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# SEGMENTATION OF SAR IMAGERY USING GIBBS DISTRIBUTION MODELS

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## ABSTRACT

This paper presents a statistical approach to modeling and segmentation of noisy images with particular applications to Synthetic Aperture Radar (SAR) imagery. It is assumed that the image is corrupted by additive independent noise and that the noise-free image is modeled as a Gibbs distributed random field or equivalently a Markov Random Field (MRF).

Based on the Gibbs Distribution (GD) models used, a maximum a posteriori (MAP) segmentation algorithm is presented, which uses a dynamic programming formulation. To bring the computational requirements to manageable levels, a near optimal version of the algorithm is implemented. The algorithm is capable of producing 2, 4, 8 or 16 level segmentations of images with relatively low SNR's. Several examples are presented on the application of the algorithm to SAR imagery.

## I. INTRODUCTION

In the recent image processing literature there has been increasing interest in use of statistical techniques for modeling and processing image data. Much of this work has been directed toward application of Markov Random Fields (MRF) (Abend et al (1965), Chellappa and Kashyap (1982), Hansen and Elliott (1982), Derin et al (1984)), and recently toward use of Gibbs Distribution (GD) models (Hassner and Sklansky (1980), Cross and Jain (1983), Elliott et al (1984), Geman and Geman (1984), Cohen and Cooper (1983), Elliott and Derin (1984)). Although interest in MRF models for tackling image processing problems can be traced to the work of Abend et al (1965), only recently have the appropriate mathematical tools for the exploitation of the full power of MRF's in image processing found their way into the computer vision literature. In particular, following the establishment of the MRF-GD equivalence in the 1970's,

the GD characterization of random fields is attracting increasing attention.

Most of the recent studies mentioned above make use of the GD characterization of MRF's. Hassner and Sklansky (1980) and Cross and Jain (1983) use GD models to characterize textures as well as blobby region formations. Elliott et al (1984), using GD models, proposed a MAP segmentation algorithm. Geman and Geman (1984) also give a MAP segmentation algorithm using GD models and a stochastic relaxation technique. Cohen and Cooper (1983), on the other hand, propose parallel and hierarchical segmentation algorithms for textured images using GD models. Elliott and Derin (1984) presented a hierarchical GD model for textured and noisy images and a segmentation algorithm for such images. They also addressed some of the parameter estimation problems that arise in GD models and segmentation algorithms.

This paper presents a MAP segmentation algorithm based on a GD model and application of this algorithm to several SAR images. The image is assumed to be corrupted by additive independent noise and the noise-free image is modeled as a particular GD. A segmentation algorithm that seeks the MAP estimate, that is a realization which maximizes the a posteriori distribution of the noise-free image given the noisy one, is presented. The maximization of the a posteriori distribution is carried out using dynamic programming formulation. Due to the formidable nature of the computational requirements, however, a near-optimal version of the algorithm, which optimally processes narrow strips of the image and combines these to yield an estimate of the whole image, is proposed.

The proposed segmentation algorithm is capable of segmenting the noisy image into 2, 4, 8 or 16 region types. For more than 2 region types, a computationally efficient version of the algorithm, consisting of a sequence of binary segmentations, is presented. Each binary segmentation is preceded by an estimation procedure, which, assuming that the image to be processed consists of two gray levels that are corrupted by additive independent Gaussian noise, yields estimates of the two gray levels and the noise variance. Thus, except for the

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parameters of the GD used, the segmentation and parameter estimation is carried out in coordination and the overall algorithm is fully automated. This algorithm is applied to several SAR images to get 2, 4 or 8 level segmentations. An earlier version of this algorithm was presented by Elliott et al (1984).

The paper is organized as follows. In Section II, the model and the segmentation problem are formulated in a statistical framework. The description of a special class of GD used in this study is also presented in Section II. The MAP segmentation algorithm is described in Section III and examples on application of the algorithm are presented in Section IV. Some comments and concluding remarks are given in Section V.

## II. PROBLEM FORMULATION AND BACKGROUND ON GIBBS DISTRIBUTIONS

### A. THE MODEL

The images of concern are all defined on a finite rectangular lattice of pixels, given by

$$L = \{(i,j): 1 \leq i \leq N_1, 1 \leq j \leq N_2\} \quad (1)$$

It is assumed that the observed image (matrix)  $y = [y_{ij}]$  is a realization from the random field  $Y = [Y_{ij}]$  defined on lattice  $L$ . In other words, the random variables  $Y_{ij}$  at each pixel  $(i,j) \in L$ , constitute the random field  $Y$ . The observed image random field  $Y$  is a function of two other random fields: (i) the "scene" (noise-free image) random field  $X$ , and (ii) the corruptive noise random field  $W$ , all three defined over the same  $N_1 \times N_2$  lattice.

The functional form of the relationship among these three random fields is given by

$$Y_{ij} = X_{ij} + W_{ij} \quad (i,j) \in L \quad (2)$$

or in matrix form

$$Y = X + W \quad (3)$$

It is assumed that the noise random field  $W = [W_{ij}]$  consists of independent identically distributed Gaussian random variables with mean 0 and variance  $\sigma^2$ , i.e.,  $W_{ij} \sim N(0, \sigma^2)$ .

