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# ISSUES IN DESIGNING GEOGRAPHIC INFORMATION SYSTEMS UNDER CONDITIONS OF INEXACTNESS

V.B. ROBINSON, A.H. STRAHLER

Hunter College/CUNY  
New York, New York

## I. ABSTRACT

Current spatial database design is based on classical Boolean logic that assumes entities, attributes, objects, and relations are given a priori in an exact manner. The inadequacy of the assumptions of exactness have recently become a major issue in design and application of geographic information systems (GIS). Locational approximation and attribute approximation are discussed as sources of inexactness in GIS's. A first attempt at formulating a consistent model for handling inexactness in geographic information systems is made by introducing a logic of inexactness. This Fuzzy Logic (FL) can serve as the basis of representing and manipulating inexactness in a spatial data base. The four distinct cases of nonfuzzy schema/nonfuzzy data, nonfuzzy schema/fuzzy data, fuzzy schema/nonfuzzy data, and fuzzy schema/fuzzy data are defined with examples. Three approaches to managing fuzzy data within a nonfuzzy schema are presented.

## II. INTRODUCTION

Current spatial database design is based on classical Boolean logic that assumes entities, attributes, objects, and relations are given a priori in an exact manner. The inadequacy of the assumptions of exactness have recently become a major issue in various kinds of geographic information systems (GIS). This paper represents a first step in an attempt at formulating a consistent model for handling inexactness in geographic information systems. First, this paper introduces a logic of inexactness that can serve as the basis of representing and manipulating spatial data that is intrinsically 'fuzzy'. Then three approaches to managing fuzzy data within a nonfuzzy schema are presented.

Locational approximation is one source of inexactness in a geographic data base. Craig (1983) presents convincing evidence that points and

boundaries are positioned in inexact locations both purposely and inadvertently. The inaccuracies are tolerable taken individually, but are potentially overwhelming when entered into a GIS and combined with other maps using overlay techniques (MacDougall, 1975). One long-term solution, for the United States entails the densification and utilization of the geodetic control network. This solution is enormously expensive, hence will be decades in coming. In the short-run it is merely suggested that producers and consumers be aware of potential problems. This paper goes further by discussing how inexactness can be incorporated into the management of geographic data. Furthermore, the assumed eventual densification and utilization of the geodetic control network is unlikely to solve all the problems of inexactness as they relate to spatial data bases.

Another source of inexactness is related to the measurement and storage of land-related attribute data. This is an especially bothersome problem in the process of land use mapping using multispectral data from satellites such as Landsat (Robinson, 1981). A fundamental source of this problem is the inconsistency that sometimes arises when trying to find a corespondence between spectral classes and information classes. The logic of this process, at present, can not accommodate the inherent inexactness of information classes that are linguistic variables that can have differing connotations depending upon objective of the mapping.

Robinson (1981) makes two recommendations of particular relevance to this paper. First, he stresses that rules of logic must be used in assigning information classes to multispectrally derived map units and the mapper must be careful to follow these rules when a posteriori rather than a priori judgments are made on the attributes of classes. It is assumed throughout that only classical Boolean logic is to be used. Second, he makes a strong case for maps of information classes derived from multispectral data to be drawn without

arbitrary boundaries so as to portray correctly to the reader the degree (or lack thereof) of heterogeneity of the classes and their attributes. Clearly the second recommendation can not be accomplished with out violating the first unless one uses an alternative logic.

Perhaps the strongest endorsement of this alternative logic was made by Bouille (1982) in the course of discussing future trends in the cartographic data bases. He makes a strong case for fuzzy set theory by emphasizing that most phenomena dealt with in map form are imperfectly organized, incompletely structured, not exactly accurate, etc. In other words, the phenomena are 'fuzzy'. The approach suggested by Bouille (1982) and taken in this paper is that rather than rejecting fuzzy data, and attempting to transform them into more exact data; we should

