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DIRECTION DEPENDENT INTERPOLATION OF AEROMAGNETIC DATA

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I. ABSTRACT

The paper describes a method for level correction and interpolation of anisotropically sampled aeromagnetic data. The level-correction is based on piece-wise comparison of adjacent flight-lines. The interpolation scheme is based on cubic splines and preserves thin linear structures in anisotropic potential field data. The design of such a method is an important building stone for the design of an image-based information system for geologic applications.

II. INTRODUCTION

In recent years large efforts have been made in the remote sensing community to develop geographic information systems as a tool in monitoring and managing environmental and natural resources. In these systems an image data base is generally supplemented with data that exist in other forms, e.g., census data, data over properties of objects in the images and so on.

In the search for new sources of nonrenewable resources satellite images alone have shown to be of little value. More often than not important information is obscured by vegetation, soil and other factors. In order to overcome these problems it is necessary to complement satellite images with geophysical measurements, either from airborne or from surface surveys.

Geophysical information over an area is, due to economical and practical reasons, anisotropic in it's nature, i.e., it exists with closely spaced data-points on widely separated datalines. A typical n*m grid cell may have a n/m ratio between 5-10; in extreme cases the n/m ratio may rise to 200 or even higher. Quadratic or close to quadratic grid cells are favourable in many applications - this holds true in particular for the integration of geophysical data into an image-based information system.

This paper deals therefore with a new method for direction dependent interpolation, cubic spline interpolation carried out in the direction of minimum variance, i.e., in geological terms, the direction of strike.

Fortunately nearly all geological features that can be detected with geophysical methods have a distinct strike within a limited area. In the absence of linear properties, however, the interpolation becomes trivial. Since interpolations is carried out in the direction of strike, it will enhance the visibility of linear features. The effect will be somewhat similar to that of contour drawing and the interpolation scheme may therefore be used for tnat purpose only.

The interpolation procedure described is designed for equidistantly spaced anisotropic data and is not suitable for irregularly distributed observations.

The initial discussion in this paper is centered around the problems of artifacts generated by interpolation. These effects are demonstrated by applying different commonly used interpolation methods to a simple artificial image. In the latter part the results of interpolation applied to magnetic data from an airborne survey in northern Sweden are shown.

The performance of the interpolation algorithm presented here is, among other things, dependent on the quality of the original data, i.e., large differences in potential field values of adjacent lines are interpreted as directional features and can therefore give rise to a substantial amount of noise in the interpolated picture.

Airborne geophysical measurements often exhibit significant differences in the mean level of adjacent flightlines due to uncontrolled drift of the measuring equipment or operator error. In section VI of this paper an algorithm for mean level correction of adjacent flightlines is discussed.

The mathematical treatment of cubic and bi-cubic splines and their application for interpolation of geophysical data is already available in the literature (De Boor, 1962 and Bhattacharyya, 1969). The method used in this work for the generation of cubic splines is described in section IV.

III. SAMPLING AND RECONSTRUCTION

Let F(x,y) denote a continuous, infinite-dimension data field representing the intensity of some physical parameter. Spatial samples would then be obtained by multiplying the data field by a sampling function composed of an infinite array of Dirac delta functions arranged in a grid of spacing (n,m). Let S(x,y) denote the sampled field, if m and n are chosen small enough, i.e., smaller than one-half period of the finest detail within the original field. It is then possible to exactly reconstruct F(x,y) by filtering S(x,y) with an appropriate filter (Pratt, 1978).

However, this ideal sampling model is not valid for geophysical measurements; the sampling array will be of finite extent, the sampling pulses will be of finite width and data may not be sampled at the Nyquist rate.

To avoid violation of the sampling theoreme in a field survey n and m must be chosen such, that the shortest anomaly period is longer than 2*n and 2*m in the x and y direction respectively, or a lowpass filter must be applied. In a ground survey with magnetic or gravimetric methods, this is impossible for economical and practical reasons.

In the case of airborne measurements the situation is better, since a lowpass filter is introduced. There are two contributors to this filter:
(a) the altitude of the airplane
(b) the integration time of the instrument.

With a n/m ratio of 5 or even higher it will still be difficult to reconstruct the datafield in the n direction. One way to solve this is to calculate the average over a number of values and let the average represent the value of an enlarged n*n grid cell. The drawback of this method is the loss of information along the data lines. The method may be used for some applications, e.g., large scale map production and lowpass filtering of noisy data.