The scene random field  $X$  is assumed to be a (finite) discrete valued random field, where each  $X_{ij}$  takes one of  $M$  values  $\{r_1, r_2, \dots, r_m\}$ , called "gray levels".  $X_{ij} = r_m$  means that pixel  $(i,j)$  is of region type  $m$ . Thus it is being assumed that the noise-free image, i.e., the scene, consists of  $M$  distinct region types. The discrete random field  $X$  is assumed to be a Gibbs distributed random field. The definitions of GD and the particular distri-

bution assumed for  $X$  are presented in the following subsections. It should be pointed out that the correlation and the spatial continuity in the scene random field  $X$  are very adequately characterized by the proposed GD models.

Finally, the scene random field  $X$  and the noise random field  $W$  are assumed to be statistically independent, although the formulation and the algorithm are amenable to a region dependent noise with no additional computational burden. Moreover, the Gaussian assumption on the noise is not essential and can be altered to any other distribution as long as the independence of noise from pixel to pixel is preserved.

### B. THE SEGMENTATION PROBLEM

For the model described above, the segmentation problem is posed as follows. It is desired to devise an estimation scheme which, based on the observed image  $y$ , a realization from the noise corrupted random field  $Y$ , will put out an estimate  $x^* = x^*(Y) |_{Y=y}$  of the noise free scene. The segmentation algorithm presented in this paper attempts to determine the maximum of the a posteriori distribution  $P(x|Y=y)$ . Due to the astronomical number of  $x$  configurations,  $M^{N_1 N_2}$  to be exact, the maximization of  $P(x|Y=y)$  can not be done by a straight-forward search. The specific algorithm proposed in this paper that attempts to determine the MAP estimate is presented in Section III.

### C. BACKGROUND ON GIBBS DISTRIBUTIONS (GD)

Based on the rectangular lattice  $L$  described above, GD are defined through the following set of definitions. First a neighborhood system on lattice  $L$  and the associated cliques are defined.

Definition 1. A collection of subsets of  $L$  described as

$$\eta = \{\eta_{ij}: (i,j) \in L, \eta_{ij} \subseteq L\} \quad (4)$$

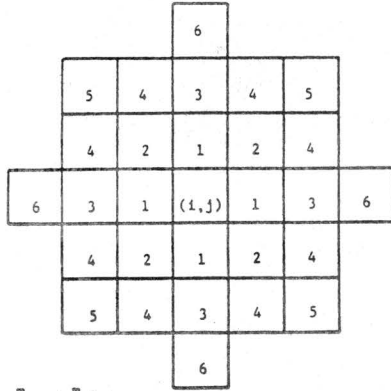
is a neighborhood system on  $L$  if and only if  $\eta_{ij}$ , the neighborhood of pixel  $(i,j)$  is such that

$$1. (i,j) \notin \eta_{ij}, \text{ and} \quad (5)$$

$$2. \text{ If } (k,l) \in \eta_{ij} \text{ then } (i,j) \in \eta_{kl} \quad (6)$$

for any  $(i,j) \in L$ .

A hierarchically ordered sequence of neighborhood systems that are commonly used in image modeling are  $\eta^1, \eta^2, \eta^3, \dots, \eta^1 = \{\eta_{ij}^1\}$  is such that for each  $(i,j) \in L$  (except for pixels on the boundaries)  $\eta_{ij}^1$  consists of the four pixels neighboring pixel  $(i,j)$ .  $\eta^2 = \{\eta_{ij}^2\}$  is such that  $\eta_{ij}^2$  consists of the eight pixels neighboring  $(i,j)$ .



$$\eta^m = \{\eta_{ij}^m\}$$

$$\eta_{ij}^m = \{k: k \leq m\}$$

Figure 1. Hierarchically arranged neighborhood systems  $\eta^m$ .

The neighborhood structure for  $\eta^1$  and  $\eta^2$ , as well as for  $\eta^3$ ,  $\eta^4$  and  $\eta^5$  are shown in Fig.1. The neighborhood system  $\eta^m$  is called the  $m$ th order neighborhood system. The "cliques" associated with a lattice-neighborhood system pair  $(L, \eta)$  is defined as follows.

**Definition 2.** A clique of the pair  $(L, \eta)$ , denoted by  $c$ , is a subset of  $L$  such that

1.  $c$  consists of a single pixel, or
2. For  $(i, j) \neq (k, l)$ ,  $(i, j) \in c$  and  $(k, l) \in c$  implies that  $(i, j) \in \eta_{kl}$ ,

The collection of all cliques of  $(L, \eta)$  is denoted by  $C = C(L, \eta)$ .

The types of cliques associated with  $\eta^1$  and  $\eta^2$  are shown in Fig.2. Now a GD can be defined as follows.