As an example, let  $U = [0, \infty]$  be the universe of discourse with  $x =$  distance (in kilometers). The membership function of 'short' may be expressed as

$$\text{short} = \int_0^{\infty} e^{-\gamma x} / x, \quad \gamma > 0 \quad (3)$$

where for each of the distances in  $U$  there is a grade of membership. The negative exponential function assigns small values in  $x$  high grades of membership in the 'short' set. Conversely large values in  $x$  are assigned low small values reflecting a low grade of membership in the set 'short'. The constant  $k$  is empirically derived and serves to calibrate the meaning of 'short'. Say that  $U = [0, 9, 25, 32, 45]$ . Then the fuzzy subset may expressed as

$$\text{short} = \sum_{i=1}^5 \mu(x_i) / x_i = 1/0 + 0.5/9 + 0.3/25 + 0.1/32 + 0/45. \quad (4)$$

keep them as fuzzy data and process them as fuzzy information by fuzzy operators producing fuzzy results. That is to say, in the past it has been generally agreed that some measure of data reliability be present in the data base. However, these measures of reliability have rarely been explicitly incorporated into the dual chores of management and retrieval of geographic data from a GIS.

### III. SOME BASIC DEFINITIONS IN FUZZY LOGIC

It is beyond the scope of this paper to present a thorough examination of fuzzy set theory. The reader is referred to Kaufmann (1975) for a comprehensive introduction to fuzzy set theory. However, it is appropriate to discuss the general framework of fuzzy set theory in order to provide a context for this paper.

#### A. FUZZY SETS AND SUBSETS

Basic to fuzzy set theory is the concept of a fuzzy subset defined as

$$A = \int_U \mu(x) / x, \quad \text{where} \quad (1)$$

$A$  is fuzzy subset in universe of discourse  $U$ . And  $\mu : U \rightarrow M$  is a membership function that takes its values in a totally ordered set  $M$  called the membership set, and  $\mu(x)$  indicates the grade of membership of  $x$  in  $A$ . The separator '/' is interpreted as 'with respect to' and is used to differentiate  $\mu(x)$  and  $x$ . The symbol  $\int$  denotes the union of fuzzy singletons  $\mu(x)/x$ . Hence when the universe of discourse is countable, the fuzzy subset  $A$  is expressed as

$$A = \mu(x_1) / x_1 + \dots + \mu(x_n) / x_n \quad (2)$$

#### B. FUZZY OPERATORS

As in normal set theory, there are several basic operations that can be performed on fuzzy sets. Some of the more relevant operations are the complement, intersection, union, algebraic product, and algebraic sum of fuzzy sets.

**Definition 1.** The complement of fuzzy set  $A$  is defined by

$$A = \int_U [1 - \mu(x)] / x \quad (5)$$

**Definition 2.** The intersection of fuzzy sets  $A$  and  $B$  is the largest fuzzy set contained at the same time in  $A$  and  $B$ , defined by

$$A \cap B = \int_U [\mu_A(x) \wedge \mu_B(x)] / x \quad (6a)$$

$$= \int_U \min[\mu_A(x), \mu_B(x)] / x. \quad (6b)$$

**Definition 3.** The union of  $A$  and  $B$  is the smallest fuzzy set containing both  $A$  and  $B$  and is defined by

$$A \cup B = \int_U [\mu_A(x) \vee \mu_B(x)] / x \quad (7a)$$

$$= \int_U \max[\mu_A(x), \mu_B(x)] / x. \quad (7b)$$

**Definition 4.** The algebraic product of  $A$  and  $B$  is defined by

$$AB = \int_U [\mu_A(x) \cdot \mu_B(x)] / x. \quad (8)$$

Definition 5. The algebraic sum of A and B is defined by

$$A \oplus B = \int_U [\mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)]/x \quad (9)$$

A useful characteristic of the intersection and union is that they correspond to the connectives 'and' and 'or' respectively. Furthermore, the algebraic product can be interpreted as the connective 'and' in a 'soft' sense. Similarly the algebraic sum can be interpreted as the soft 'or.'

### III. FUZZY SCHEMA MODELING AND INFORMATION

A relative accuracy, or membership value, may be assigned to each record of a relational or flat file thereby quantifying the concept of fuzzy records. For the moment we are not concerned with the process by which this membership value was arrived at. After Sack et al (1983), we will briefly consider the four distinct cases where there is a nonfuzzy schema/nonfuzzy data, nonfuzzy schema/fuzzy data, fuzzy schema/nonfuzzy data, and fuzzy schema/fuzzy data.

