Evaluation of Projection Errors using Commercial Satellite Imagery

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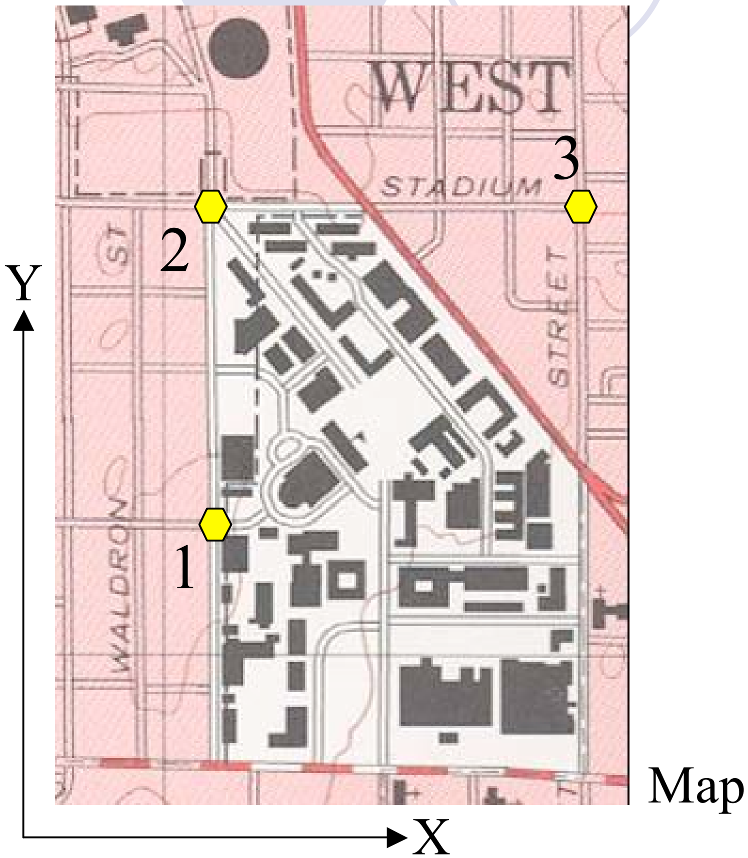
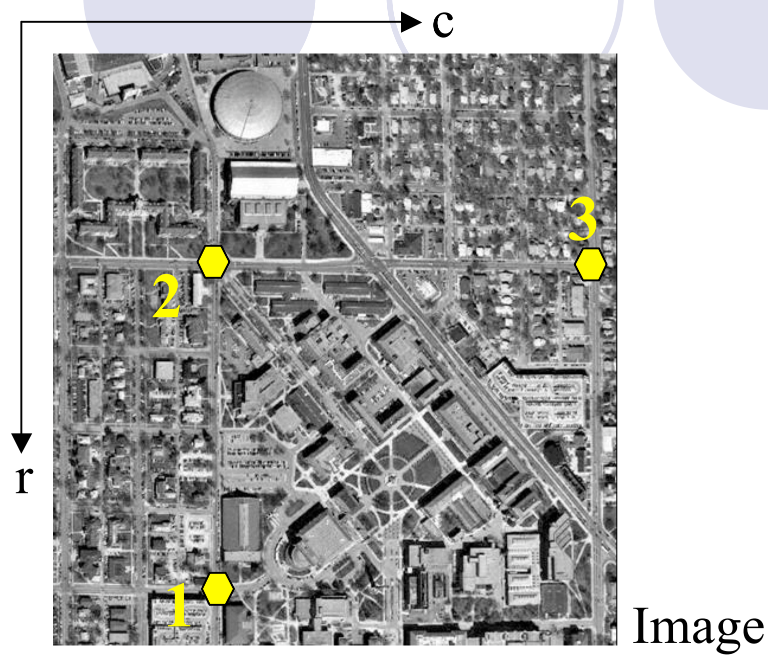
Outline of Presentation

- Traditional Mapping Polynomial, Image Warping, Rubber Sheeting Approach
- Description of Photogrammetry Approach: Physical Model and Replacement Model
- Example of Physical Model
- Example of Replacement Model
- Evaluation of Projection Errors Using Vendor Supplied Replacement Model
- Conclusions

Mapping Polynomials or Rubber Sheeting





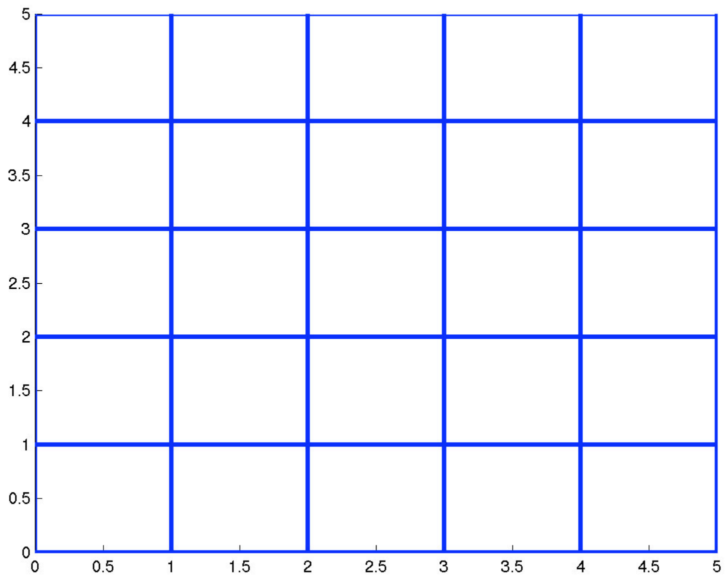
For each point we create two equations. We need at least as many equations as unkowns. If more, then we use least squares. It is like a regression problem: linear, easy. But we are confounding the effects of sensor, platform motion, and terrain relief. What should be the order of the polynomial ?

$$r = a_0 + a_1X + a_2Y + a_3XY + a_4X^2 + a_5Y^2$$

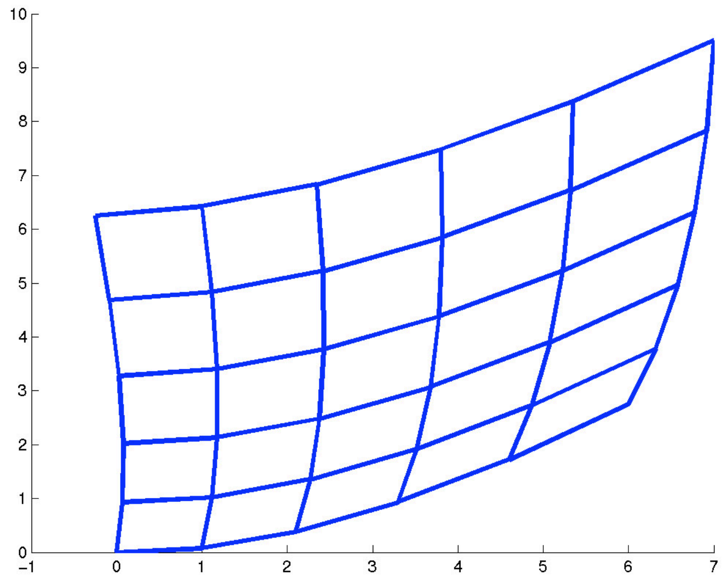
$$c = b_0 + b_1X + b_2Y + b_3XY + b_4X^2 + b_5Y^2$$

Graphical View of Rubber Sheet Transformation (2^{nd} order, 12-parameter)

Reference grid



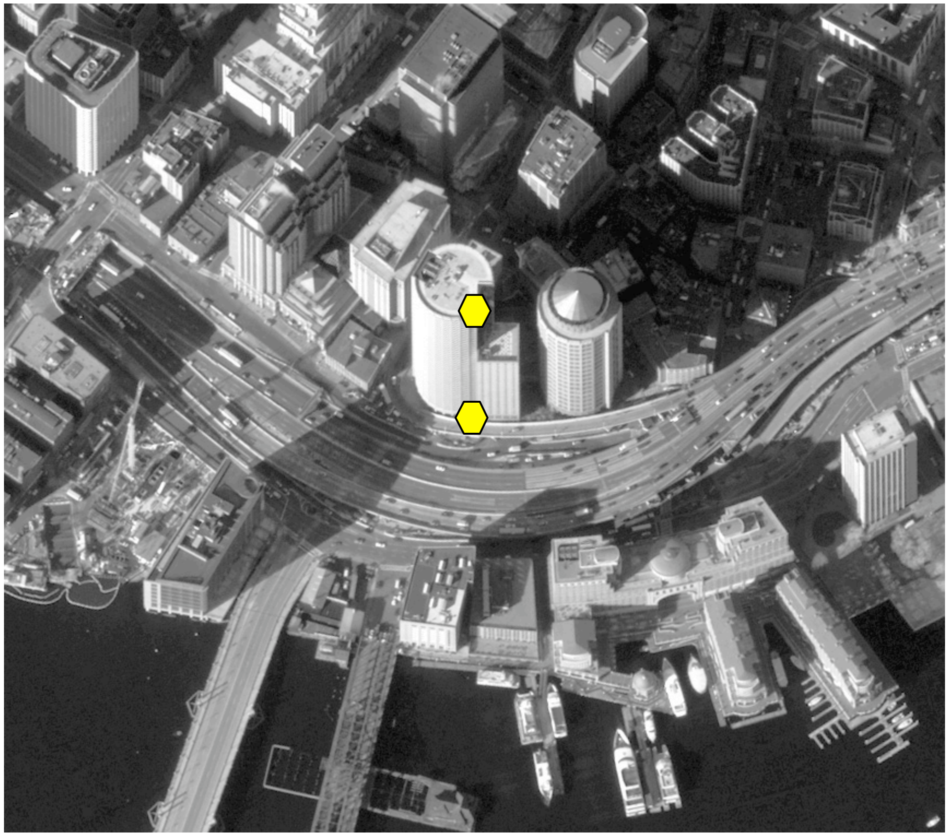
Transformed grid



Mapping Polynomials or Rubber Sheeting

If the terrain is flat, the sensor has narrow field of view, the sensor is nadir looking, and the ground sample distance is large, then *you can get reasonable results using the approach of mapping polynomials.*

The accompanying Quickbird image (0.61m pixel) shows the pitfalls of mapping polynomials when the above conditions do not apply. The two marked points have the same XY and they would get mapped into the same (row, col), but clearly that is wrong. You could expand the polynomial by adding some Z-terms. But that would not work. *Modeling the actual physical imaging process is the only way.*



Photogrammetric Approach to Image Geometry

Physical Sensor and Platform Models

•Sensor (Camera): aperture size, focal length, scanning elements, optical distortion

•Platform : orbit parameters: altitude, velocity, etc., orientation / attitude, rates and accelerations

OR

Use **Directly**

Use **Indirectly** through Rational Polynomials

Use physical model **Directly**, vendor or manufacturer defines the generic model: equations and constants. For a particular image, numerical values come from

- **Vendor:** support data supplied with image, metadata, ephemeris data, etc.
- **User:** obtain numerical values using ground control points

Advantage: parameters have physical meaning, flexibility

Disadvantage: vendor may not want to share, each one different

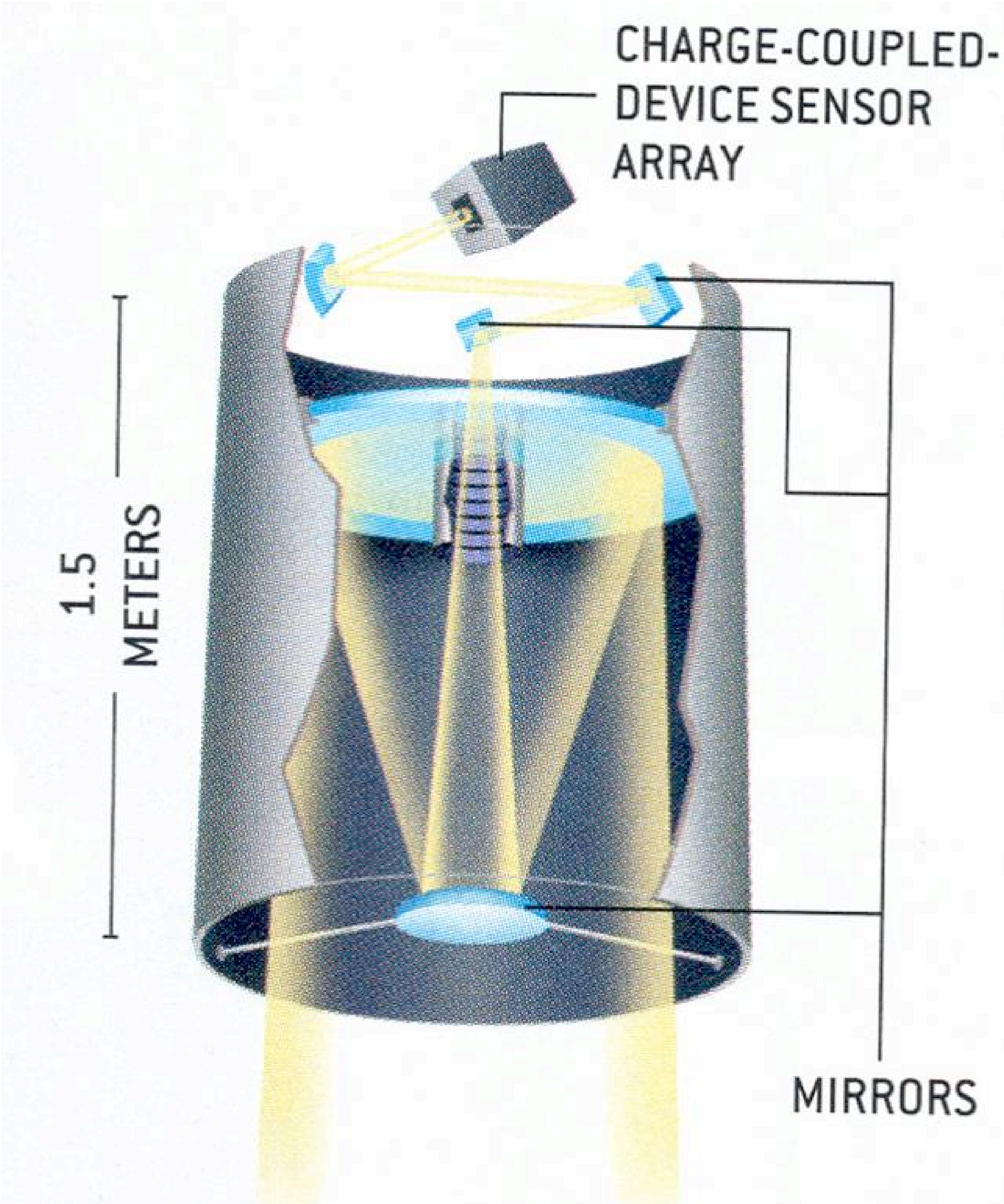
Use physical model **Indirectly** via Rational Polynomial Coefficients (Replacement). For a particular image, numerical values come from