**Definition 3.** Let  $\eta$  be a neighborhood system defined over the finite lattice  $L$ . A random field  $X = \{X_{ij}\}$  defined on  $L$  has Gibbs Distribution (GD) or equivalently is a Gibbs Random Field (GRF) with respect to  $\eta$  if and only if its joint distribution is of the form

$$P(X=x) = \frac{1}{Z} e^{-U(x)} \quad (7)$$

where  $U(x) = \sum_{c \in C} V_c(x)$  energy function (8)

$$V_c(x) = \text{potential assoc. with clique } c \quad (9)$$

$$Z = \sum_x e^{-U(x)} \quad \text{partition function} \quad (10)$$

The partition function  $Z$  is simply a normalizing

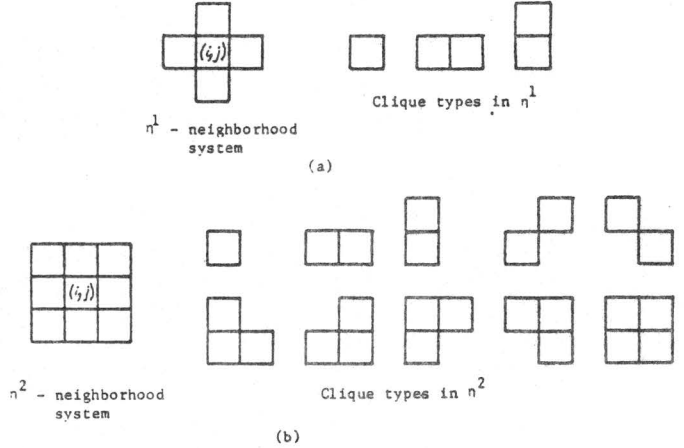


Figure 2. Neighborhood systems  $\eta^1$  and  $\eta^2$ , and their associated clique types.

constant, so that the sum of the probabilities of all realizations  $x$ , add to unity. The only condition on the otherwise totally arbitrary clique potential  $V_c(x)$  is that it depends only on the pixel values in clique  $c$ .

The origins of GD lie in physics and statistical mechanics literature. Ising (1925) used a special GD, now known as the Ising Model, to describe the magnetic properties of ferromagnets. The source of the revived interest in GD, especially in the context of image modeling and processing is an important result known as the Hammersley-Clifford Theorem. This result proven in the 1970's independently by several researchers, establishes a one-to-one correspondence between MRF's and GRF's. Unlike the MRF characterization, the GD characterization provides the joint distribution of the random field, is free from consistency problems and provides a more workable spatial model. For a detailed treatment the reader is referred to Besag (1974).

#### D. A CLASS OF GIBBS DISTRIBUTIONS

In this subsection, a particular class of GD, which is used to model the scene random field  $X$ , is presented. In concurrence with the model described above, the random field  $X$  consists of  $M$ -valued discrete random variables  $\{X_{ij}\}$  taking values in  $R = \{r_1, r_2, \dots, r_m\}$ . To define a GD it suffices to specify the neighborhood system  $\eta$ , the associated cliques and the clique potentials,  $V_c(x)$ . Here, it is assumed that the random field is homogeneous, that is the clique potentials depend only on the clique type and the pixel values in the clique, but not on the position of the clique in lattice  $L$ . The distribution is specified in terms of the second order neighborhood system,  $\eta^2$ . The extension to larger neighborhood systems and the restriction to smaller ones are evident.

The neighborhood system is  $n^2$  and cliques are those associated with  $n^2$ . The clique potentials are defined as follows. A parameter is assigned to each clique type, except for the single pixel clique. In other words,

$$\begin{aligned}
 & [^{**}, \beta_1], [^*, \beta_2], [^{**}, \beta_3], [^*, \beta_4] \\
 & [^{**}, \gamma_1], [^{**}, \gamma_2], [^{**}, \gamma_3], [^{**}, \gamma_4], [^{**}, \epsilon_1]
 \end{aligned} \tag{11}$$

The associated clique potentials are defined as

$$V_c(x) = \begin{cases} \theta & \text{if all } x_{ij} \text{ in } c \text{ are equal} \\ -\theta & \text{otherwise} \end{cases} \tag{12}$$

where  $\theta$  is the parameter specified for the clique type  $c$ . For the single pixel cliques, the clique potential is defined as

$$V_c(x) = \alpha_k \quad \text{for } x_{ij} = r_k \tag{13}$$

The  $\alpha_k$  parameters can be used to control the percentage of pixels in each region type, while the other parameters can be used to control the size and direction of clustering.

This class of GD has been used extensively by the authors for modeling wide variety of images, both as region formation models and as texture models. For a more detailed treatment of this class of GD, the reader is referred to Elliott and Derin (1984) and Derin (1985-a).

### III. MAP SEGMENTATION ALGORITHM

In this section, first the (optimal) MAP segmentation algorithm and then a near-optimal version of it, necessitated by computational concerns, are described.

#### A. MAP ALGORITHM

As pointed out in the previous section, MAP estimate of the scene is the realization  $x^*$  that maximizes the a posteriori distribution  $P(X=x|Y=y)$  based on the observed image  $y$ . Using Bayes' rule

$$P(X=x|Y=y) = \frac{P(Y=y|X=x) P(X=x)}{P(Y=y)} \tag{14}$$

Since  $P(Y=y)$  is independent of the maximization over  $x$ , equivalently, the numerator is to be maximized. That is,  $x=x^*$  that maximizes

$$P(X=x, Y=y) = P(Y=y|X=x) P(X=x) \tag{15}$$

or equivalently that maximizes

$$\ln P(X=x, Y=y) = \ln P(Y=y|X=x) + \ln P(X=x) \tag{16}$$

is sought.