#### A. NONFUZZY SCHEMA AND NONFUZZY DATA

This case of nonfuzzy schema and nonfuzzy data is like the standard relational data base using flat files. Typically this is the conceptual data model upon which most GIS's are based. In this case domains are assumed to be perfectly independent of one another thus allowing their manipulation using predicate logic. Likewise, the data residing in each record or tuple are assumed to exactly measure or describe a particular location. Considering what is known about the correlation between domains and issues of data reliability, both of these assumptions are not very realistic. Since this is the model underlying the design of GIS's under conditions of exactness we will not consider this model in any further depth. However, we should point out that this model is a special case of the more general case of fuzzy schema and fuzzy data.

#### B. NONFUZZY SCHEMA AND FUZZY DATA

Here the semantics of the data model are expressed by precise logical constraints but information is difficult or essentially impossible to capture exactly. This is the most common model used for fuzzy data representation and management. There are two distinct approaches to the management of information under these conditions (e.g., Buckles and Petry, 1982; Sack et al, 1983). Both will be discussed later in more detail.

#### C. FUZZY SCHEMA AND NONFUZZY DATA

This model has a fuzzy intention while the data can be captured exactly. That is to say that while that data is captured exactly, domains may be related to one another so that one domain can be imperfectly reproduced from data in another domain. To illustrate this concept, consider a trivial example of a typical data model used in resource-oriented GIS's where the number of acres Wet, Dry Meadowland and Sage Brush describe each record --

Location	Wet Meadowland	Dry Meadowland
2 10	29	60
2 11	25	62

In this case it is easy to appreciate that Wet Meadowland and Dry Meadowland are not likely to be mutually exclusive information classes in an exact sense. Thus there is likely to be some overlap between the two domains. Although, this situation is not uncommon in resource oriented GIS's, it has not been the subject of much research the fuzzy representation and management of data.

#### D. FUZZY SCHEMA AND FUZZY DATA

This represents the ultimate case of inexactness in GIS design. This case incorporate both the fuzzy domains and fuzzy data in the model of data management and retrieval. Furthermore, it represents the most general case that encompasses the previous three cases as special cases where either data or schema are defined by identity relations.

### IV. MODELS OF FUZZY DATA/NONFUZZY SCHEMA

Three approaches to the representation and manipulation of inexact data in a relational data base are discussed. Each of the three approaches below are considered as variations on the theme of representation and manipulation of fuzzy data within a nonfuzzy schema. Thus, for the purposes

#### A. FUZZY DATA/NONFUZZY SCHEMA MODEL I

The first FDNS model to be discussed was developed by Sack et al (1983). Theirs is a fuzzy relational data base management system where each tuple has attached to it a membership value. This approach is dependent upon a data base composed of many data files, a common characteristic of geographic data bases. Thus, using the simple relational model an example relevant to designing a GIS could be -

Spatial Data File 1 (SDF1)

Location	Land Cover	$\mu$
10 10	Wet Meadow	.8
10 11	Dry Meadow	.4
10 12	Dry Meadow	.9
11 10	Sage Brush	.5
11 11	Sage Brush	.7
:	:	:
.	.	.

Spatial Data File 2 (SDF2)

Location	Slope	$\mu$
10 10	Flat	.6
10 11	Flat	.7
10 12	Gentle	.4
11 10	Gentle	.6
11 11	Flat	.6
:	:	:
.	.	.

In this scheme the membership value,  $\mu$ , is attached to each record. Thus, a measure of belief is directly attached to each tuple. This presumes that the correspondence of the tuple to the intended purpose of the data file is known a priori. We will return to this point momentarily, but for now let us consider examples of some fundamental operators on our trivial fuzzy data sets above.

