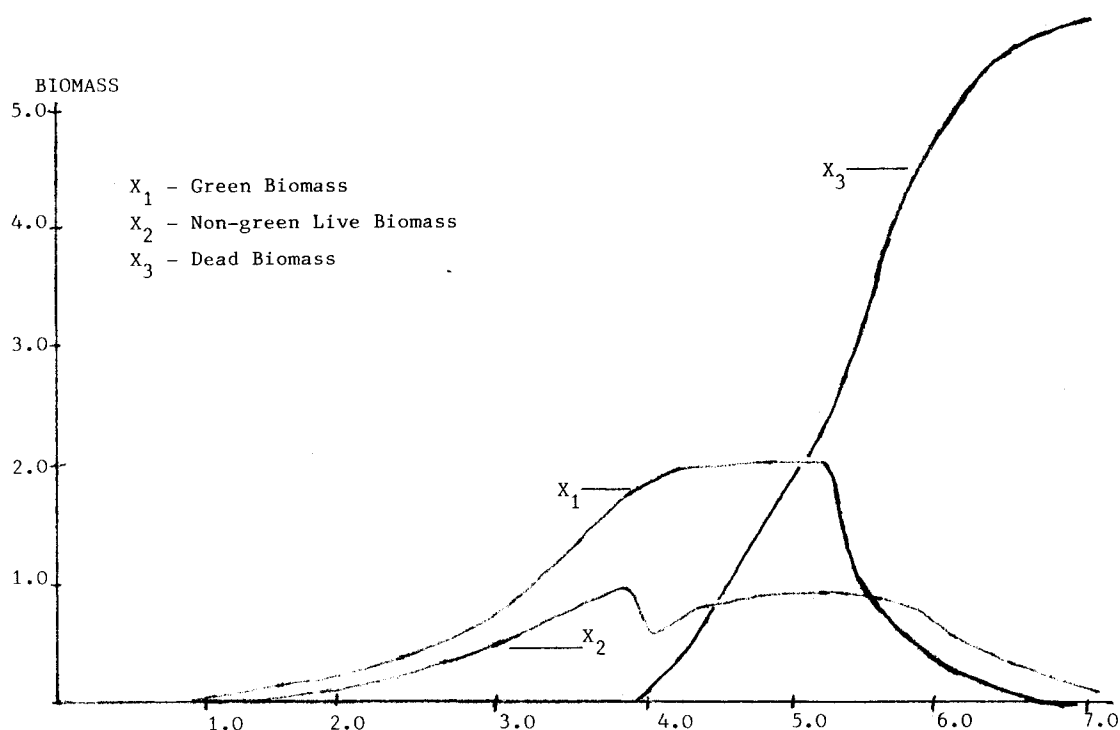
BIOMASS

X_1 – Green Biomass
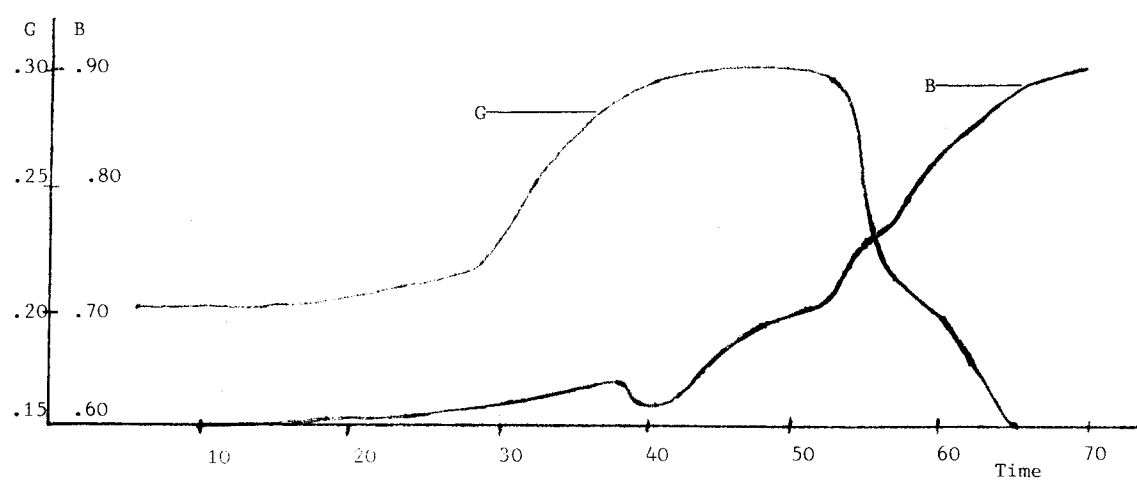X_2 – Non-green Live Biomass
X_3 – Dead Biomass

Figure 1. Biomass vs. Time

Figure 2. Greeness and Brightness vs. Time