The first term in the RHS of (16) is the "data" term and is merely the sum-log of marginal Gaussians. The second term is the "model" term and is given in (7)-(10), with clique potentials defined in (11)-(13). Both terms can be arranged in recursion, thus making dynamic programming an attractive tool for the maximization of the joint log-likelihood. This however involves an  $M^{N_1}$  state dynamic programming algorithm and is computationally prohibitive for  $N_1 > 4$ . So, a near-optimal version of the algorithm is adopted instead. The optimal MAP algorithm is conveniently expressed as a particular case of the near-optimal version. Therefore, the near-optimal algorithm is described first, and the particular case that corresponds to the optimal algorithm is specified afterwards.

#### B. NEAR-OPTIMAL MAP SEGMENTATION

In this subsection, an optimal algorithm is posed for processing images consisting of a few rows. The complete near-optimal algorithm is then obtained by applying this optimal processor on overlapping strips of the larger  $N_1 \times N_2$  image. This algorithm will be near-optimal, when correlation between the random variables drops rapidly as the distance between them increases. This assumption is reasonable in the sense that it can often be shown for 1-D Markov processes. However, calculation of correlations in a MRF is extremely difficult even for the simplest case, and is an unresolved problem in statistics literature.

First consider the problem of segmenting a  $D \times N_2$  image, where  $D \ll N_1$ . Using the Gibbs likelihood for  $X$  and the Gaussian i.i.d. conditional likelihood for data, the two components of the joint likelihood given in (16) can be expressed as

$$\ln P(X=x) = -\ln Z + \sum_c V_c(x) \tag{17}$$

$$\begin{aligned}
 \ln P(Y=y|X=x) = & -\frac{DN_2}{2} \ln(2\pi\sigma^2) \\
 & - \sum_{m=1}^M \sum_{(i,j) \in S_m} \frac{1}{2\sigma^2} (y_{ij} - r_m)^2
 \end{aligned} \tag{18}$$

where  $S_m = \{(i,j) : x_{ij} = r_m\}$ .

It is noted here, that the joint log-likelihood given in (16), with the two components given in (17) and (18) can be calculated recursively as follows

$$\ell_0 = -\frac{ND_2}{2} \ln(2\pi\sigma^2) - \ln Z \tag{19}$$

$$\ell_k = \ell_{k-1} + \sum_{c \in C^{k-1,k}} V_c(x) - \sum_{m=1}^M \sum_{(i,j) \in S_m^k} \frac{1}{2\sigma^2} (y_{ij} - r_m)^2 \quad (20)$$

$$\ell_{N_2} = \ln P(X=x, Y=y) \quad (21)$$

where

$$C^{k-1,k} = \{c : c \text{ contains only pixels in column } k \text{ or only pixels in columns } k-1 \text{ and } k\} \quad (22)$$

$$S_m^k = \{(i,j) : x_{ij} = r_m, 1 \leq i \leq D, j = k\} \quad (23)$$

This recursion in conjunction with the principle of optimality allows formulation of a forward dynamic programming algorithm for finding  $x^*$  that maximizes  $\ell_{N_2}$ .

The state space associated with the dynamic programming algorithm has dimension  $M^D$ , since there are  $M^D$  possible segmentations of each column of the  $D \times N_2$  scene. This implies that the algorithm would have  $N_2$  iterations with on the order of  $M^{2D}$  calculations during each iteration. Thus, this algorithm may be computationally tractable only for small values of  $M$  and  $D$ , e.g.,  $2 \leq M, D \leq 4$ . Using the hierarchical segmentation scheme that is described below, this algorithm can be applied iteratively to segment images for which  $M > 4$ .

The above algorithm is a standard dynamic programming application, hence details are not given here. However, the following remarks are in order. The value of  $\ell_0$  is independent of any segmentation  $x$ , and hence the algorithm can be initialized by setting  $\ell_0 = 0$ . In particular, it is not necessary to undertake the difficult task of calculating the partition function  $Z$ . It is also noted that in processing the  $D \times N_2$  strip if scene values at pixels neighboring the strip are available, this information can be used as a boundary condition for the MRF. If such a boundary condition is not available, then either a fictitious one can be hypothesized or the pixels of the strip along the boundary can be assumed to have smaller neighborhoods.

The dynamic programming algorithm described above is capable of optimally processing a  $D \times N_2$  strip of an  $N_1 \times N_2$  image, where  $D < N_1$ . It gives the optimal MAP segmentation of the  $D \times N_2$  strip based on

the image data on that strip. Thus, for  $D = N_1$  this algorithm gives the optimal MAP estimate. But for reasons explained above the algorithm can not be implemented unless for small  $D$ . Hence, the near-optimal version described below was devised.

In order to use this optimal processor for a strip to obtain segmentation of the whole image, it is assumed that the random variables  $X_{ij}$  and  $X_{i+D,j}$  for all  $(i,j)$  have negligible correlation. Thus, the impact of the image data on and below row  $i+D$ , on the segmentation of row  $i$  is assumed to be negligible. In view of this, the complete segmentation procedure is described by the following algorithm.