Union. In this operator, when two identical records appear the supremum is employed as final membership value. For example, the union of Dry Meadows with Flat slopes, i.e., Locations with Dry Meadows or Flat slopes yields the table -

Location	Land Cover	Slope	$\mu$
10 11	Dry Meadow	Flat	.7

Intersection. This is used for searching for records that are common to both the relations. When identical records are found, the minima is used to find the final membership value. For example, the intersection of Dry Meadows and Gentle slopes yields the table

Location	Land Cover	Slope	$\mu$
10 12	Dry Meadow	Gentle	.4

Join. This operator is often of great importance in the management of spatial data. In this operator  $U(t) = \min[U_r(t), U_s(t)]$ . Consider the common case where there are two referenced data layers, each containing one variable (e.g, SDF1 and SDF2). The natural join (SDF1, SDF2) on Location would result in -

Location	Land Cover	Slope	$\mu$
10 10	Wet Meadow	Flat	.6
10 11	Dry Meadow	Flat	.4
10 12	Dry Meadow	Gentle	.4
11 10	Sage Brush	Gentle	.5
11 11	Sage Brush	Flat	.6
:	:	:	:
.	.	.	.

Queries using FL and linguistic variables are made less flexible in this system since the membership values presume that the correspondence of the tuple to the intended purpose of the data base is known a priori.

B. FUZZY DATA/NONFUZZY SCHEMA MODEL II

Buckles and Petry (1982) describe an alternate scheme for representing inexact information in the form of a relational database. They show that two critical properties possessed by ordinary relational databases also exist in their scheme. First, no two tuples have identical interpretations. Second, each relational operation has a unique result. These are of importance when specifying mechanisms the manipulation of the information via queries that preserve the information and the nuances fuzzy uncertainty.

The scheme of Buckles and Petry (1982) differs from ordinary relational databases in important respects. First, components of tuples need not be single values. For example, it is not uncommon for a pixel to be classified differently as a function of classifier used. Illustrate the relevance of this concept to cover data bases derived from remote sensing techniques let us consider the problem of classification accuracy reported in Pettinger (1982) study of the Blackfoot River Watershed. Both digital classification and photo interpretation were used to arrive at the land resource classes and evaluate the accuracy of the digital classification. In a subset of pixels taken from Pettinger (1982, p.21) the following results were reported -

Location	Photo Interpretation	Digital Classification
3 1	Dry Meadow	Riparian Hardwoods
3 2	Riparian Hardwoods	Riparian Hardwoods
3 3	Dry Meadow	Wet Meadow
3 4	Riparian Hardwoods	Wet Meadow
3 5	Dry Meadow	Sage Brush

These results can be used to create a fuzzy land cover data base where

Location	Land Cover
3 1	Dry Meadow, Riparian Hardwoods
3 2	Riparian Hardwoods
3 3	Dry Meadow, Wet Meadow
3 4	Riparian Hardwoods, Wet Meadow
3 5	Dry Meadow, Sage Brush

The second distinguishing characteristic of this scheme is that for each domain set a

similarity relation is defined over the set elements. Note that  $d[i,j]$  are ordinary, not fuzzy sets. The similarity relation is the mechanism for the introduction of 'fuzziness' in the this approach. A similarity relation is a generalization of an equivalence relation in which the similarity relation is reflexive, symmetric, and transitive. For example, consider that the final form of a land cover data base may be of the form -

Location	Land Cover	Slope
3 1	Dry Meadow (DM)	Gentle (G)
3 2	Riparian Hardwoods (RH)	Flat(F)
3 3	Dry Meadow (DM)	Steep (S)
3 4	Wet Meadow (WM)	Flat (F)
3 5	Sage Brush (SB)	Gentle (G)

Land Cover  
Similarity Relation:

	WM	DM	RH	SB
WM	1.0	0.6	0.4	0.2
DM	0.6	1.0	0.2	0.4
RH	0.4	0.2	1.0	0.3
SB	0.2	0.4	0.3	1.0

Slope  
Similarity Relation:

	F	G	S
F	1.0	0.6	0.2
G	0.6	1.0	0.4
S	0.2	0.4	1.0

It is worthwhile at this point to emphasize that, in terms of the nonfuzzy schema/nonfuzzy data model, the implied Land Cover similarity relation would be an identity relation. The point is that this approach is a generalization of the classic relational data model. A fundamental implication of this generalization is that precise responses can not be formulated to queries based on fuzzy data. Consider a query posed to the above data file to the effect of 'Locate areas of Wet Meadow AND Steep Slopes' would result in the assignment of a 'truth value' to each tuple in the following manner -