- **Vendor:** for certain products, vendor supplies 80 term RPC
- **User:** can obtain via regression using a dense 3D grid and corresponding image points, based on physical model

Advantage: same parameters for all sensors, easy for software applications

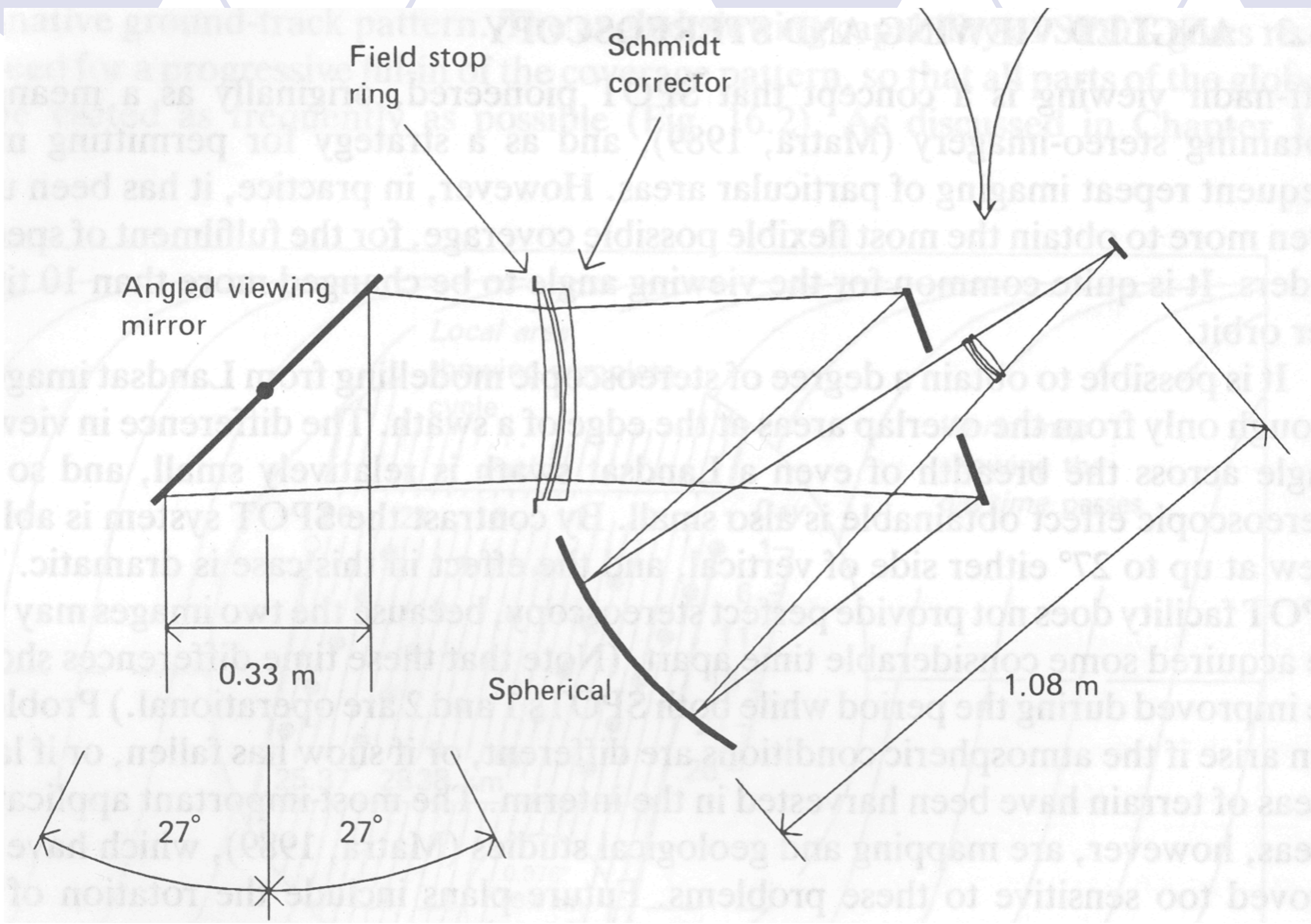
Disadvantage: no physical meaning to the coefficients

Physical Model



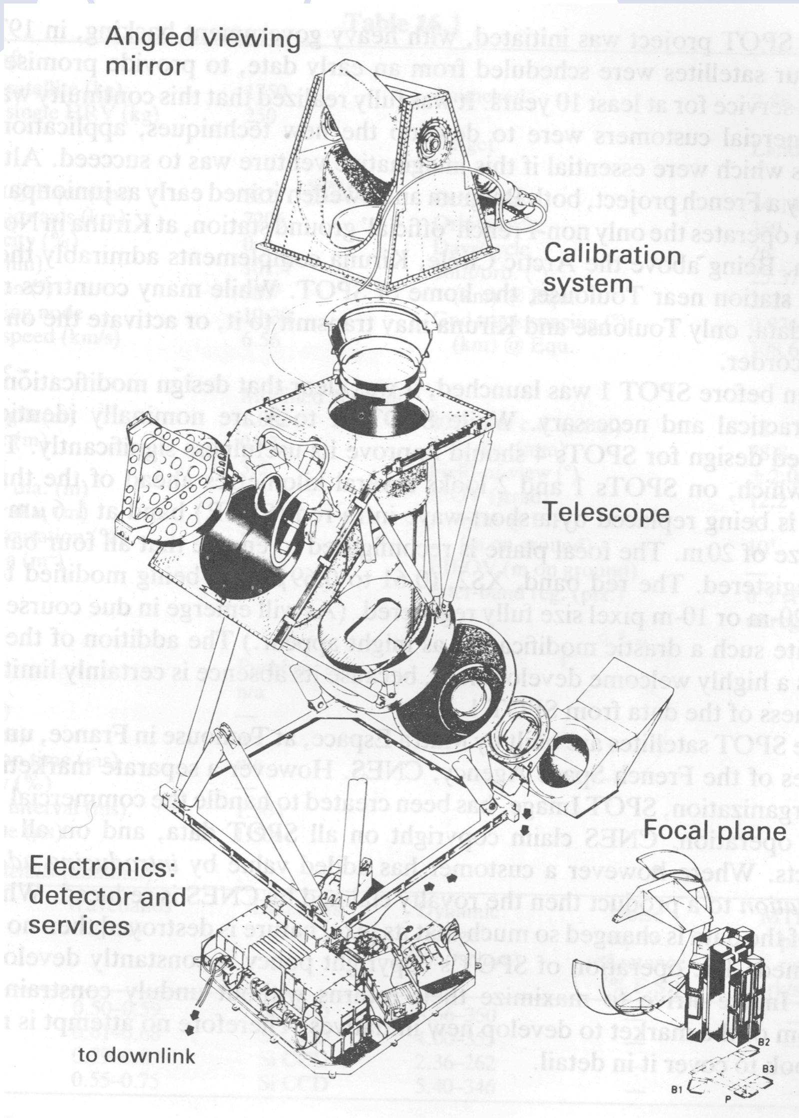
Schematic of telescope optics layout for modern remote sensing camera

Schematic of Spot Optics



From Pease, Satellite Imaging Instruments

Cutaway Drawing of Spot Sensor



Dimensional stability is very important to maintain good focus. The structural tubes are made from carbon fiber material with a small negative thermal expansion coefficient. The titanium fittings have a positive thermal expansion coefficient that *just cancels* the tubes. (That is good engineering!)

From Pease, Satellite Imaging Instruments

Attitude Sensing

Quickbird Assembly

Compare

Spot: 0.2 deg @ 820 km => 2870m

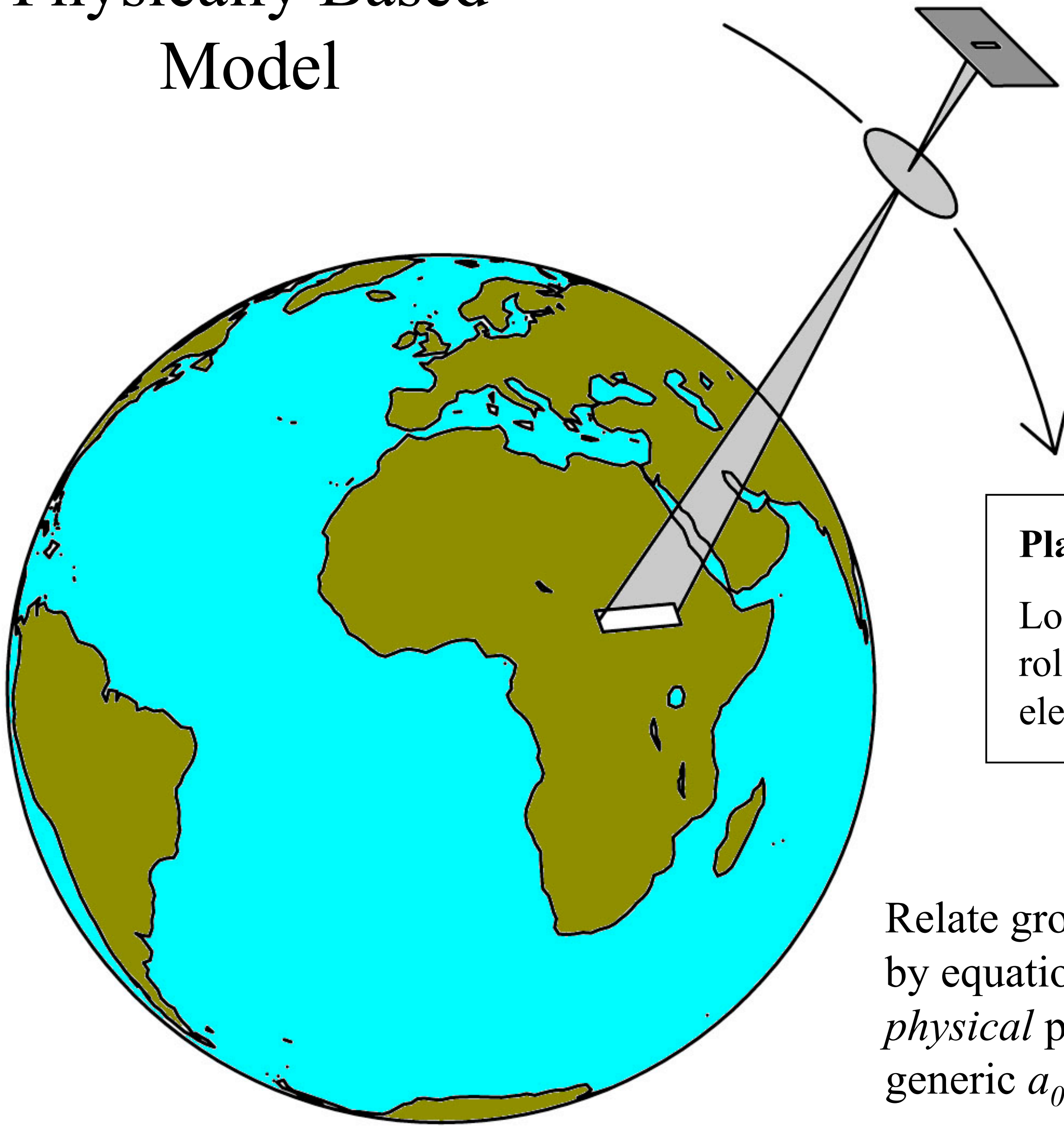
Quickbird: 3 sec @ 450 km => 7m

Star Tracking
Camera CT-601
from Ball Aerospace

3 arc second
accuracy

Terrestrial Photograph of Orion

Physically Based Model



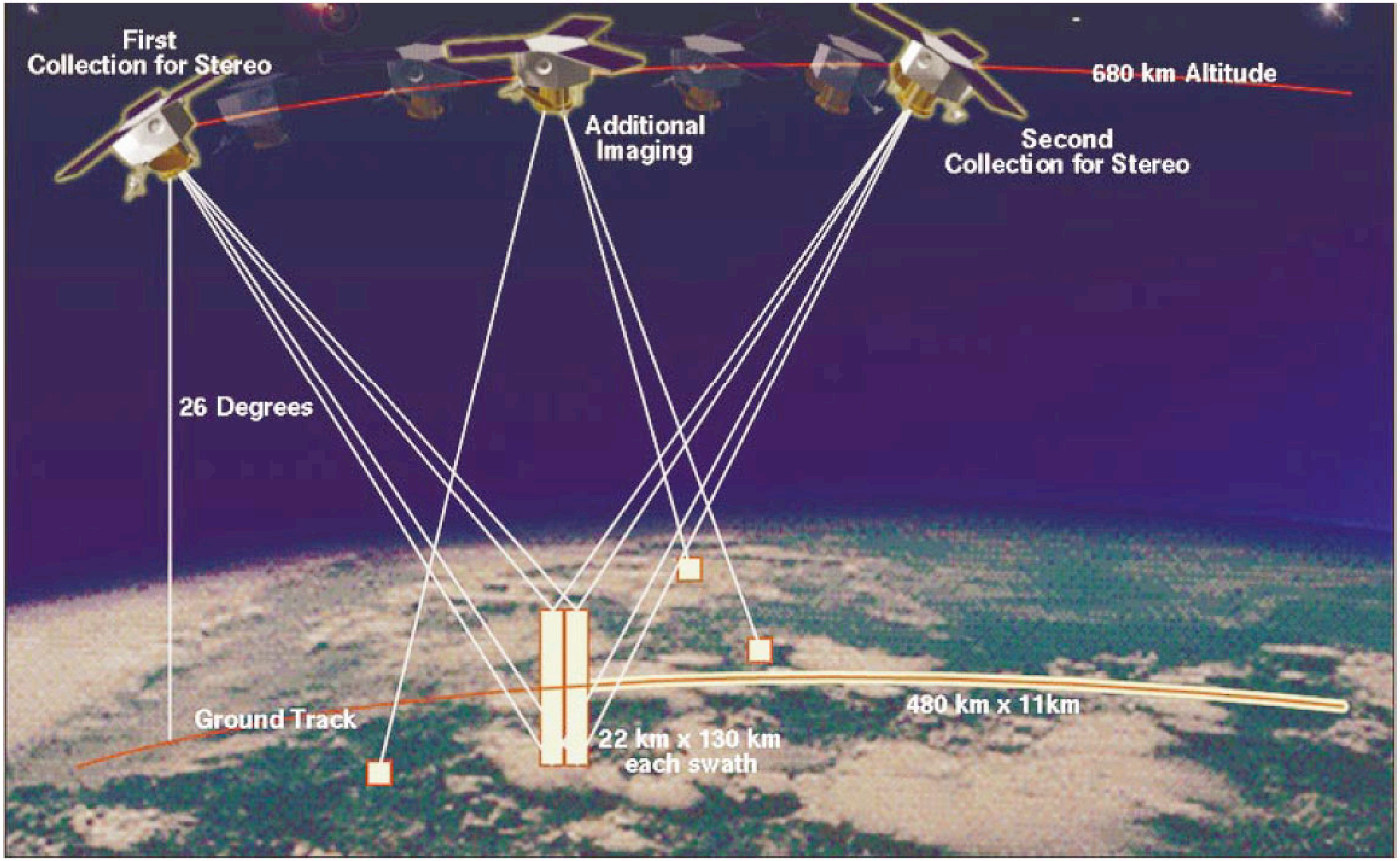
Sensor parameters:

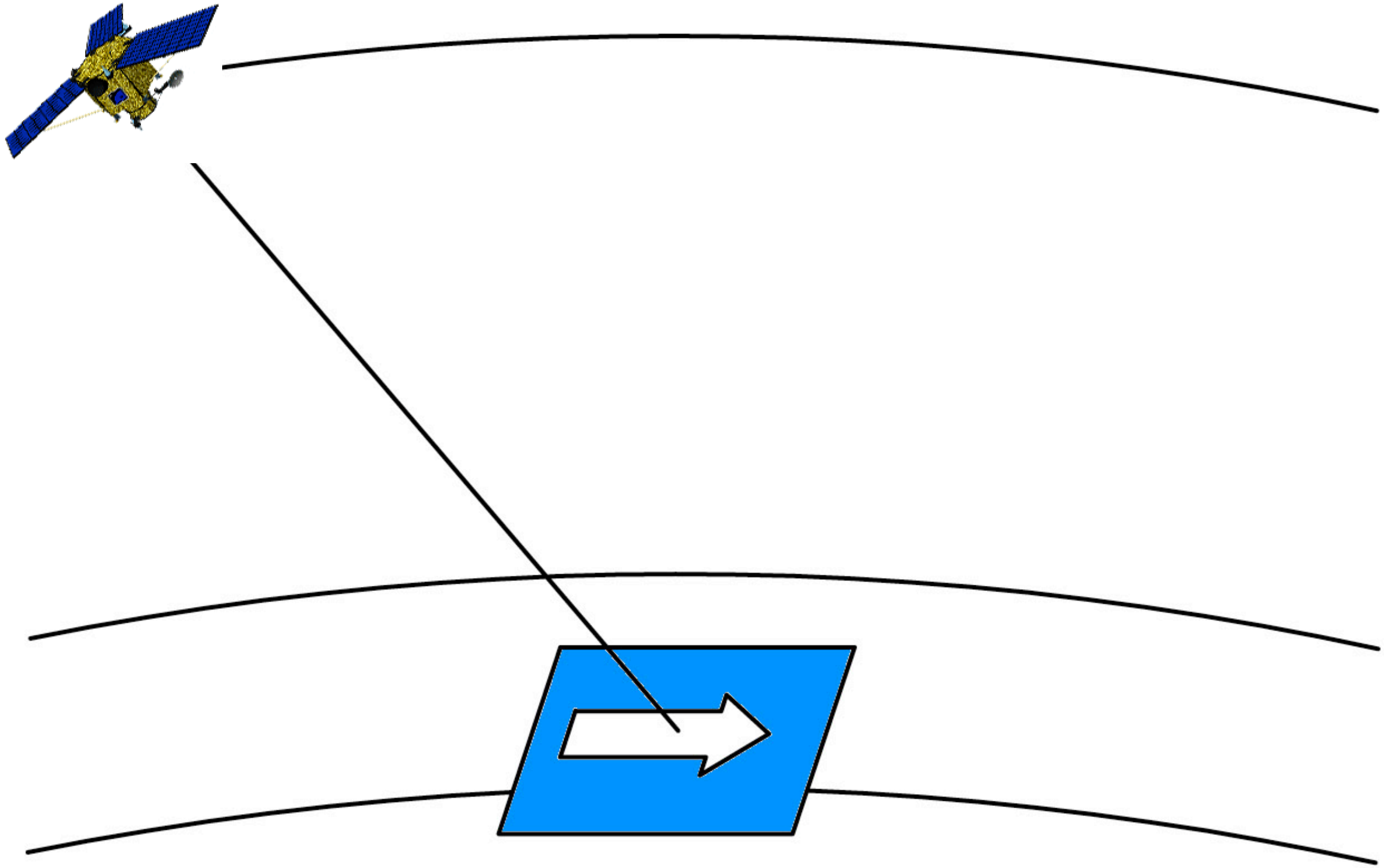
Focal length, principal point location, lens distortion, line rate, detector (pixel) size

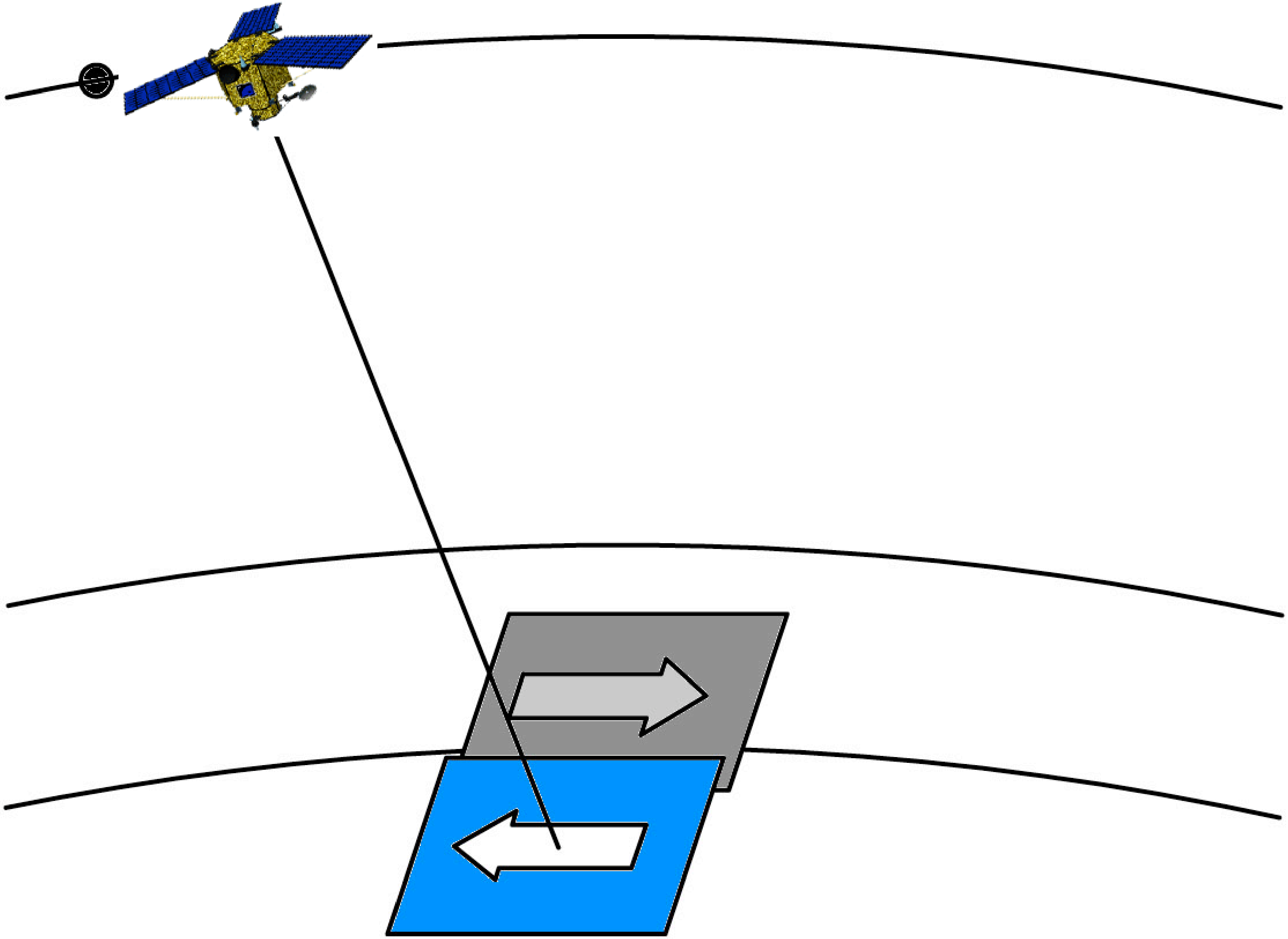
Platform parameters:

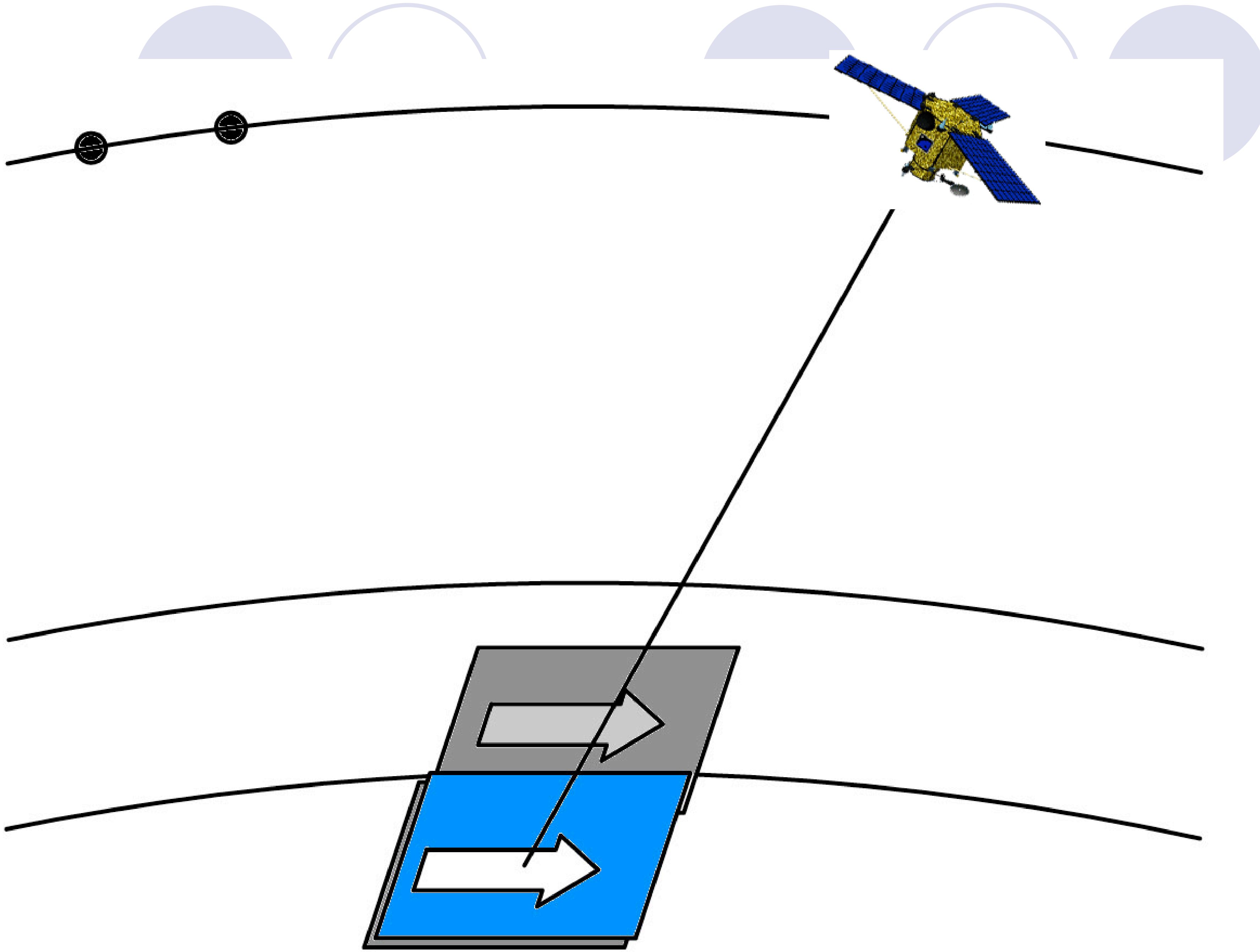
Location X,Y,Z, time, attitude roll, pitch, yaw, kepler orbit elements (*a,e,i,*W,w,n)

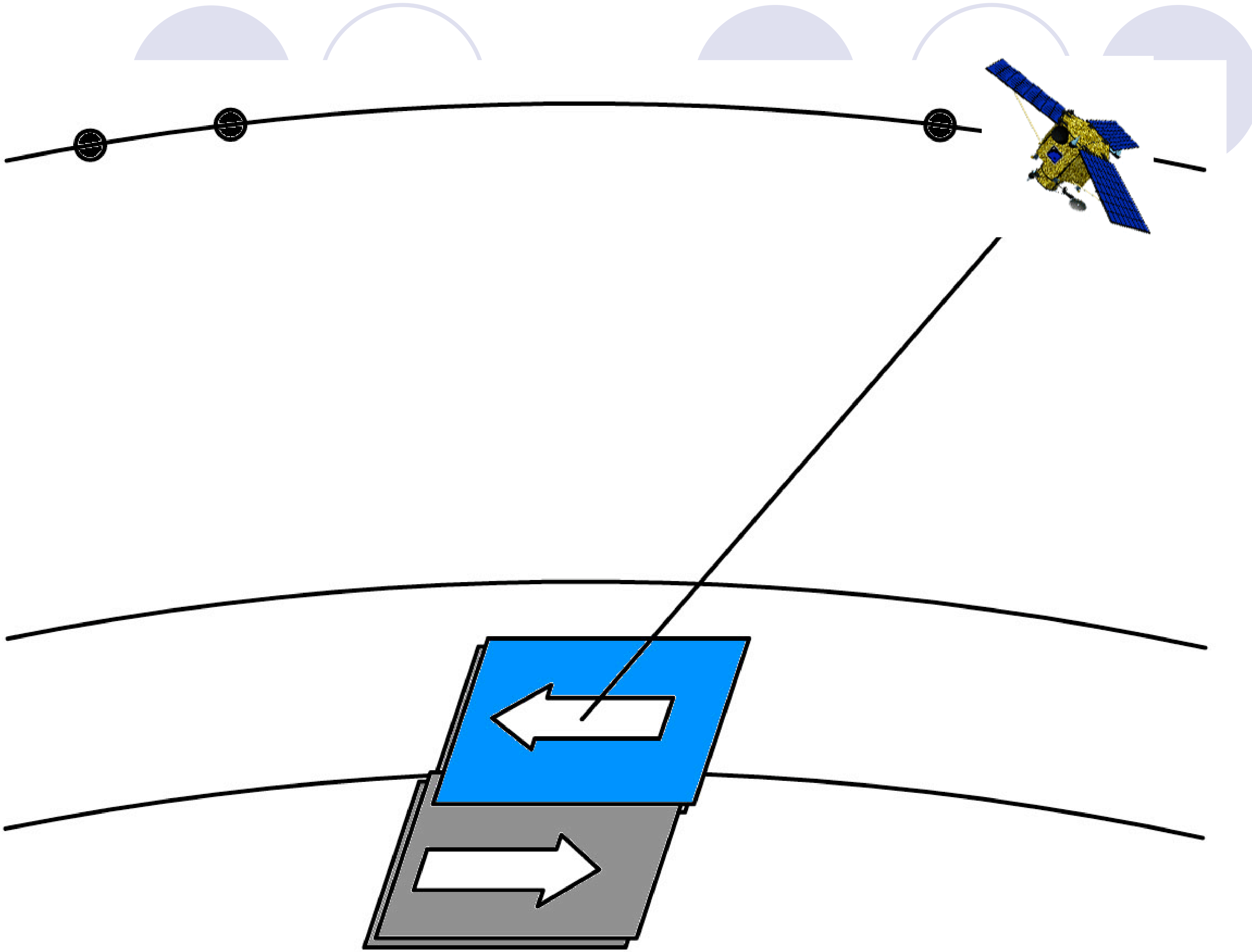
Relate ground point and image point by equations with the above *actual physical* parameters, rather than the generic a_0 , a_1 , a_2 , ... parameters.