#### Segmentation Algorithm

- Step 0 Choose a value for  $D$ ,  $2 < D \leq 4$ .
- Step 1 Set  $I = 1$
- Step 2 Apply the optimal strip processor (dynamic programming algorithm) to rows  $I$  through  $I+D-1$ .
- Step 3 If  $I+D-1 = N_1$ , store the segmentation for rows  $I$  through  $I+D-1$  and go to Step 7.
- Step 4 Store the segmentation for row  $I$  and discard the rest.
- Step 5 Set  $I = I+1$ .
- Step 6 Go to Step 2.
- Step 7 Stop.

To summarize, the dynamic programming algorithm is applied to overlapping image strips of width  $D$ , but only the segmentation of the first row of each strip is used. For example, the processing of rows 1 through  $D$  yields a segmentation of row 1, and the processing of rows 2 through  $D+1$  yields a segmentation of row 2. Under the correlation assumption above, this algorithm is near-optimal since the data in row  $I+D$  and below will have little impact on the segmentation of row  $I$ .

As pointed out above, the strip dynamic programming algorithm can make use of a boundary condition if one is available. In the overall segmentation algorithm described above, while processing the  $I$ th strip consisting of rows  $I$  through  $I+D-1$ , the segmentation of row  $I-1$ , which was obtained during processing the previous strip, is used as a boundary condition. For  $I=1$ , however, there is no such previous segmentation; therefore for  $I=1$ , a boundary condition is either hypothesized or not used at all.

As it was alluded to above, when  $M$ , the number of region types, is larger than 4, direct application of the algorithm is computationally not feasible. Therefore, a hierarchical segmentation scheme can be used instead. The hierarchical segmentation scheme being proposed consists of successive applications of the binary segmentation. The image is first segmented into two region types by binary segmentation ( $M=2$ ) and then each region is segmented into two region types and so on. Thus, using binary segmentation algorithm only,  $M$  level



segmentation can be obtained by M-1 applications of the binary segmentation algorithm. As it is shown by examples in Section IV, the hierarchical segmentation scheme outlined here yields extremely good segmentation results for M=4 and M=8.

It should be pointed out that in any application of the segmentation algorithm - binary or M-ary - the intensity levels  $r_m$ 's and noise variance  $\sigma^2$  (or  $\sigma_m^2$ 's if noise is region dependent) must be known. If these parameters are not known in advance, they are estimated using the method of moments for estimating the component parameters of Gaussian mixtures. For a detailed treatment of the method of moments the reader is referred to Cohen (1967) for 2 component mixtures and to Derin (1985-b) for 2 or more component mixtures.

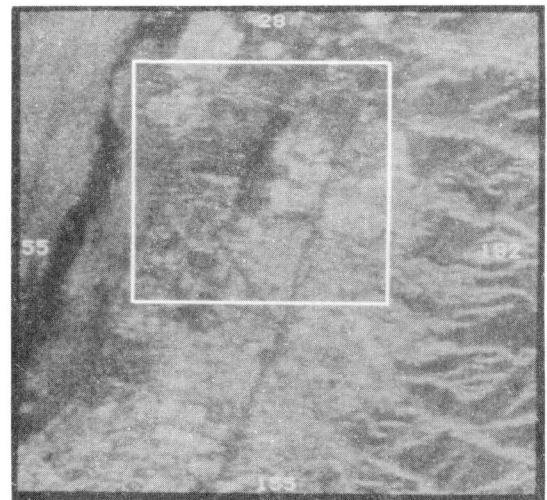
Finally, the segmentation algorithm described above can easily be revised to accommodate for region dependent noise, possibly non-Gaussian noise, potential functions  $V_c$ 's other than those given in (11)-(13) and neighborhood systems larger than the  $n^2$  used here. The computational requirements of the algorithm, however, will increase for larger neighborhood systems, but will not be significantly effected by the change in the other assumptions.

#### IV. EXAMPLES

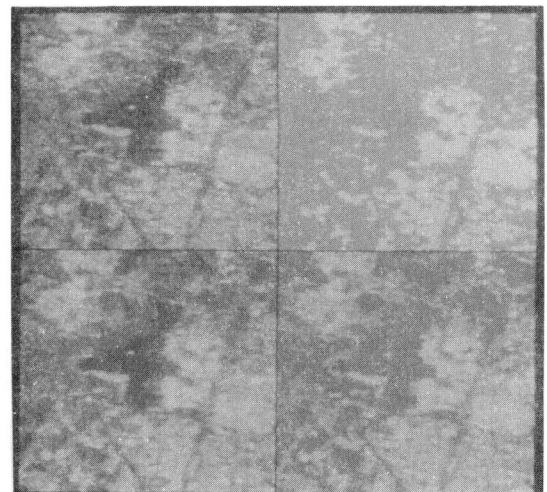
In this section, examples are presented on the application of the near-optimal MAP segmentation algorithm described above to several SAR images. The SAR images considered are all somewhat noisy, and hence no additional noise is added. They are assumed to be realizations from a scene random field corrupted by additive independent noise, as described in Section II. The scene is an M-valued GD of the class described in Section II and the noise is i.i.d. Gaussian.

Using the hierarchical segmentation scheme described above, 2, 4 and 8 level segmentations of SAR images are obtained. The binary segmentation algorithm works as follows. The noisy image data is processed and estimates for the two gray levels and the noise standard deviation are obtained using the method of moments. These estimates are then used to obtain the MAP - dynamic programming segmentation of the image. Other parameters necessary for the segmentation algorithm are the strip width D and the GD clique potential parameters given in (11)-(13). In extensive experiments, strip width D=3 was found to be adequate. D=4 did not result in a significant improvement over the D=3 case. Hence, in all segmentation results presented here D=3 was used for strip width.