Another approach to the problem is to reconstruct parts of the original datafield properties using a more or less sophisticated interpolation procedure.

IV. INTERPOLATION

Interpolation has been aptly described as "reading between the lines of a data table". It is essentially the process of replacing a function f() whose value is given at only a finite number of points, by a continuous function F() which is equal or very nearly equal to f() at those points. A simple example of an interpolating function is the formula for linear interpolation. This function approximates the graph of f() between two consecutive points by the chord joining these points. If the interval between the data points is small enough, then the error resulting from this method will be negligible. In the case of potential field data from an airborne survey, linear interpolation is suitable along the flightlines but not perpendicular to the lines.

In order to construct an interpolation method for undersampled data, it is essential to use existing a priori information on the properties of the original potential field, e.g., continuity of the derivatives, linearity etc. It is also necessary to know in beforehand, what features should be maintained with a minimum of distortion. For magnetic and gravimetric data these features are: shape and resolution of individual anomalies, magnitude and location of maximas and minimas and the continuity and steepness of the field gradients. A very attractive interpolating function, which will meet most of these requirements, is the bi-cubic spline. The method for the generation of splines is described by De Boor, 1962. The interpolation scheme essentially generates piecewise cubic polynomials representing the field function in every interval between the points of observation. For this, it utilizes the continuity of the function and its first two successive derivatives at all data points. The strength of this scheme lies in the fact that the function is made continuous, in particular its slope and curvature. Due to the undersampling of data, however, bi-cubic spline interpolation has a tendency of breaking up thin elongated anomalies into shorter widened ones. This is a highly undesirable artifact especially in the magnetic case, where both diabases and faults may give rise to this type of anomaly.

To solve this problem without loss of all the attractive properties of the bi-cubic splines, an interpolation scheme which utilizes linearized spline interpolation (see Fig. 2) in the direction of minimum variance was designed. This interpolation method will copy the features of the surrounding lines with a small displacement due to the direction of strike. It will also make the function continuous in this direction (Fig. 1) as long as there is no change in strike.

The interpolation scheme is carried out as follows:

1. The strike direction is determined by one of the methods described in section VII.
2. A line is computed through the point to be interpolated and where it intercepts the flightlines. Notice, that the direction line will not necessarily intercept the flightlines at a data point. When necessary, data values at the point of intercept are computed by linear interpolation along the lines (Fig. 1).
3. Finally, the desired value is calculated by cubic spline interpolation. The interpolation function used is

   $$F(x) = C_0 + C_1x + C_2x^2 + C_3x^3.$$

   The constants C_0, C_1, C_2 and C_3 are computed from $F(x_1)$, $F(x_2)$, $F'(x_1)$ and $F'(x_2)$. $F'(x_1)$ and $F'(x_2)$ are determined in the simplest possible way

   $$(F'(x_C) = (F(x_{C+1})-F(x_{C-1})/(x_{C+1}-x_{C-1}))$$

   (Fig. 2).

It must be kept in mind that the values generated by interpolation of undersampled data are synthetic values and should be used with great care, especially for quantitative interpretation.

V. ARTIFACTS PRODUCED BY INTERPOLATION

The simple artificial image (Fig 3a) that is used to show the effects of interpolation, has the same anisotropic properties as real geophysical data from airborne measurements carried out by the Geological Survey of Sweden, i.e. a n/m ratio of five where the horizontal side of the grid cell is one length unit. This simple image, however, contains one of the most difficult features for computer interpolation, the diagonal line.

In Fig 3b the effects of a cubic spline polynomial applied perpendicularly to the data lines are shown. Notice, how the linear apperence of the diagonal line degenerates from a continuous line to a line of "islands" elongated in the direction of interpolation, where the peak value of each island represents the original datapoint. Also notice, how the line is widened and the edges are blurred. To generate the interpolated values in Fig 3c, an interpolation routine from the NAG Fortran library was used. This is a bi-cubic spline routine where four original data lines with eleven points/line are used to compute the interpolation polynomial. The mathematical background for the algorithm used in the NAG-routine E01ACF is described in detail in (Handscomb, 1966 and Hayes, 1970). As can be seen, there is a slight improvement from Fig. 3b, but the result is far from perfect due to the undersampling of the original image between the data lines.