First
Collection for Stereo
680 km Altitude
Additional
Imaging
Second
Collection for Stereo
26 Degrees
Ground Track
480 km x 11km
22 km x 130 km
each swath



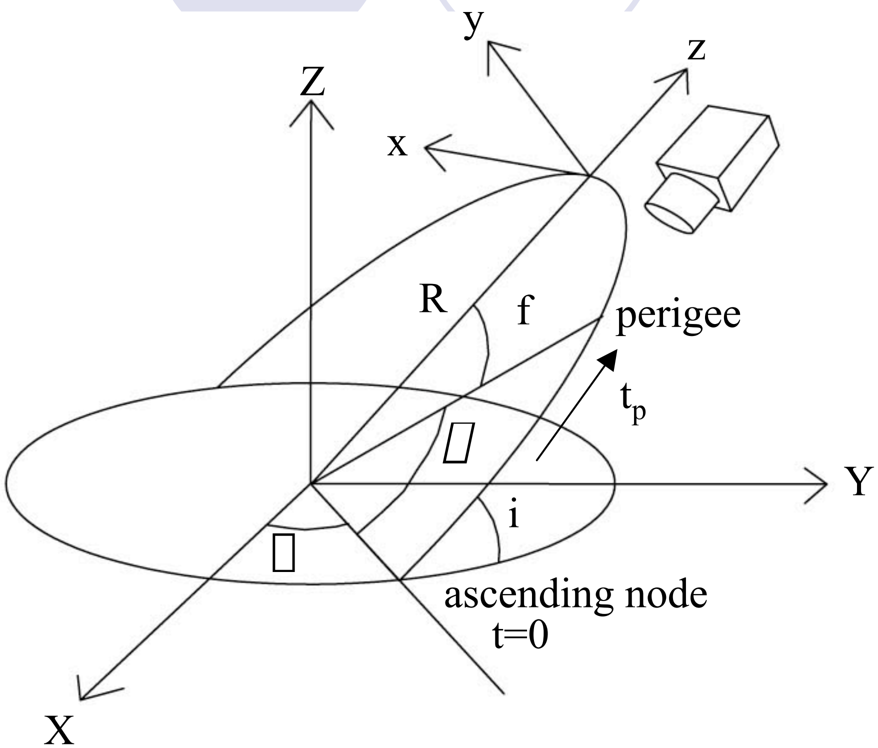






Development of the Condition Equations for a Space Based Pushbroom Camera (Using SPOT as an Example)

Development of SPOT Condition Equation – Good Model for Generic Pushbroom Camera from LEO



Must have approximations for

Ω, i, ω, a, e

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

t_f: time at frame center, in header (metadata)

delta-t: delta-time from frame center, equals 0.001504 sec * line,

$$t = t_f + \Delta t$$

orbit period, $\tau = 2\pi\sqrt{\frac{a^3}{GMe}}$

GMe = 398600.5E09 m^3/s^2

$a_s = r_e + alt_s$,6378137m + 822000m = 7200137m

$$\tau = 2\pi\sqrt{\frac{7200137^3}{398600.5E09}}$$

τ = 6080.259 min

τ = 101.338 sec

t_p: time from ascending node to perigee

Condition Equation cont'd.

$$t_p = \frac{\tau}{\pi}\tan^{-1}\left[\frac{\sqrt{1-e}}{\sqrt{1+e}}\tan(\omega/2)\right] - \frac{\tau}{2\pi}\frac{e\sqrt{1-e^2}\sin\omega}{1+e\cos\omega}$$

$$\Delta t_p = t - t_p$$

$$M_n = \frac{2\pi\Delta t_p}{\tau}, \text{ mean anomoly}$$

$$E = e\sin E + M_n, \text{ (kepler equation, E : eccentric anomaly)}$$

solve iteratively for E

$$R_s = a(1 - e\cos E) \text{ ; vector from earth center to satellite}$$

$$\begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix} = \mathbf{M_b}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

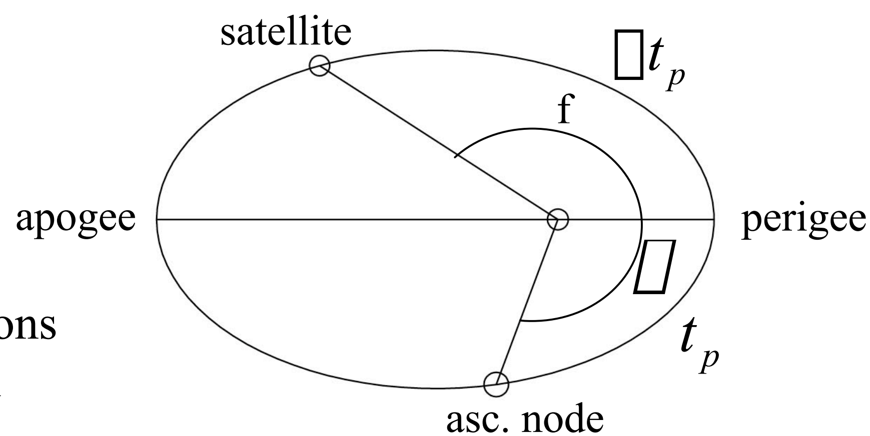
Construct $\mathbf{M_b}$ from 3 sequential rotations applied to XYZ (ECEF) to bring them parallel to xyz (instantaneous satellite system)

f: true anomaly

$$\sin f = \frac{\sqrt{1-e^2}\sin E}{1 - e\cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e\cos E}$$

$$f = \tan^{-1}\left(\frac{\sin f}{\cos f}\right)$$



Condition Equation, cont'd.

The XYZ obtained in this way will be only approximately correct and we must allow for refinements, modeled as second order polynomials of time:

$$\square X = \square X_0 + \square X_1 \square t + \square X_2 \square t^2$$

$$\square Y = \square Y_0 + \square Y_1 \square t + \square Y_2 \square t^2$$

$$\square Z = \square Z_0 + \square Z_1 \square t + dZ_2 \square t^2$$

Likewise the attitude (orientation) produced by the prior rotation matrix will be only approximately correct and we must allow for refinements to the attitude, again modeled as second order polynomials of time:

$$\square\square = \square\square_0 + \square\square_1 \square t + \square\square_2 \square t^2$$

$$\square\square = \square\square_0 + \square\square_1 \square t + \square\square_2 \square t^2$$

$$\square\square = \square\square_0 + \square\square_1 \square t + \square\square_2 \square t^2$$

Condition Equation, cont'd.

We put these small refinement rotations into matrix as follows:

$$\mathbf{M_a} = \mathbf{M}_{\square\square}\mathbf{M}_{\square\square}\mathbf{M}_{\square\square}$$

We must also account for a tilt or inclination of the camera. In the case of SPOT this is a cross track tilt (+/- 27 degrees) about the x (motion) axis, implemented by a stationary (but moveable) mirror:

$$\mathbf{M_t} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\square & \sin\square \\ 0 & \square\sin\square & \cos\square \end{bmatrix}$$

In the case of an *agile* spacecraft such as IKONOS or Quickbird, this pointing can be any arbitrary cross-track, in-track, or spin attitude, and thus requires 3 rotations:

$$\mathbf{M_t} = \mathbf{M_z}(\square)\mathbf{M_y}(\square)\mathbf{M_x}(\square)$$

Note that we are over parameterized with rotations here. You cannot carry all as unknowns. But it may be convenient to separate in this way to make if clear which physical effect the parameter refers to.

Collecting all of this into the collinearity condition equation:

$$\begin{bmatrix} 0 \\ y \\ -f \end{bmatrix} = \lambda \mathbf{M_t} \mathbf{M_a} \mathbf{M_b} \left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \left(\mathbf{M_b^T} \begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \right) \right)$$

Image space info

Scale

Rotations

Ground point (ECEF)

Instantaneous exposure station for this line (nominal part from the time and a small correction)

Combine terms, eliminate scale

$$\begin{bmatrix} 0 \\ y \\ -f \end{bmatrix} = \lambda \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$0 = -f \frac{U}{W}$$

$$y = -f \frac{V}{W}$$

$$F_x = f \frac{U}{W}$$

$$F_y = y + f \frac{V}{W}$$

We can also add some other inner orientation parameters such as lens distortion, principal point offset, etc.

So how many parameters do we have? There are 5 groups,

- **Orbit parameters** $\square, i, \square, a, e, t_f$ (6)
- **Position corrections** $\square X_0, \square X_1, \square X_2, \square Y_0, \square Y_1, \square Y_2, \square Z_0, \square Z_1, \square Z_2$ (9)
- **Attitude corrections** $\square\square_0, \square\square_1, \square\square_2, \square\square_0, \square\square_1, \square\square_2, \square\square_0, \square\square_1, \square\square_2$ (9)
- **Pointing** $\square_t$ (1)
- **Inner orientation** x_0, y_0, f, k_1 (4)

Total here is 29, some will be held constant (maybe at zero), we may add some. Stochastic treatment is guided by redundancy, geometric strength of figure (parameters known to be highly correlated will probably not both be carried as unknowns), and by uncertainties

For SPOT we get an approximation of the off-nadir attitude from the angle readout of the mirror position. For Quickbird, we have the attitude described by quaternion elements, throughout the scene.

Depending on the source of information about ground control points, we may need to do some prior transformations such as,

$$\begin{bmatrix} X \\ Y \\ H \end{bmatrix} \rightarrow \begin{bmatrix} \phi \\ \lambda \\ h \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Map projection coordinates and orthometric (sea level) height

Geodetic coordinates and ellipsoid height, need info about geoid undulation

Geocentric, ECEF

Replacement Model

RPC Model

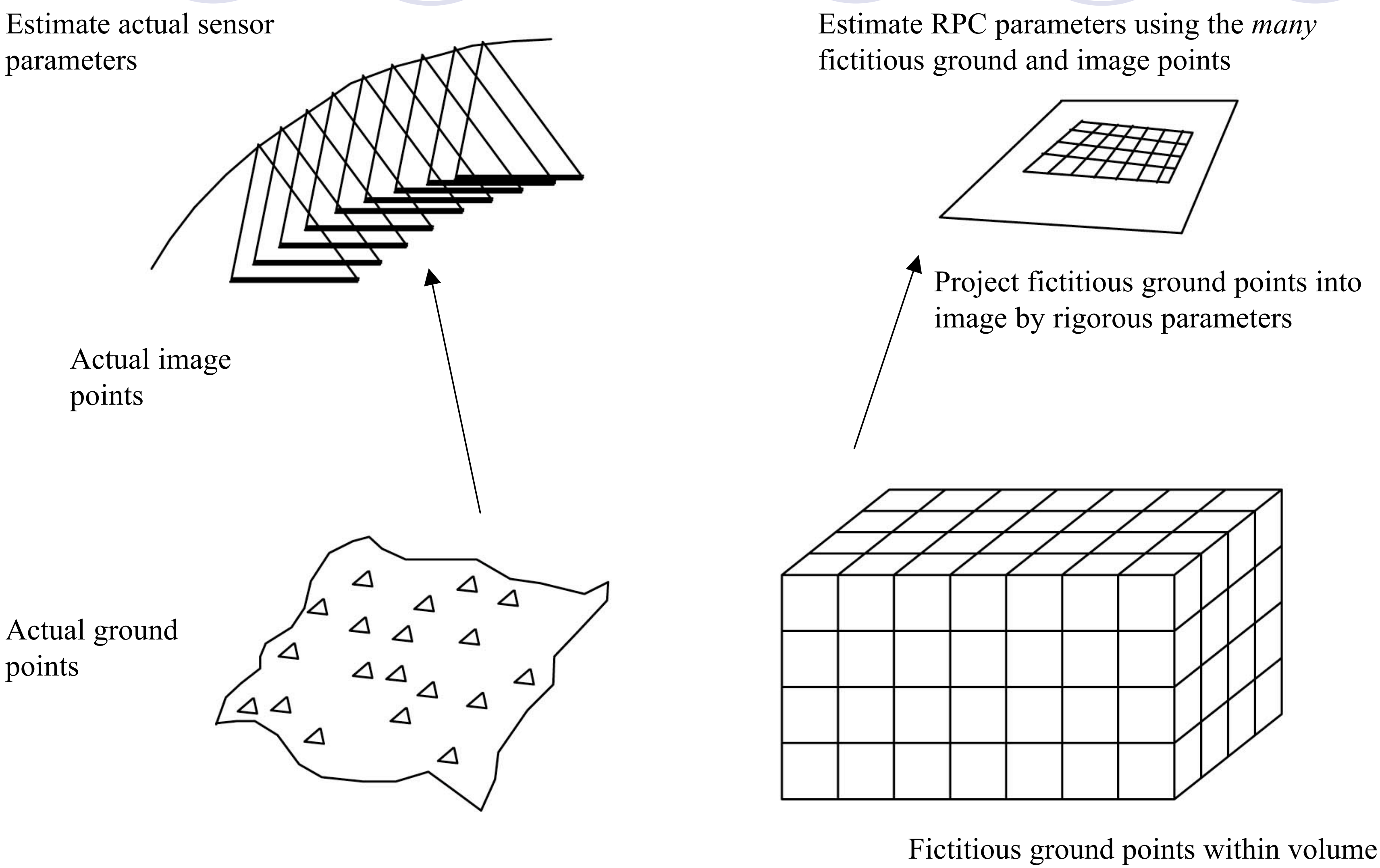
$$r = \frac{p1(X,Y,Z)}{p2(X,Y,Z)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} a_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{n1}\sum_{j=0}^{n2}\sum_{k=0}^{n3} b_{ijk} X^i Y^j Z^k}$$