For the clique potential parameters of (11)-(13), the following assumptions are made. All  $\alpha_i$ 's are taken as 0, thus prior distribution is assumed to be uniform. Moreover, all  $\gamma_i$ 's and  $\xi_1$  are also



(a)



(b)

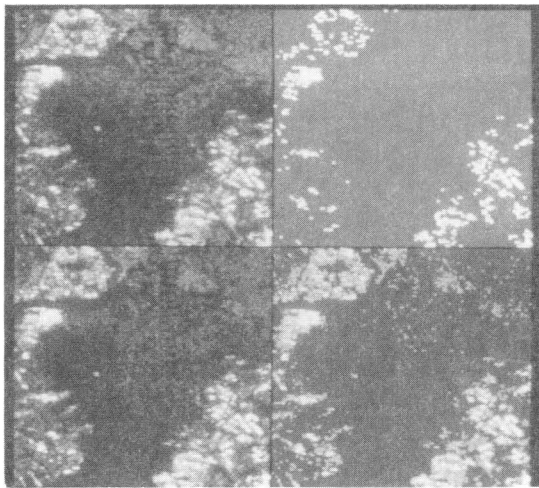
Figure 3. (a) SAR image of Santa Barbara, (b) 128x128 section from (a), its 2, 4, 8 level segmentations with  $\beta=0.15$ .(\*)

assumed to be 0. Thus, the only non-zero clique potentials are those of the pair cliques. For the examples in this paper, all pair clique parameters  $\beta_i = \beta$  for  $i=1, \dots, 4$ . Experimentations with different  $\beta$  values indicate that the segmentation is quite sensitive to  $\beta$  values. Larger  $\beta$  values eliminate noisy data, but in the process image detail is also eliminated. Conversely, small  $\beta$  preserves image detail but leaves noise speckles uncorrected. This argument is substantiated by some examples presented below.

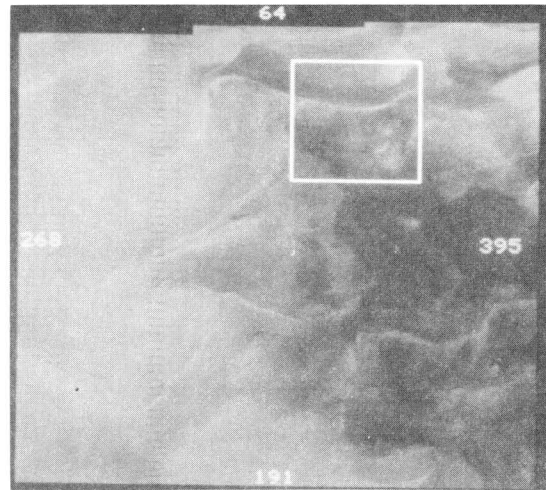
The sensitivity of the segmentation to parameters, and the seemingly casual assignment of parameter values is somewhat disconcerting. This fact emphasizes the strong need for parameter



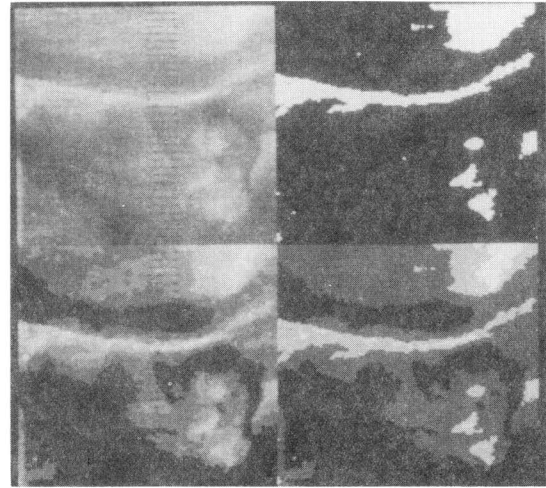
(a)



(b)



(a)



(b)

Figure 4. (a) SAR image of Baltimore Harbor, (b) 128x128 section from (a), its 2, 4, 8 level segmentations with  $\beta=0.1$ .(\*)

Figure 5. (a) SAR image of Ocean Scene, (b) 128x128 section from (a), its 2, 4, 8 level segmentations with  $\beta=0.2$ .(\*)

estimation techniques in GD models. There are a few studies on this important problem (Besag (1974), Cross and Jain (1983), Derin et al (1985)). In this work, however, parameter estimation on the GD model was not considered. Values for the  $\beta$  parameters that experimentally give good segmentation results were used. The  $\beta$  parameter used in each case is specified.