To solve this problem, an interpolation routine that computes the direction of minimum variance was designed. The polynomial used for interpolation is the same as was used in Fig 3b, but now it is applied in the direction of minimum variance. The result from this routine is shown in Fig. 3d.

Figures 3b-3e must not be seen as a test of the accuracy of the demonstrated interpolation methods. They should merely be seen as examples of the artifacts produced in the process of reconstructing undersampled data, using different preset constraints. The only conclusion that can be drawn is that the continuity of the slope is not a suitable constraint for the reconstruction of thin lines.

VI. LEVEL CORRECTION

In particular in aeromagnetic measurements, the mean level of adjacent flightlines can exhibit large variations. These variations can be assumed to have their origin in faulty measurements based on the physical nature of the data. Potential fields seldom exhibit a behaviour of sudden change in overall intensity, but homogeneous transitions are expected. Furthermore, these sudden changes in overall intensity give rise to "false" structures and influence the result of the interpolation algorithm in an undesirable fashion.

The problem was remedied with the following procedure:

Consider two adjacent flightlines with number n and n+1. Each flightline is divided into a sufficiently large number of intervals of equal size. For each of the intervals the mean value of the field and the associated variance are computed.

Two adjacent flightlines are then compared in the following fashion:

The 15 intervals with the highest variance are deleted. The mean values of the remaining parts are then compared with each other. Of these, the half with the largest differences are discarded. From the rest an additive correction is computed that is applied to flightline n+1. The procedure is then repeated for lines n+1 and n+2 and so forth until all lines have been corrected. A weakness of the method is that the first line has to be considered as reference line and all other lines are subsequently corrected with respect to that reference, while in reality line x has correct values.

The authors consider the errors introduced by the particular choice of reference line negligible.

VII. THE METHOD USED FOR STRIKE DETERMINATION

A number of geometrical different masks were tested for the determination of strike. The mask finally used consists of 3 data lines and 9 datapoints/line (Fig. 4). The mask is rotated from -54.5 to +54.5 degrees in 15 steps where point P on line LIN is center of rotation. The sum of the variances for the 9 lines within the mask is computed for each step. The direction with the minimum variance sum is then the direction of strike at point P on line LIN.

Using this direction, however, will divide the interpolated image into blocks as can be seen in Fig. 6d. To avoid this highly undersirable effect the direction values for LIN-1 and LIN+1 are computed prior to interpolation and the average direction over three data lines in the direction of strike at point P is used. This is the method used to interpolate Fig. 3d.

VIII. RESULTS

To illustrate the effects of different interpolation methods and different strike determining routines on real values, data from aeromagnetic measurements made by SGU (Geological Survey of Sweden) were chosen. The flightlines are oriented E-W with the lines 200 m apart and measured with a sampling rate of one data point every 40 meters.

The area is suitable for tests of interpolation methods since it contains rock types with varying magnetic and geometric properties. This can be seen on the original magnetic map (Fig. 5a). Notice especially the diagonal lines represented by the layered intermediate to acid volcanic rocks in the NW part of the map.

All parts of Fig. 5 are produced on a bi-level display device using integer dither technique. This technique, unfortunately, produces some smoothing effects. Therefore the differences in level between single flightlines cannot be seen easily.

Fig. 5a shows the original magnetic map without any type of interpolation or correction. Fig. 5b is interpolated with a simple cubic spline polynomial applied perpendicular to the flightlines. As is expected from the results of interpolation of the syntetic image in Fig. 3b, all thin linear features that are not oriented in the direction of interpolation degenerate to lines of islands and their edges are blurred. Fig. 5c shows the result of the NAG-library bi-cubic spline interpolation routine. Notice the severe artifacts produced in the western part of the magnetic map.

Fig. 5d and 5e present the results of the direction dependent methods. It can be seen in Fig. 5d how this routine tends to divide the inhomogeneous magnetic area in the NE part of the map into blocks with boundaries with a 45 degree strike.

IX. CONCLUSIONS AND DISCUSSION

It is evident from the examples shown above, that, even if it is impossible to reconstruct a data field which is undersampled in one direction, it is possible to reconstruct and enhance certain properties of that field by the use of an appropriate filter.

The drawbacks of such filters are, that they will introduce artifacts, e.g., the blocks produced by the strike dependent interpolation methods. This happens in areas where no distinct strike exists, i.e., objects are only "visible" on one dataline due to the undersampling. A possible way to minimize these artifacts may be to scan the original data with masks of varying geometry in a neighbourhood of the value to be interpolated. This procedure is described by Nagao et. al., 1980, who used the method for an edge preserving smoothing filter.