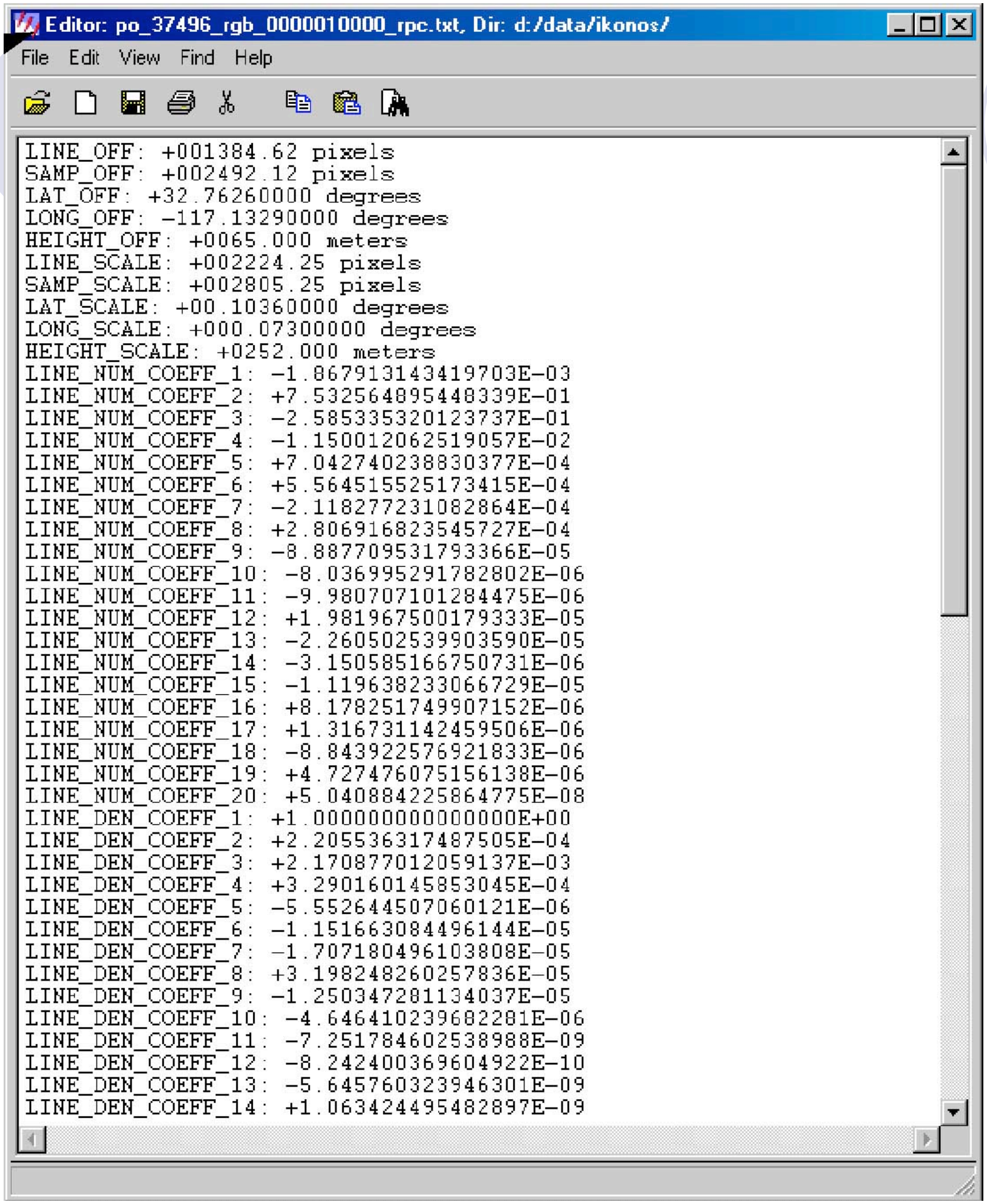
$$c = \frac{p3(X,Y,Z)}{p4(X,Y,Z)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} c_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{n1}\sum_{j=0}^{n2}\sum_{k=0}^{n3} d_{ijk} X^i Y^j Z^k}$$

For the third order model, only terms with i+j+k <= 3 are allowed. Those terms are shown below.

$$1, x, y, z, x^2, y^2, z^2, xy, xz, yz, x^2y, xy^2, x^2z, xz^2, y^2z, yz^2, x^3, y^3, z^3, xyz$$

Rigorous Sensor Model Parameter Estimation & RPC Parameter Estimation



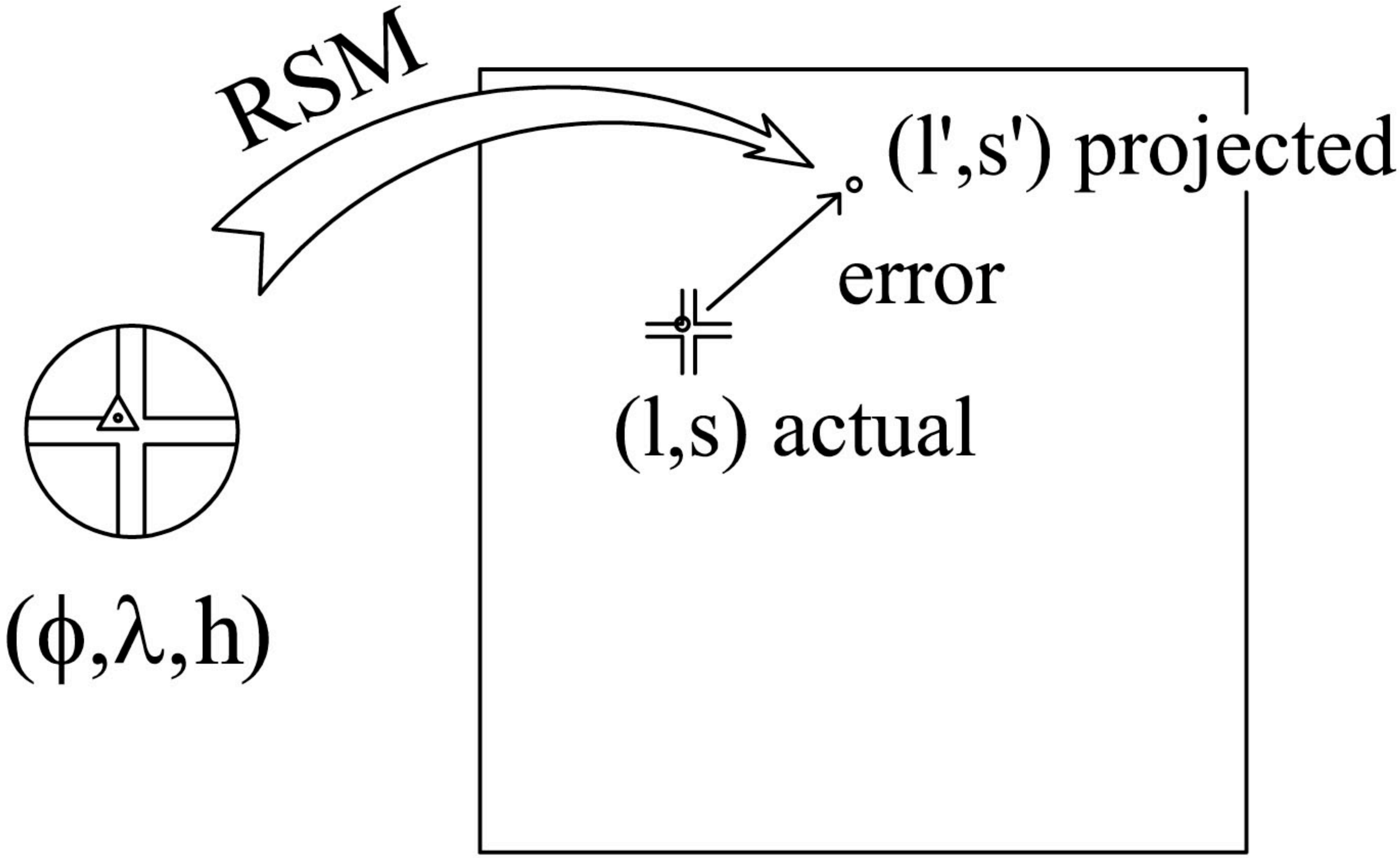



```
LINE_OFF: +001384.62 pixels
SAMP_OFF: +002492.12 pixels
LAT_OFF: +32.76260000 degrees
LONG_OFF: -117.13290000 degrees
HEIGHT_OFF: +0065.000 meters
LINE_SCALE: +002224.25 pixels
SAMP_SCALE: +002805.25 pixels
LAT_SCALE: +00.10360000 degrees
LONG_SCALE: +000.07300000 degrees
HEIGHT_SCALE: +0252.000 meters
LINE_NUM_COEFF_1: -1.867913143419703E-03
LINE_NUM_COEFF_2: +7.532564895448339E-01
LINE_NUM_COEFF_3: -2.585335320123737E-01
LINE_NUM_COEFF_4: -1.150012062519057E-02
LINE_NUM_COEFF_5: +7.042740238830377E-04
LINE_NUM_COEFF_6: +5.564515525173415E-04
LINE_NUM_COEFF_7: -2.118277231082864E-04
LINE_NUM_COEFF_8: +2.806916823545727E-04
LINE_NUM_COEFF_9: -8.887709531793366E-05
LINE_NUM_COEFF_10: -8.036995291782802E-06
LINE_NUM_COEFF_11: -9.980707101284475E-06
LINE_NUM_COEFF_12: +1.981967500179333E-05
LINE_NUM_COEFF_13: -2.260502539903590E-05
LINE_NUM_COEFF_14: -3.150585166750731E-06
LINE_NUM_COEFF_15: -1.119638233066729E-05
LINE_NUM_COEFF_16: +8.178251749907152E-06
LINE_NUM_COEFF_17: +1.316731142459506E-06
LINE_NUM_COEFF_18: -8.843922576921833E-06
LINE_NUM_COEFF_19: +4.727476075156138E-06
LINE_NUM_COEFF_20: +5.040884225864775E-08
LINE_DEN_COEFF_1: +1.000000000000000E+00
LINE_DEN_COEFF_2: +2.205536317487505E-04
LINE_DEN_COEFF_3: +2.170877012059137E-03
LINE_DEN_COEFF_4: +3.290160145853045E-04
LINE_DEN_COEFF_5: -5.552644507060121E-06
LINE_DEN_COEFF_6: -1.151663084496144E-05
LINE_DEN_COEFF_7: -1.707180496103808E-05
LINE_DEN_COEFF_8: +3.198248260257836E-05
LINE_DEN_COEFF_9: -1.250347281134037E-05
LINE_DEN_COEFF_10: -4.646410239682281E-06
LINE_DEN_COEFF_11: -7.251784602538988E-09
LINE_DEN_COEFF_12: -8.242400369604922E-10
LINE_DEN_COEFF_13: -5.645760323946301E-09
LINE_DEN_COEFF_14: +1.063424495482897E-09
```

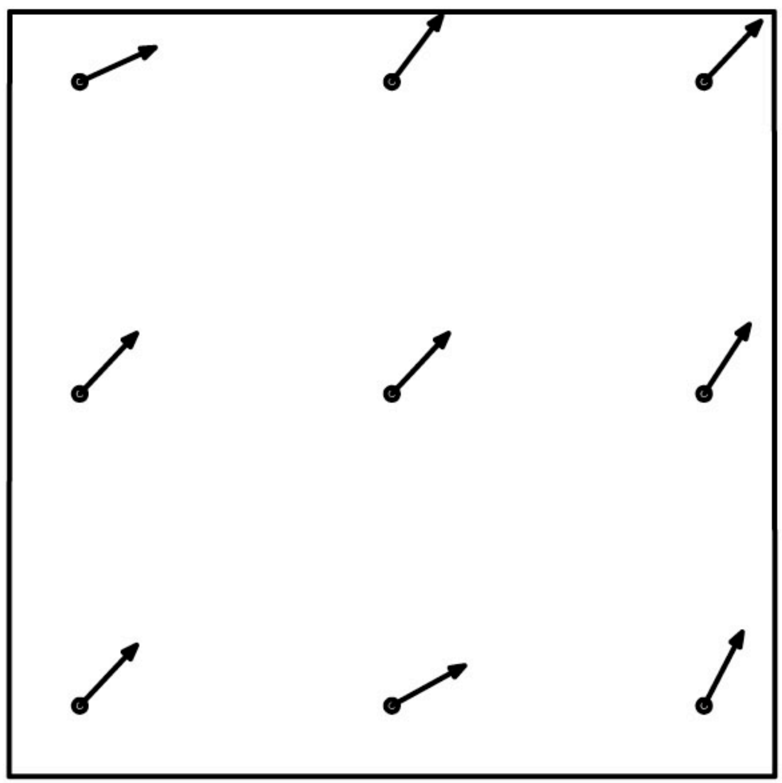
Erdas Imagine / Orthobase support for IKONOS RPC data – note the line_numerator coefficients go up to #20, this implies a 3rd order polynomial

Evaluation of Projection Errors Using Vendor Supplied RPC Rational Polynomial Coefficients