Examples presented here are segmentations of sections from 4 SAR images that were provided by ONR. These images are:

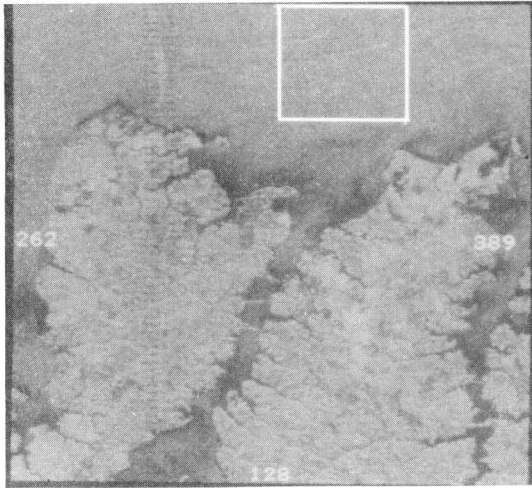
1. Santa Barbara (256x256),
2. Baltimore Harbor (256x256),
3. Ocean Scene (512x512),
4. Chesapeake Bay (512x512).

\* Stands for "in clockwise order, starting with the upper left quadrant".

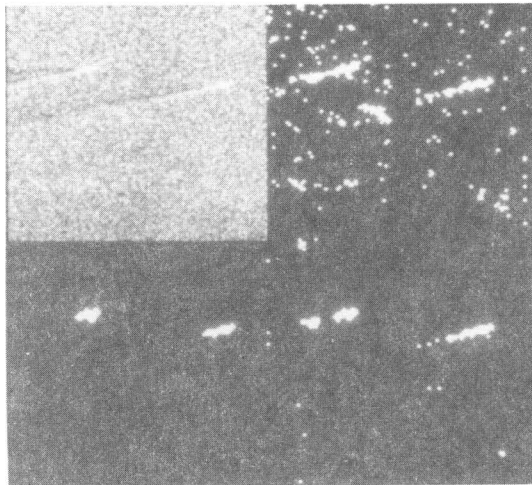
128x128 sections from these images were selected and segmented.

For the first three of these images, a 128x128 section from each one is selected, which are depicted in Figs. 3.a, 4.a and 5.a. Each one of these 128x128 SAR images are segmented to 2, 4 and 8 regions. The corresponding segmentation results are shown in Figs. 3.b, 4.b and 5.b. From the fourth, i.e. Chesapeake Bay image, 3 different 128x128 sections are selected (Figs. 6.a, 7.a and 8.a). Each one is first segmented to 2 regions with 3 sets of  $\beta$  parameters (Figs. 6.b, 7.b and 8.b) and then segmented to 2, 4 and 8 regions (Figs. 6.c, 7.c and 8.c). It should be noted that, in some instances, although a 4 or 8 level segmentation is sought, the algorithm yields fewer significantly

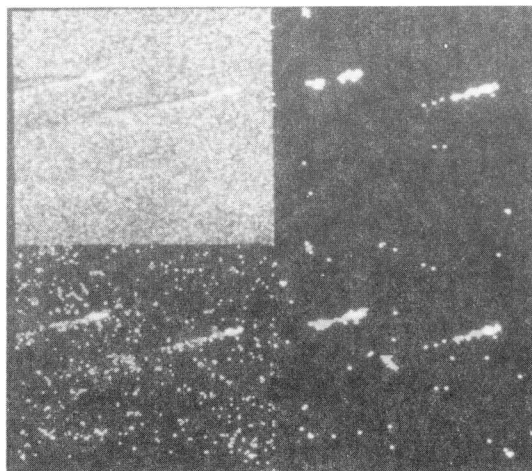




(a)

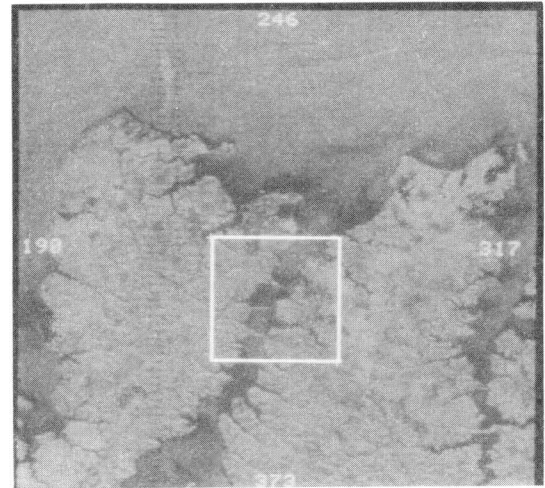


(b)

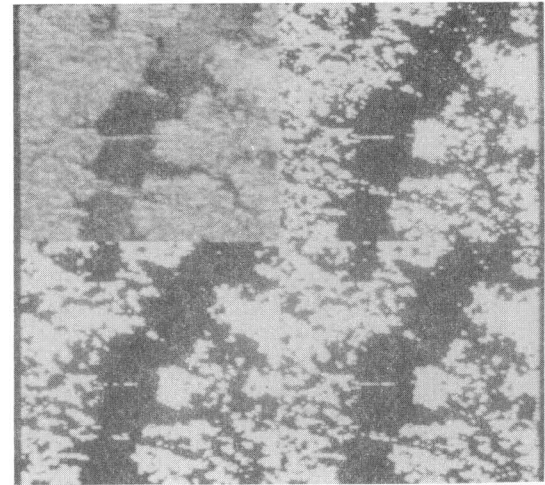


(c)