If the scanning is carried out in a proper way, it should be possible to classify the area and then apply the interpolation method that will give the best result for that specific class.

The danger with such sophisticated interpolation methods is, that it will be extremly difficult to discriminate the artifacts produced by interpolation from real physical phenomena. This in turn may lead to serious misinterpretations. It is important, therefore, that the user of interpolated data is familiar with the artifacts that may be produced by interpolation.

In spite of these drawbacks, however, a properly applied reconstruction filter is essential for the detection of subtle features both for manual and machine interpretation. It is also clear that for the interpolation of thin linear structures it is essential to determine their strike.

The authors do not claim the methods for strike determination described in this paper to be optimal. They should be seen as examples of the possibilities of strike dependent reconstruction filters.

An alternative method is used by Geologinen Tutkimuslaitos (Geological Survey of Finland). Instead of calculating the strike between flightlines, they have two or three magnetometers mounted in the airplane, wingtips and tail (J. Korhonen, private communication). This set up eliminates the need for computation of the strike, since the horizontal derivatives are given by this procedure.

X. ACKNOWLEDGEMENTS

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XI. REFERENCES

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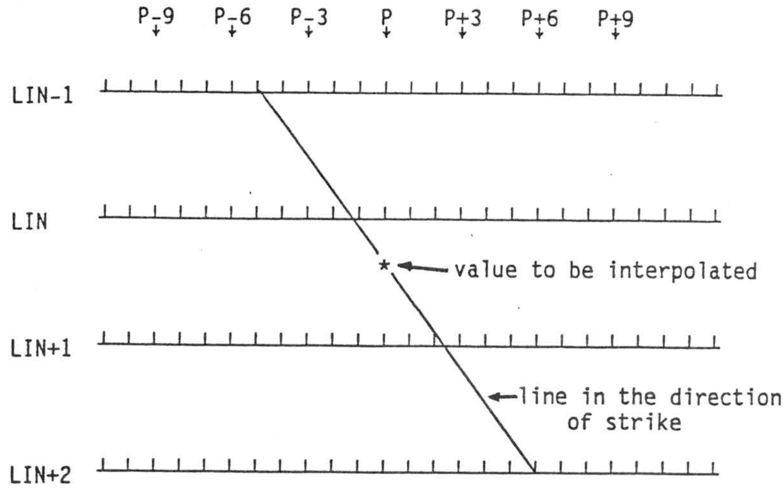

Figure 1. The setup for interpolation in the strike direction. Notice that four lines are used for the interpolation independent of the method used to determine the strike.

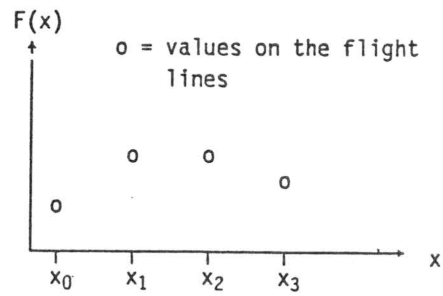

Figure 2. The set up for interpolation of F(x) for $x_1 < x < x_2$.

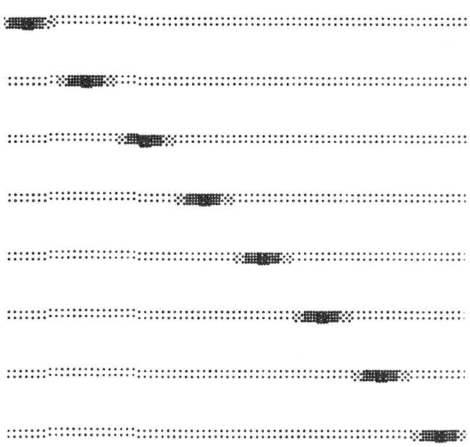

Figure 3a. The original image.

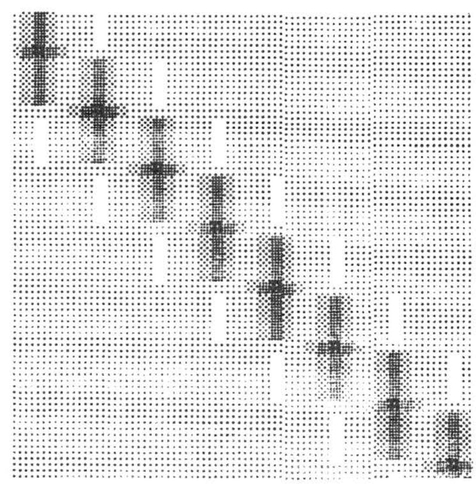

Figure 3b. Cubic spline interpolation perpendiculary to the data lines.