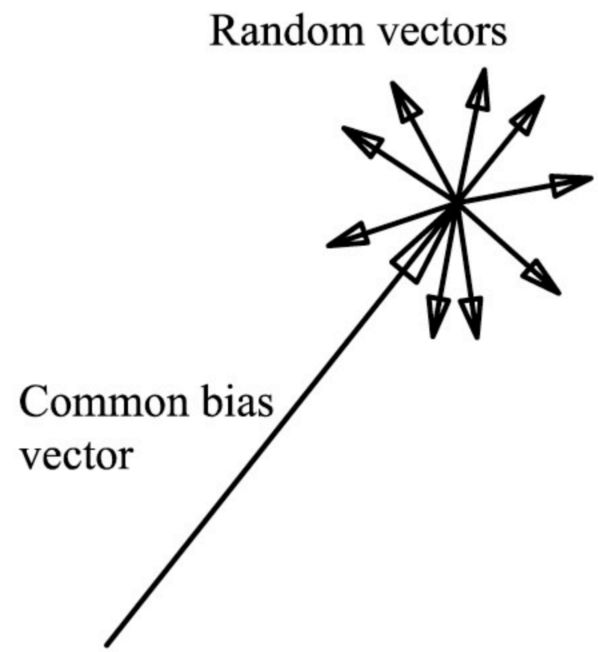
Control Point Projected to Incorrect Location in the Image – Suspect Similar Errors (magnitude & direction) Occur Everywhere in This Image



Error Pattern for This Image – Seems to be Common Bias plus Smaller Random Part

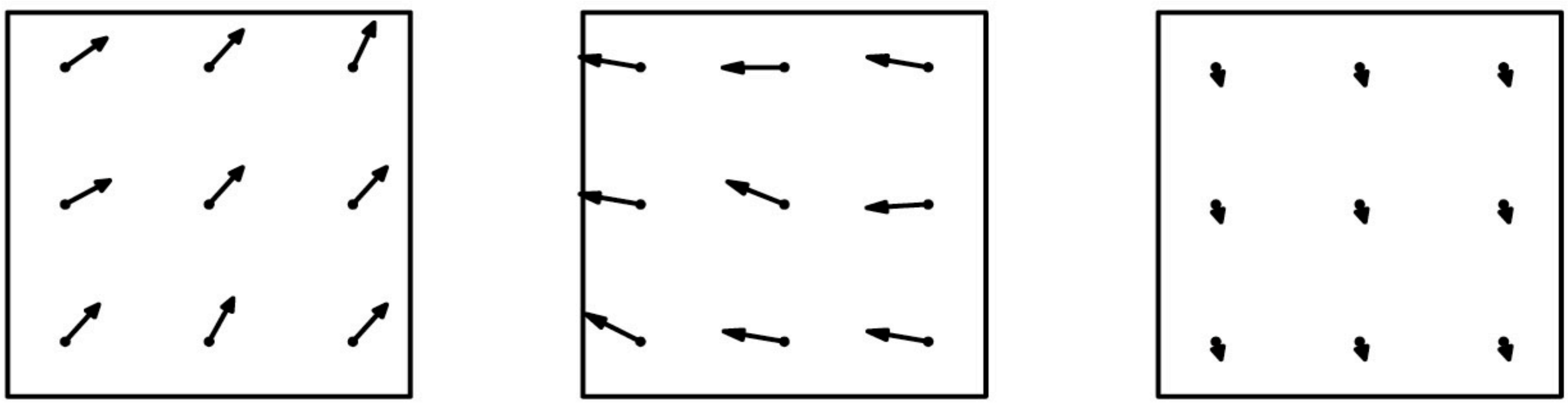


Errors from control checkpoints

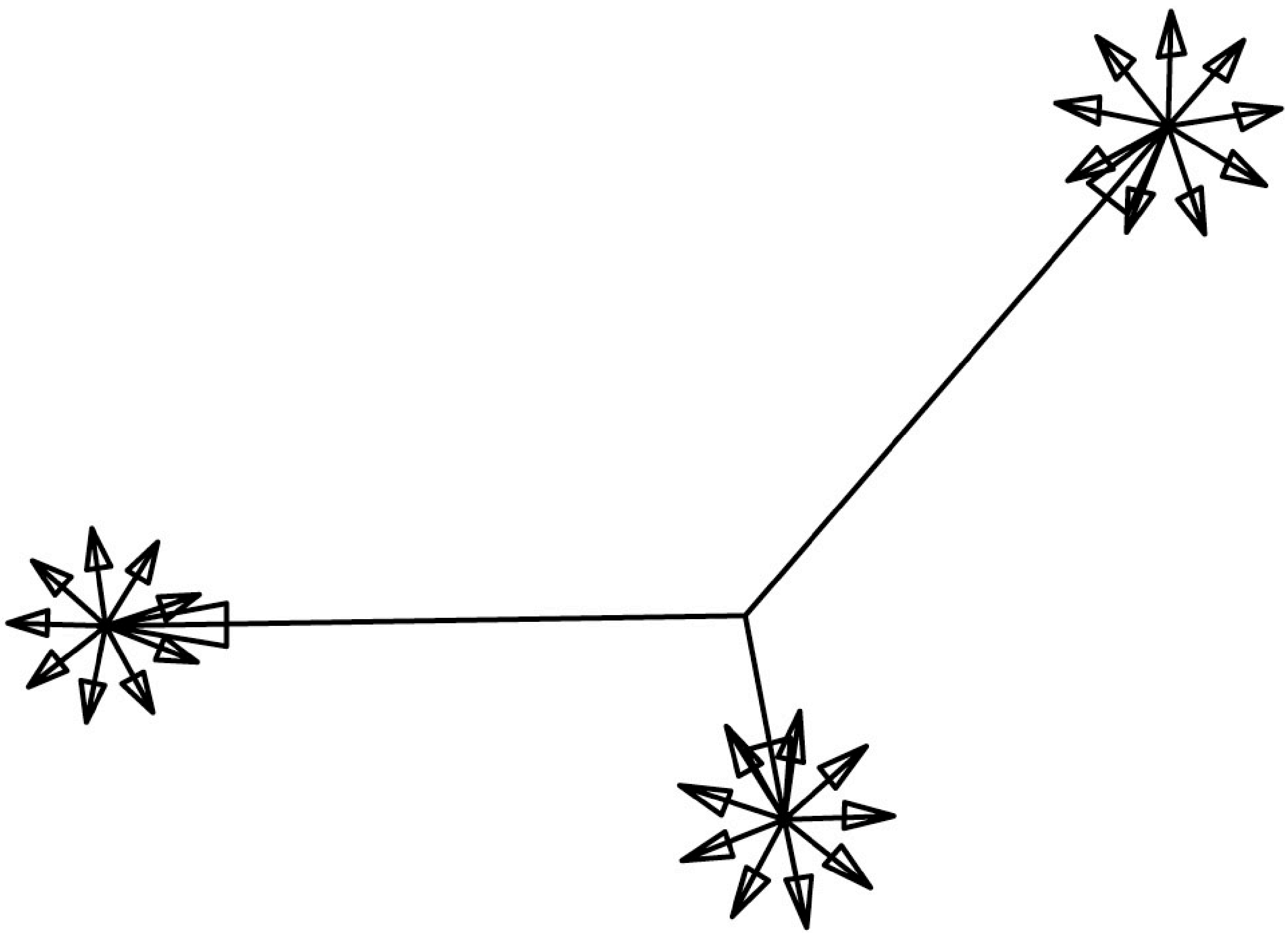


Error vectors decomposed into bias and random parts

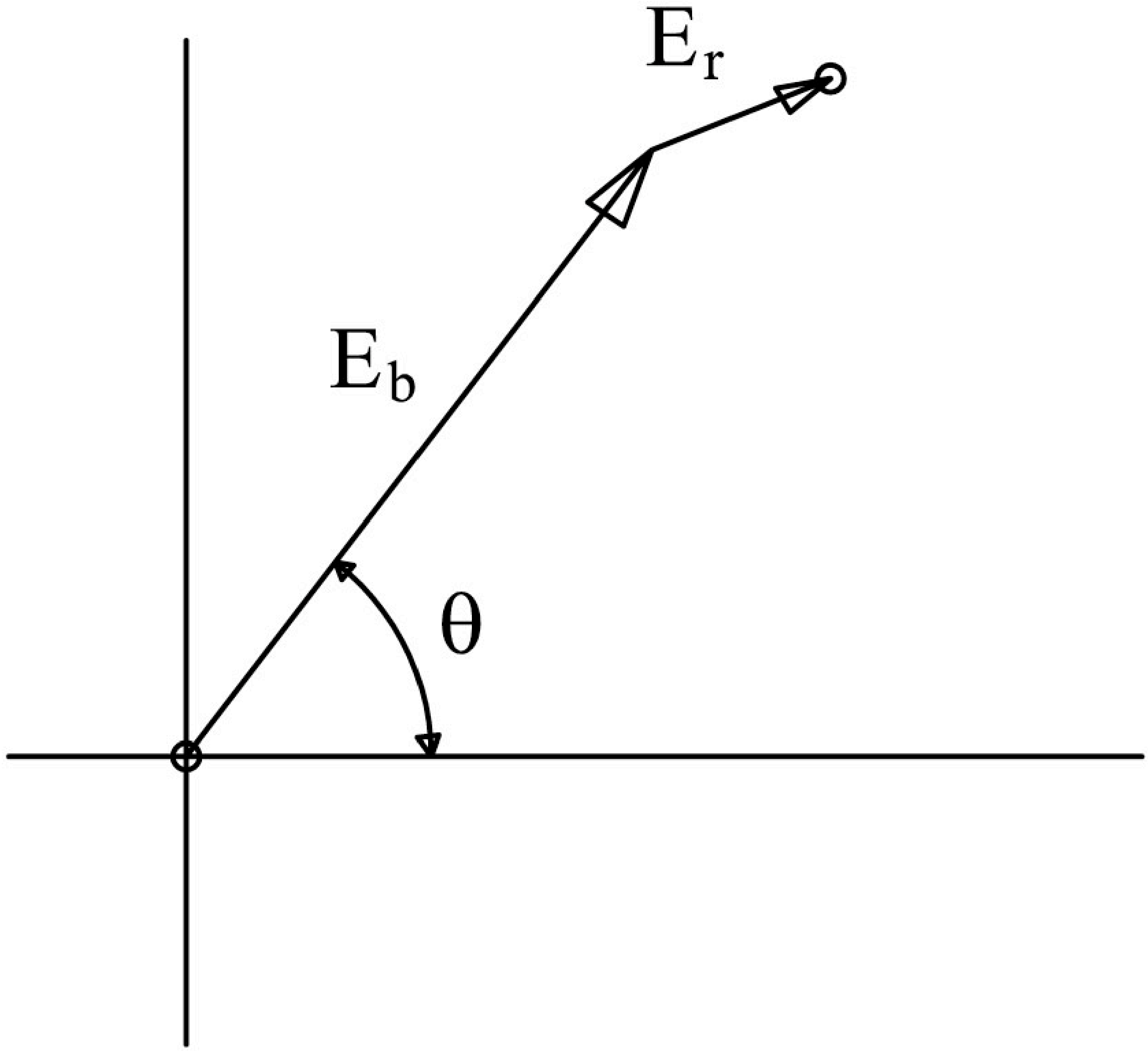
Conjecture: The bias is itself a random vector that is consistent within an image but different between images (corollary: the biases between “same orbit” images might be correlated)



Graphical Depiction of Errors from Three Images (Conjecture – verify with actual images & GCPs)



Another Way to Visualize an Individual Control Point Error, Decomposed into a Bias and a Random Component



For a single image with GCPs, can estimate bias and random part this way

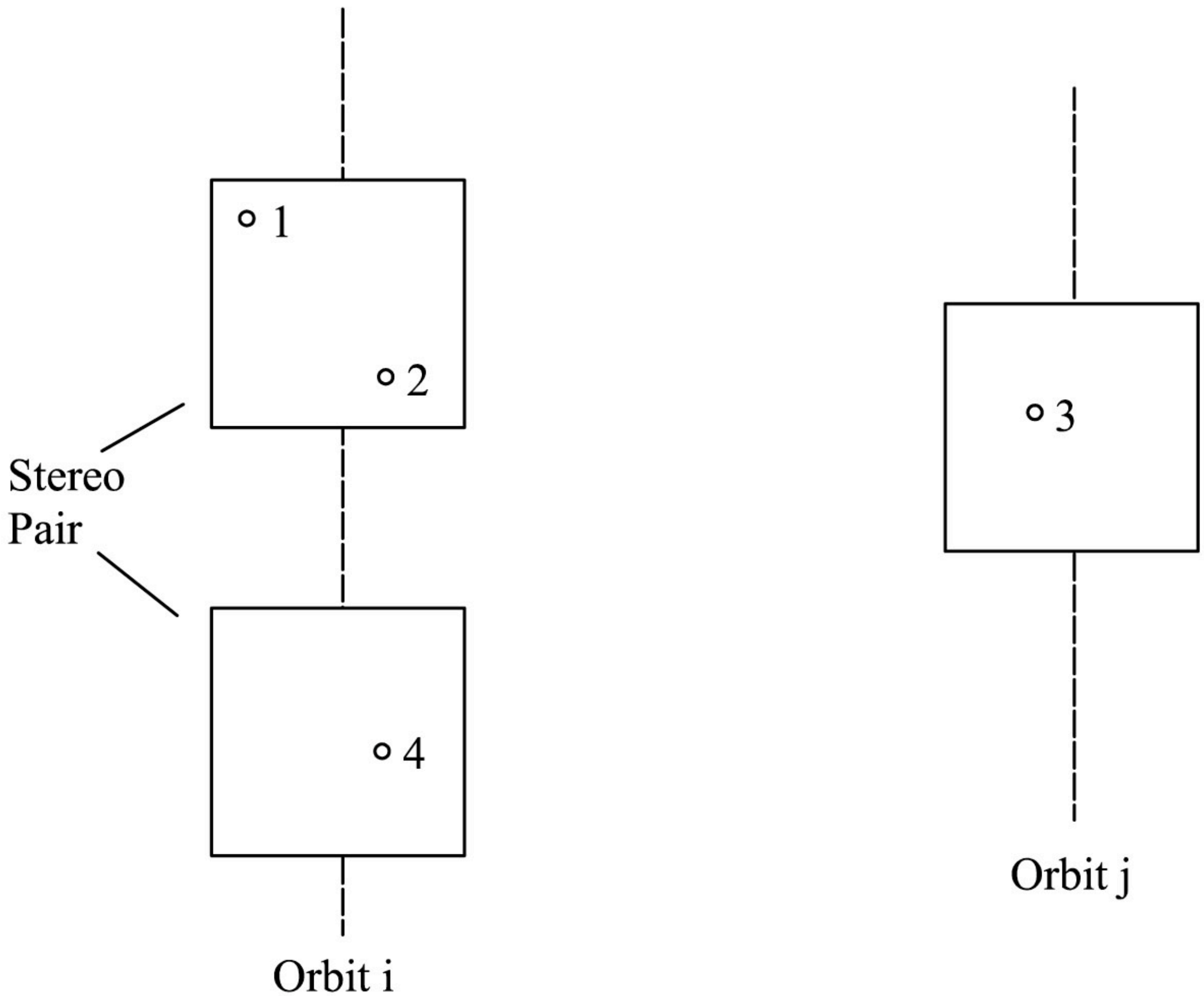
$$\begin{bmatrix} e_x \\ e_y \end{bmatrix} = \lambda_{bias} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \end{bmatrix}$$

$$a = \lambda_{bias} \cos\theta$$

$$b = \lambda_{bias} \sin\theta$$

Want to estimate one lambda and theta for each image, making the a,b substitution allows one to use a linear estimation model