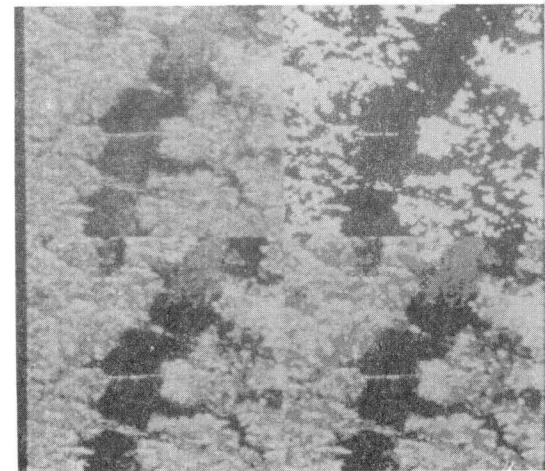
Figure 6. (a) SAR image of Chesapeake Bay, (b) 128x128 section from (a), its 2 level segmentations with  $\beta=0.3, 0.5, 0.8, (*)$ , (c) 128x128 section from (a), its 2, 4, 8 level segmentations with  $\beta=0.5, 0.4, 0.2.(*)$



(a)

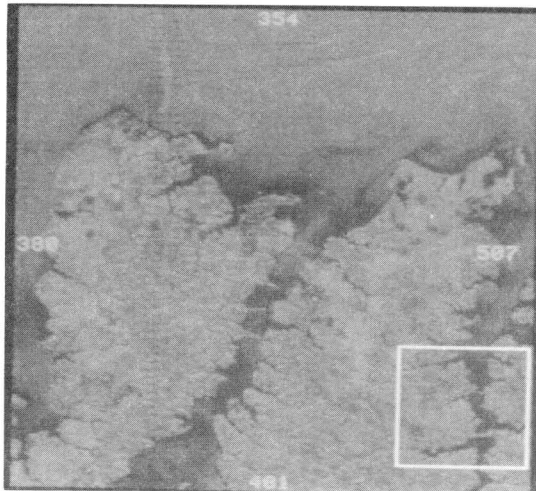


(b)

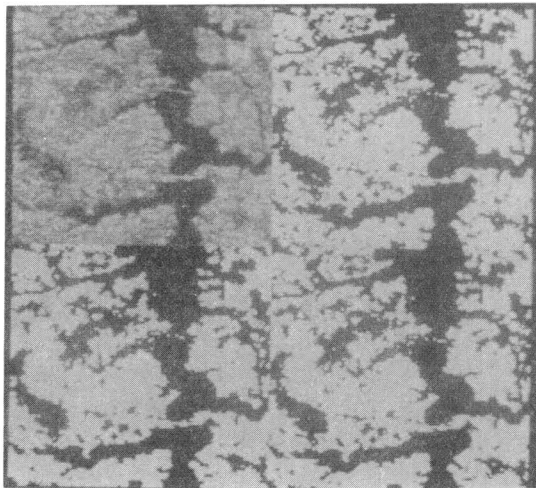


(c)

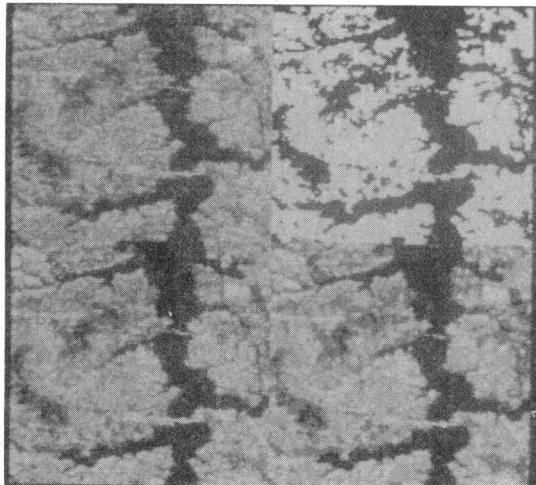
Figure 7. (a) SAR image of Chesapeake Bay, (b) 128x128 section from (a), its 2 level segmentations with  $\beta=0.1, 0.15, 0.2,(*)$ , (c) 128x128 section from (a), its 2, 4, 8 level segmentations with  $\beta=0.15.(*)$



(a)



(b)



(c)

Figure 8. (a) SAR image of Chesapeake Bay, (b) 128x128 section from (a), its 2 level segmentations with  $\beta=0.1, 0.14, 0.16, (*)$ , (c) 128x128 section from (a), its 2, 4, 8 level segmentations with  $\beta=0.14, (*)$

distinct levels. If two levels are very close in value, then segmentation into these two levels is not carried out. For example, in Fig. 6.c, the 4 level segmentation yields 3 distinct levels and the 8 level segmentation yields 5 distinct levels.

Since noise-free scene is not available, it is not possible to assess the goodness of the segmentations presented here. But our experimentations with test images indicate that the algorithm yields segmentations that are quite accurate even for low SNR images.

#### V. CONCLUDING REMARKS

This paper presents a new approach to segmentation of noisy images, in particular SAR images. It uses the GD to model spatial continuity or clustering properties of regions. The algorithm is recursive in nature, requires a single pass over the data, and works well at low signal to noise ratios. The extension of the algorithm to the problem of segmentation of textured images is already implemented and the preliminary results are promising.

Areas that require future work are the parameter estimation problem in GD and development of GD models to classes of images, which are of interest.

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