Location	Land Cover	Slope	Compatibility with Query
3 1	DM	G	$\min[0.6, 0.4] = 0.4$
3 2	RH	F	$\min[0.4, 0.2] = 0.2$
3 3	DM	S	$\min[0.6, 1.0] = 0.6$
3 4	WM	F	$\min[1.0, 0.2] = 0.2$
3 5	SB	G	$\min[0.2, 0.4] = 0.4$

Note that the tuple most closely corresponding to the query is that with Dry Meadow on a Steep Slope. In a typical GIS the lack of an exactly matching tuple(s) would result in the user modifying the query in an attempt to reach an approximate response from a system based on exactness. This resulted from a lack of tuples exactly matching those specified in the query, thus similarity relations represent the compatibility of a proposition or query with a geographic data base.

### C. FUZZY DATA/NONFUZZY SCHEMA MODEL III

This third approach addresses the representation of inexact relations using PRUF and test-score semantics (Zadeh, 1981). The

Possibilistic Relational Uniform Fuzzy (PRUF) meaning representation language is based on the assumption that the imprecision intrinsic in natural languages is possibilistic in nature. Hence the logic underlying PRUF is a Fuzzy Logic in which the truth values of propositions are linguistic. PRUF provides a basis for question-answering and inference from fuzzy premises, i.e. approximate reasoning. In addition, PRUF may be employed as a language for the representation of imprecise knowledge. In general, an expression in PRUF is viewed as a procedure that - given a set of relations in a database - returns a fuzzy relation, a possibility distribution, or a possibility assignment equation.

In PRUF a relational database is a collection of fuzzy relations that are characterized in various ways by tables, predicates, recognition algorithms, generation algorithms, etc. Since an expression in PRUF is a procedure, it involves, in general, not the relations in the database but only their frames. For example, the frame of the database is comprised of:

PLACE | Name | , NEAR | Name1 | Name2 |

Correspondingly, an expression in PRUF such as

NEAR[Name1=Indianapolis]  
Name2×u

represents a procedure that returns the fuzzy subset of PLACE comprising names of places that are near Indianapolis.

Underlying PRUF are test-score semantics (TSS). The basic idea of TSS is that an entity in linguistic discourse has the effect of inducing elastic constraints on a set of objects or relations in the universe of discourse. The meaning of the entity may be defined by (a) identifying constraints that are induced by the entity; (b) describing tests that must be

performed to ascertain the degree to which each constraint is satisfied; and (c) specifying the manner in which the partial test scores are to be aggregated to yield an overall test score. Meaning of a linguistic entity is identified by testing elastic constraints. Testing constraints is accomplished using the tools of test-score semantics and fuzzy logic can be used to assess the compatibility of a linguistic summary with a given database.

The process of meaning representation in test-score semantics involves three distinct phases that are summarized as:

Phase 1 - an explanatory database frame or EDF is constructed.

Phase 2 - a test procedure is constructed that acts on relations in the explanatory data base (EDB), yielding test scores representing the degree to which the elastic constraints are satisfied.

Phase 3 - partial test scores are aggregated into an overall test score that is a vector measuring compatibility of the semantic entity with the EDB.

In test score semantics the testing of constraints induced by a proposition or query is performed on a collection of fuzzy relations which constitute an **explanatory database (ED)**. In an indirect way, the testing and aggregation procedures in test-score semantics are viewed as a description of a process by which the meaning of the proposition or query is composed from the meanings of the constituent relations in the explanatory database.

$$\Pi_{\text{Location}(\text{PROPERTY}(\text{Strahler's Meadow}))} = ( \text{NEAR}[\text{Place.Name1} = \text{Indianapolis}] \wedge ( \text{SLOPE}[\text{Place.Name} = \mu] = \text{GENTLE}[\text{Slope} = \mu] ) )$$

In PRUF, the translation of a proposition may be either **focused** or **unfocused**. Focused translation generally leads to a possibility assignment equation. Unfocused translation is a collection of tests that are performed on the database and a set of rules for aggregating the partial test scores into an overall test score that represents the compatibility of a proposition with the database.

A simple example of each type of translation will serve to illustrate the differences.