Simulation of RPC projection errors using conjectured model



$$\mathbf{P} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}$$

$$\sigma_{bias} = 5$$

$$\sigma_{random} = 2$$

$$\rho_{12} = 1.00$$

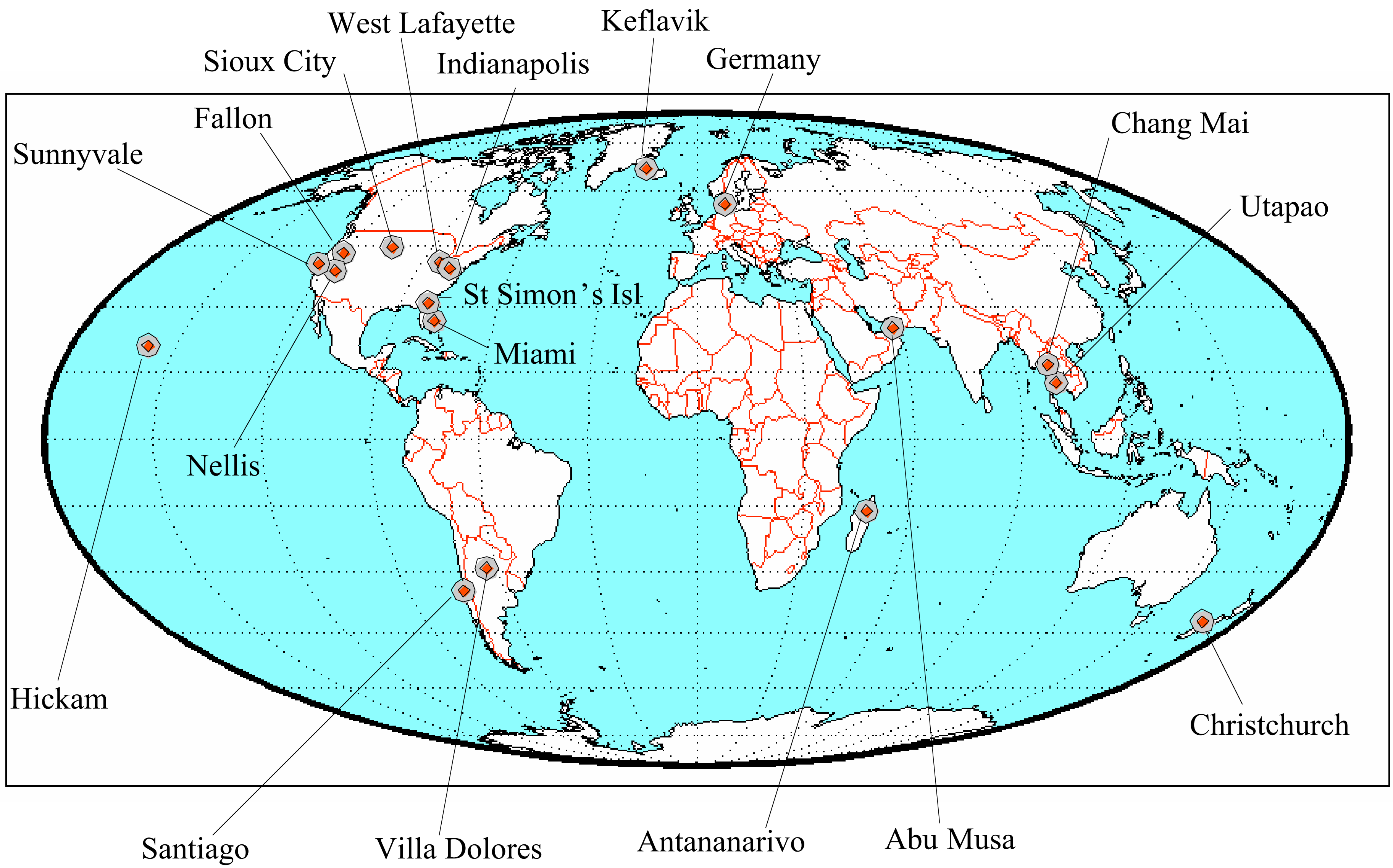
$$\rho_{14} = \rho_{24} = 0.5$$

$$\rho_{13} = \rho_{23} = \rho_{43} = 0.0$$

The Study

(Supported by NGA)

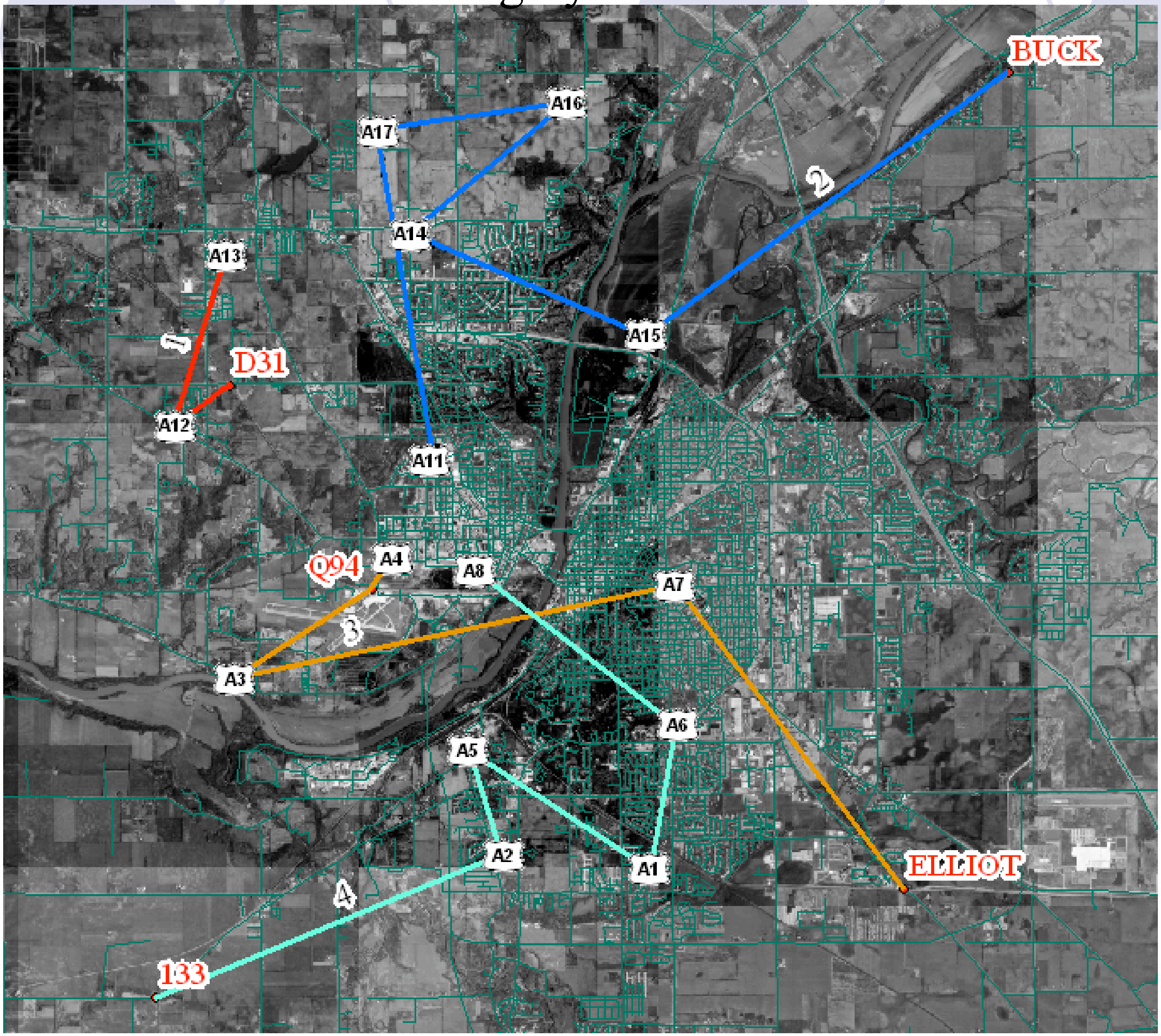
Imagery Sites



Purdue, IKONOS, stereo pair (in 2 segments)

Indianapolis, Quickbird, single image

GPS Survey, 4 receivers, 8 sessions, NGS control and photo-ID points for imagery evaluation



NGS CP D31

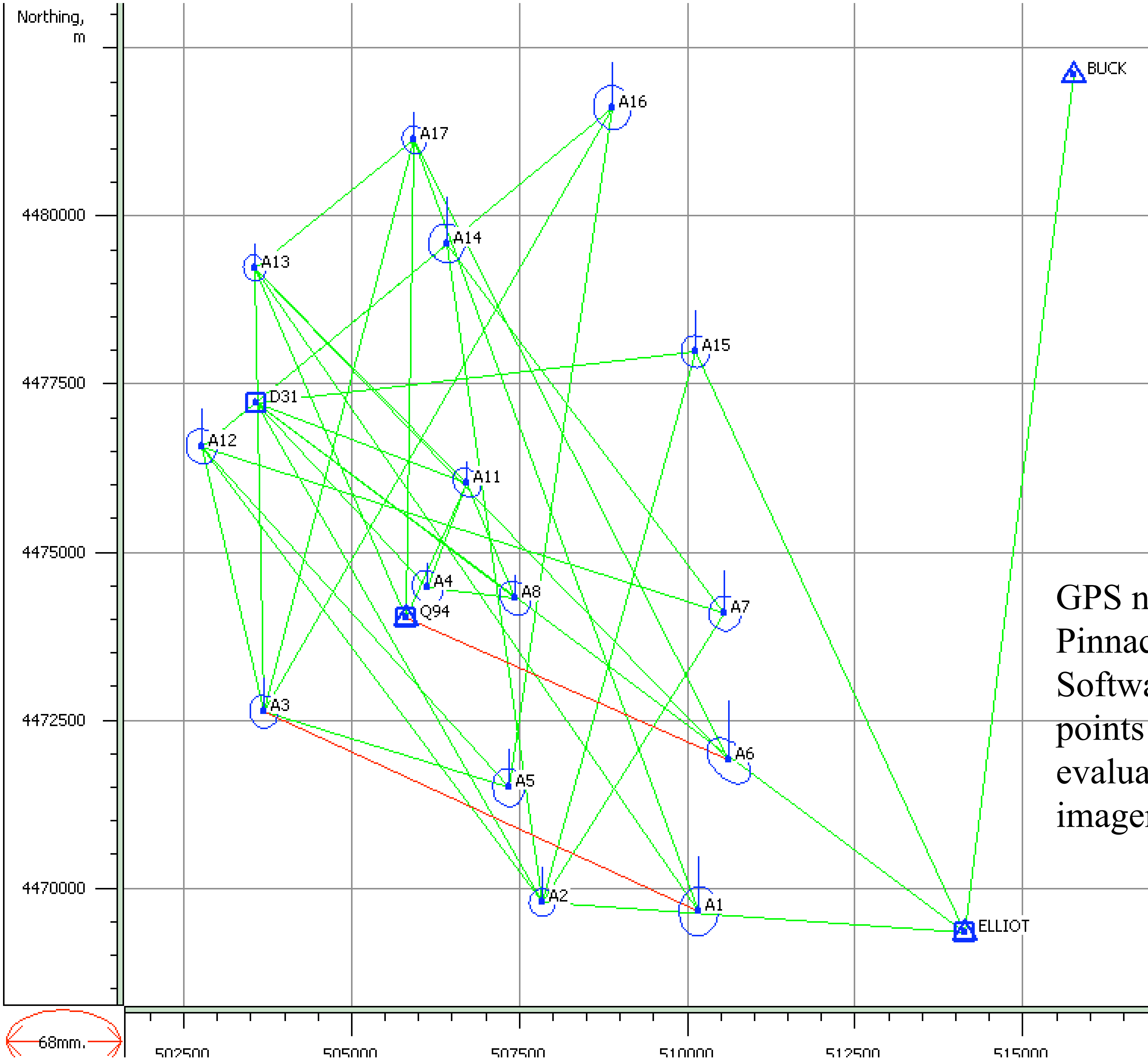
Photo ID A12

Photo ID A13

NGS CP BUCK

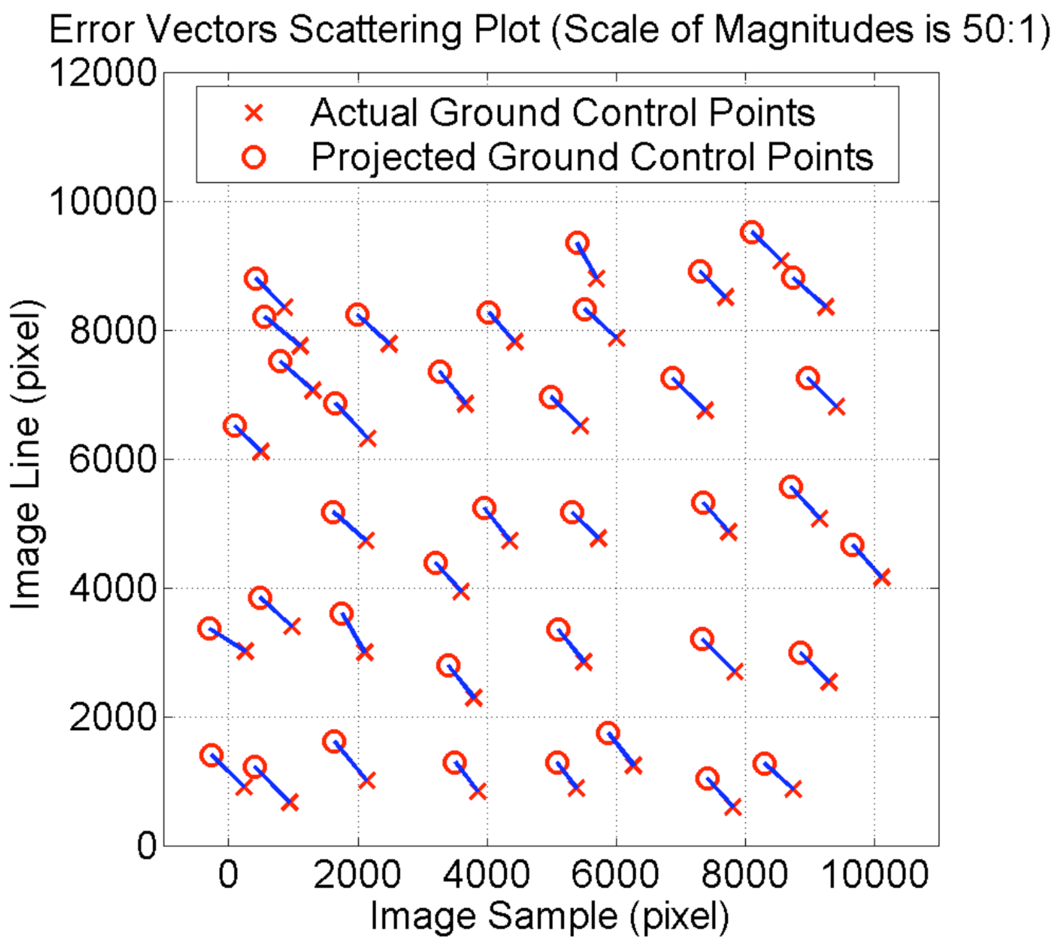
NGS CP BUCK

Selected occupations of NGS control points and photo ID points.