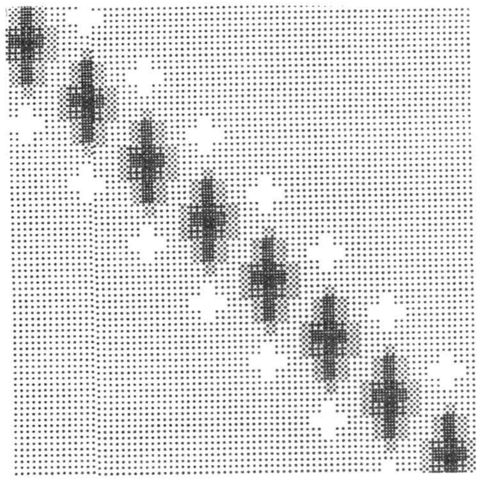

Figure 3c. Bi-cubic spline interpolation NAG-lib.

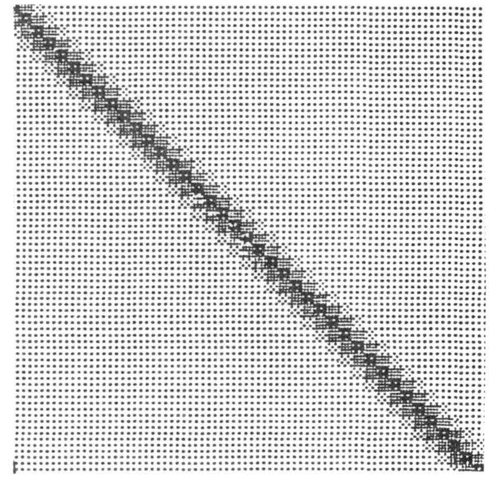

Figure 3d. Cubic spline interpolation in the direction of minimum variation.

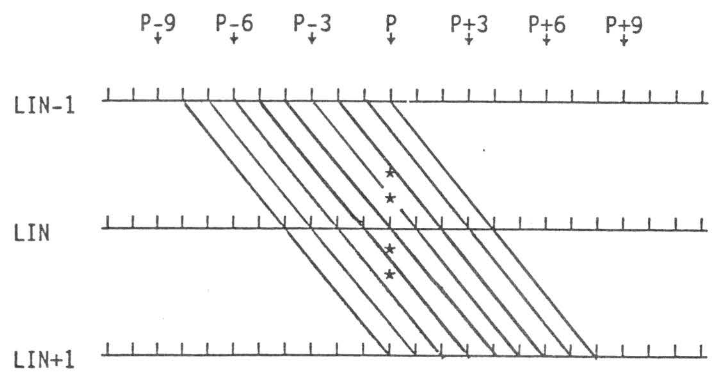

Figure 4. * = values to be interpolated.

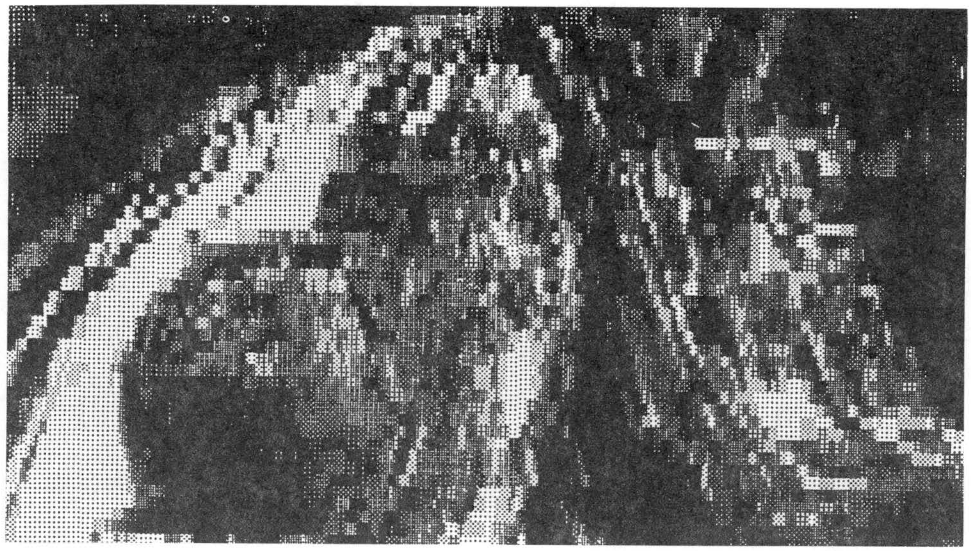

Figure 5a. Original magnetic data (light = high values).