Consider the proposition - Strahler's meadow is on gentle slopes near Indianapolis. The unfocused translation is -

1. Define the data frame (DF) needed to test proposition :

$$\text{DF} \triangleq \text{PROPERTY}[\text{Place.Name}] + \text{SLOPE}[\text{Place.Name}; \text{Slope}] + \text{GENTLE}[\text{Slope}; \mu] + \text{NEAR}[\text{Place.Name1}; \text{Place.Name2}; \mu]$$

2. Find the name of the location of Strahler's Meadow:

$$a \triangleq \text{PROPERTY}[\text{Name} = \text{Strahler's Meadow}]. \text{Place.Name}$$

3. Find the slope of the meadow of Strahler:

$$b \triangleq \text{SLOPE}[\text{Place.Name} = a]. \text{Slope}$$

4. Test the constraint induced by GENTLE :

$$t_1 = \text{GENTLE}[\text{Slope} = b]. \mu$$

5. Test the constraint induced by NEAR :

$$t_2 = \text{NEAR}[\text{Place.Name1} = \text{Indianapolis}, \text{Place.Name2} = a]. \mu$$

6. Aggregate  $t_1$  and  $t_2$  :

$$t = t_1 \wedge t_2$$

Suppose that the relation PROPERTY does not contain Strahler's Meadow or it does not exist in the database. Then the focused translation of the proposition is the possibility assignment equation

The above focused translation equation conveys the information that the possibility distribution of  $\text{Location}(\text{PROPERTY}(\text{Strahler's Meadow}))$  is the intersection of two possibility distributions. One reflects the constraint that it is near Indianapolis while the other reflects the constraint that it is on a gentle slope. Thus, it is easy to see that the focused translation most closely resembles the typical spatial query.

## V. CONCLUDING DISCUSSION

This paper began by considering the discussion that has occurred regarding inexactness in GIS's. Essentially, we view the problem as one of nonfuzzy schema and fuzzy data. This will probably remain the ruling paradigm of the field for some time. Inexactness was GIS was shown to be due largely to an 'information concept' not exactly represented by the data. In this sense data items are to be assigned membership values based on the degree of correspondence. However, one of the remaining fundamental issues is how to arrive at these membership values. One method is to use standard membership functions. This approach ignores the subjectivity of inherent in the meaning of the 'information concepts.' The problem of acquiring the subjective representation of concepts and relations from users of GIS's is central to implementing any of the FDNS models. Recently, Robinson (1984) has proposed an intelligent, Spatial Relations Acquisition Station (SRAS) that can be incorporated into a GIS. The SRAS would acquire a user's representation of a spatial concept and store it in an explanatory data base.

An important implication of this paper is that GIS's should be thought of as containing inexact representations of segments of reality. These segments of reality have been measured and stored with the purpose of answering questions that are ill-specified before design of the system. The typical use of a GIS is to test some proposition (or query) against data stored in the GIS. The outcome is the compatibility of the proposition or query with that segment of reality as represented in the geographic data base.

Finally, there is the question of how to graphically display the results of imprecise queries directed to a fuzzy data base. This poses unique problems GIS's since the information product of a GIS is typically a map. The continuous-tone choropleth map is ideally suited to solving this problem for those desiring digital plotter output. Benson (1982) describes an approach to data display in which membership is modeled by exploiting the fuzzy nature of color categories. Thus, methods for displaying inexactness in GIS's are developing and should be integrated into any system that seeks to explicitly manage inexactness.

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## VII. AUTHORS BIOGRAPHICAL DATA

Vincent B. Robinson is an Associate Professor of Geography at Hunter College. He received his Ph.D. from Kent State University. Current research interest is in the incorporation of natural language understanding, artificial intelligence, and spatial modeling into geoprocessing systems. Many ideas presented here are a result of research conducted during a NASA/IEEE Faculty Summer Fellowship experience at Goddard Space Flight Center, 1983.

Alan H. Strahler is Professor and Chairman in the Department of Geology & Geography. He has extensive experience in the remote sensing of vegetation. His recent research is focuses on canopy modeling of conifer forests and fundamental research into the development of spatial modeling concepts for application in remote sensing.