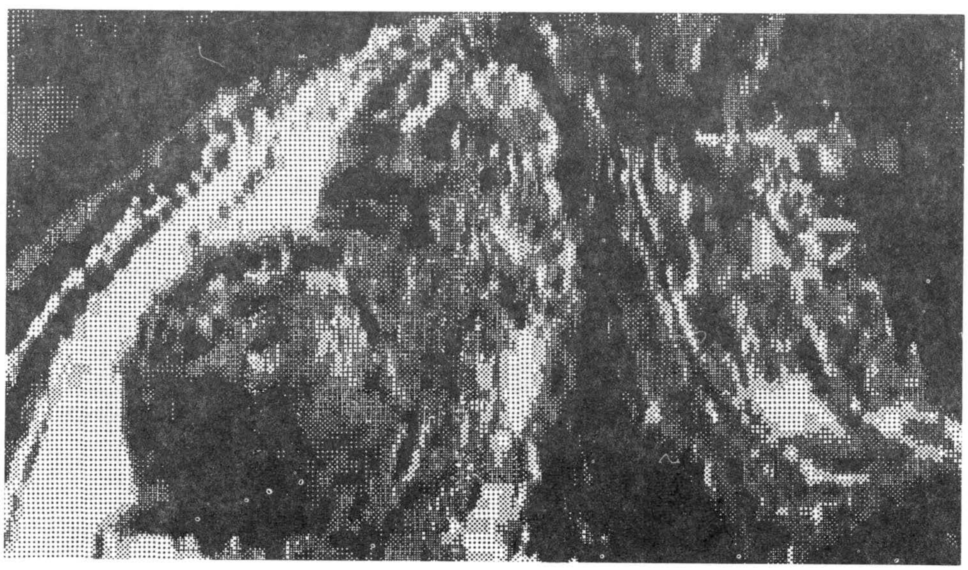

Figure 5b. Interpolated with cubic spline perpendiculary to the flight lines.

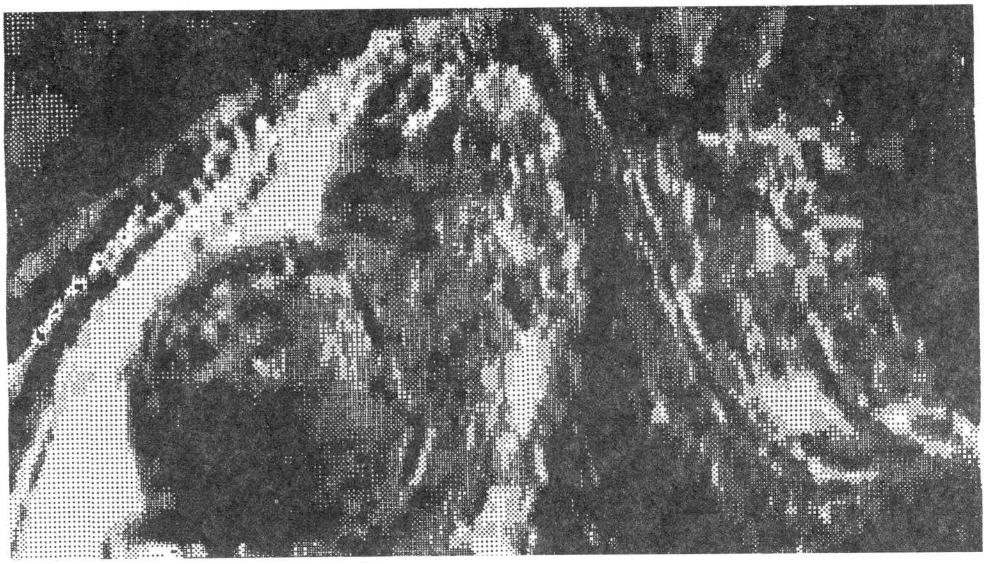

Figure 5c. Interpolated with NAG-lib bicubic spline.

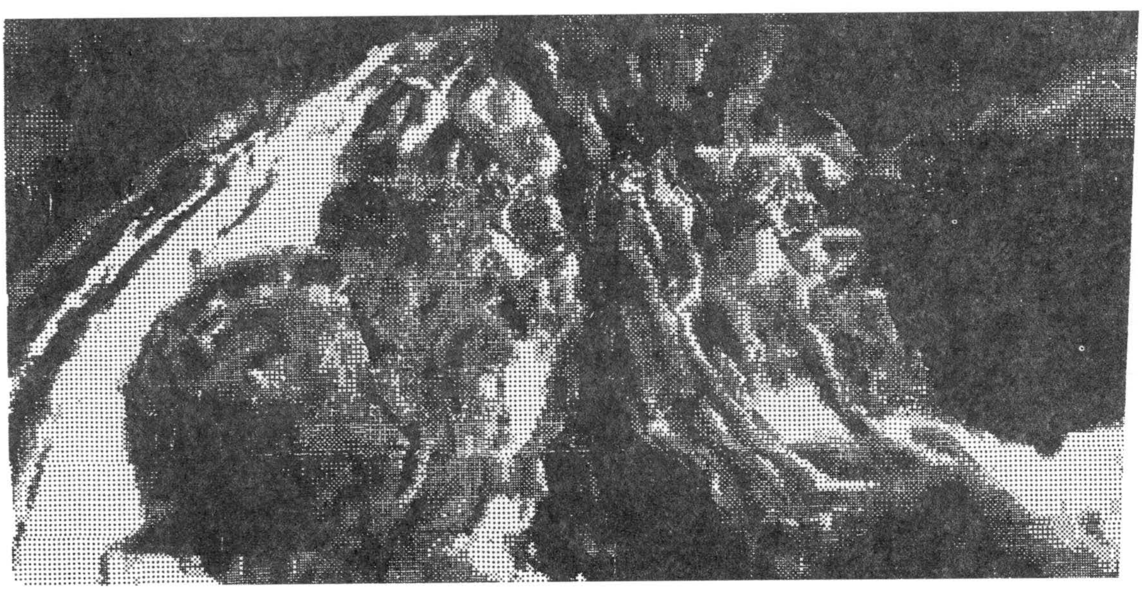

Figure 5d. Interpolated with direction dependent cubic spline, no average of the direction of strike.

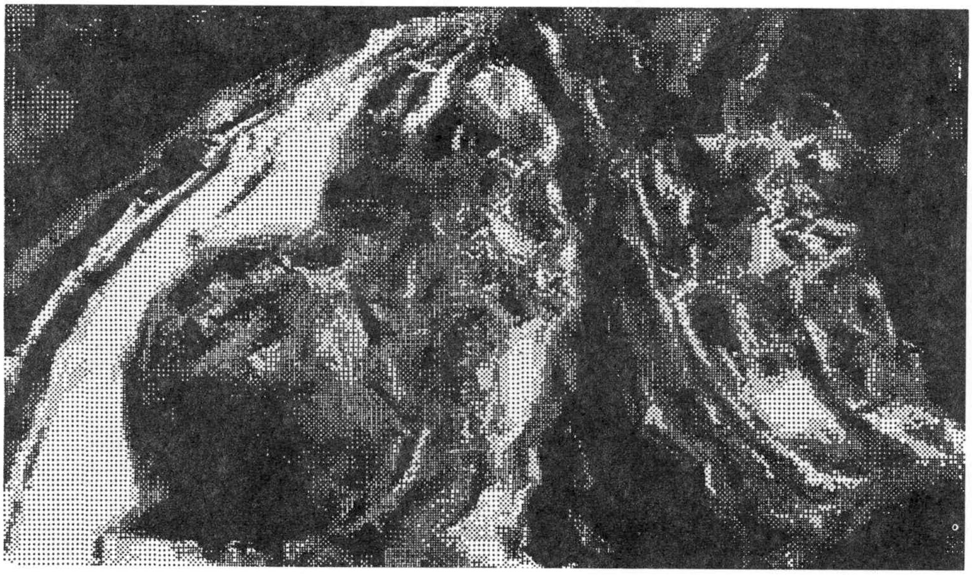

Figure 5e. Interpolated with direction dependent cubic spline, utilizing the average strike over three lines.

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