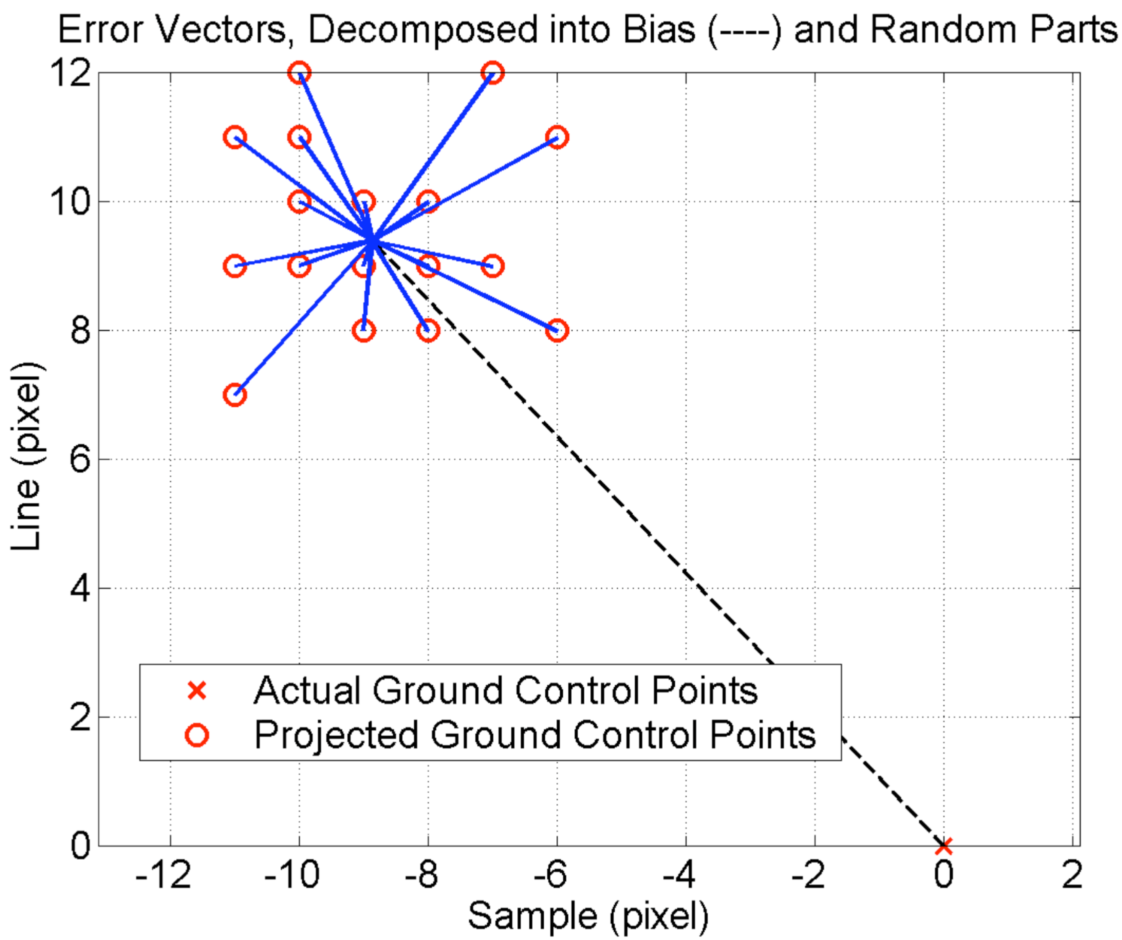


GPS network, from Pinnacle Adjustment Software, 15 photo ID points to use for evaluation of IKONOS imagery

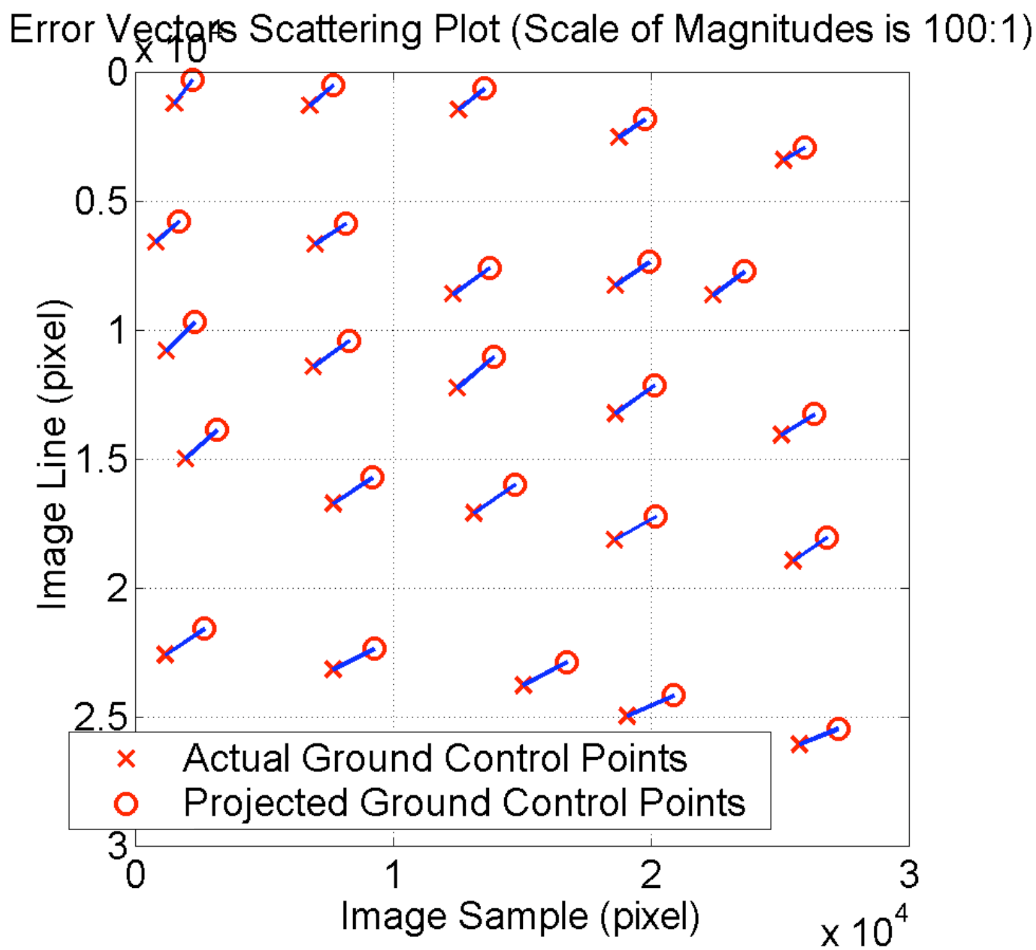
Error Vectors of Thailand Imagery 1 (Left)



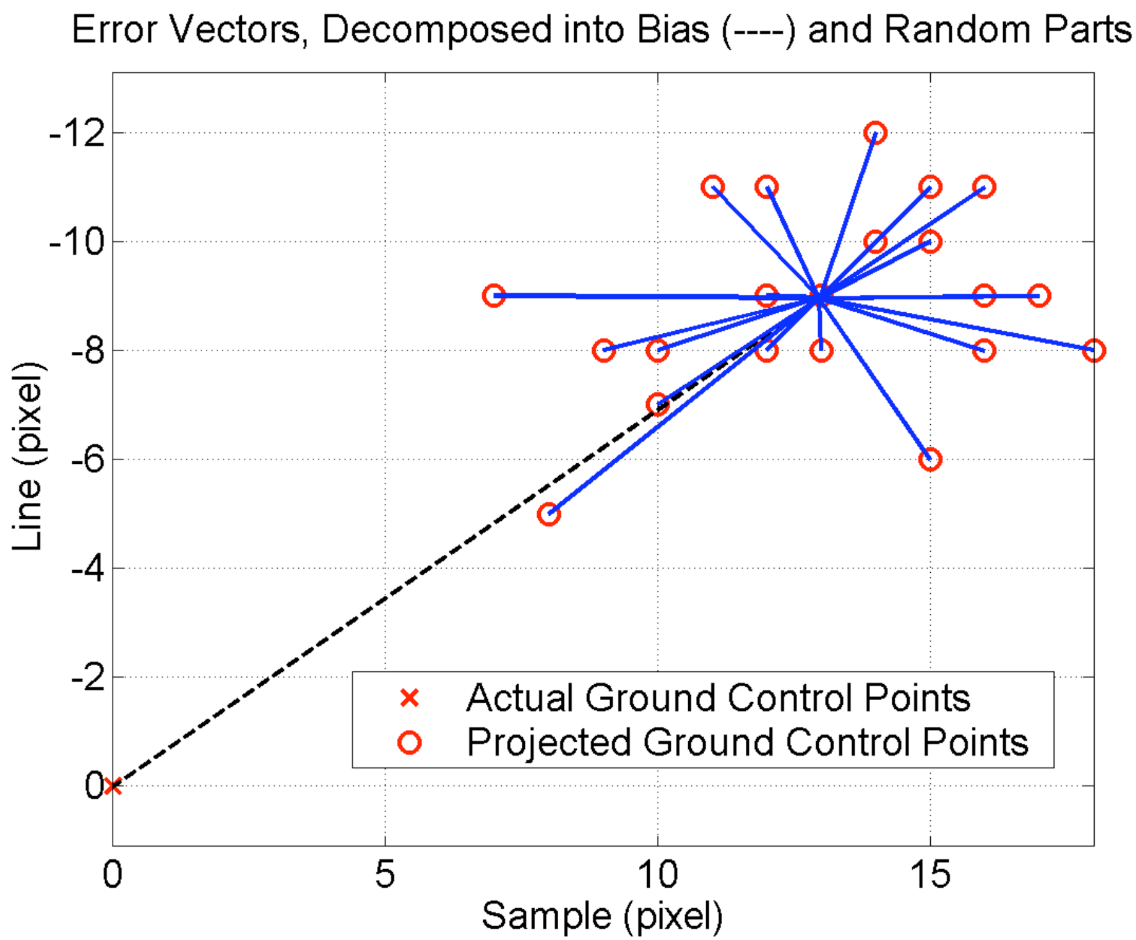
Error Vectors of Thailand Imagery 1 (Left)



Error Vectors of Fallon Imagery (Left)

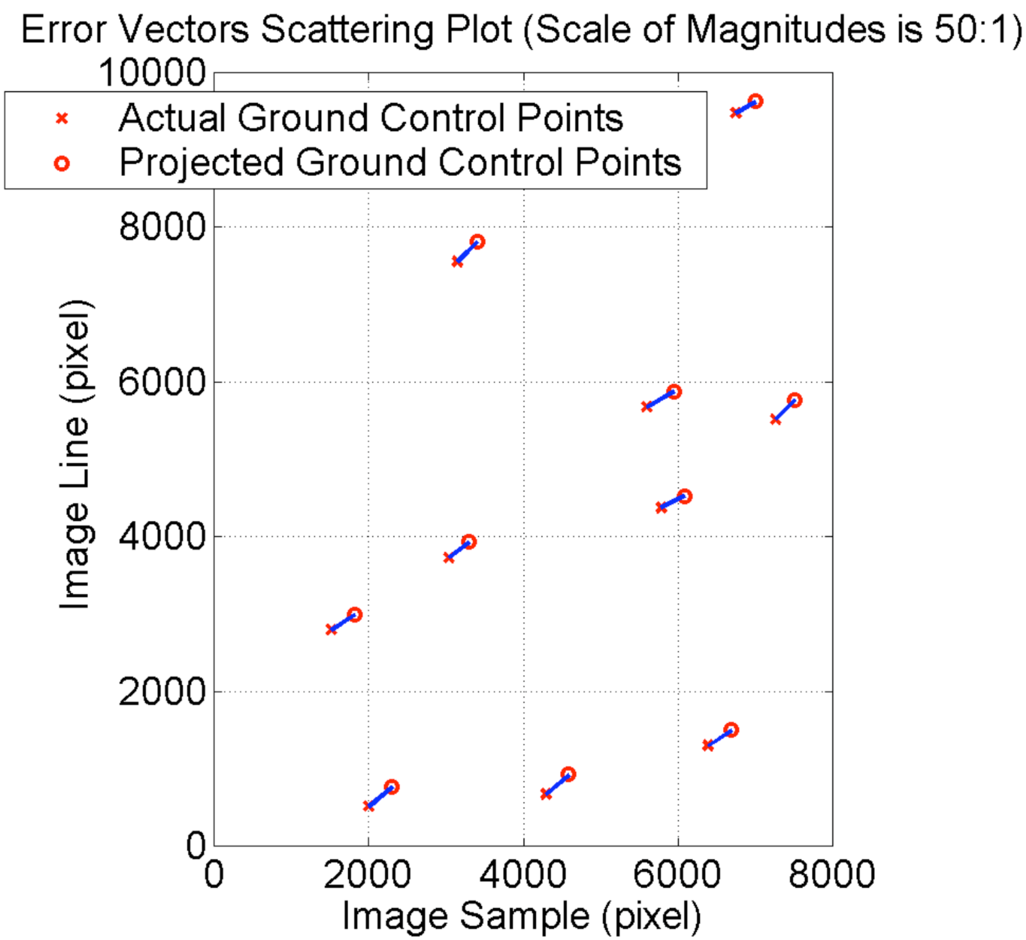


Error Vectors of Fallon Imagery (Left)

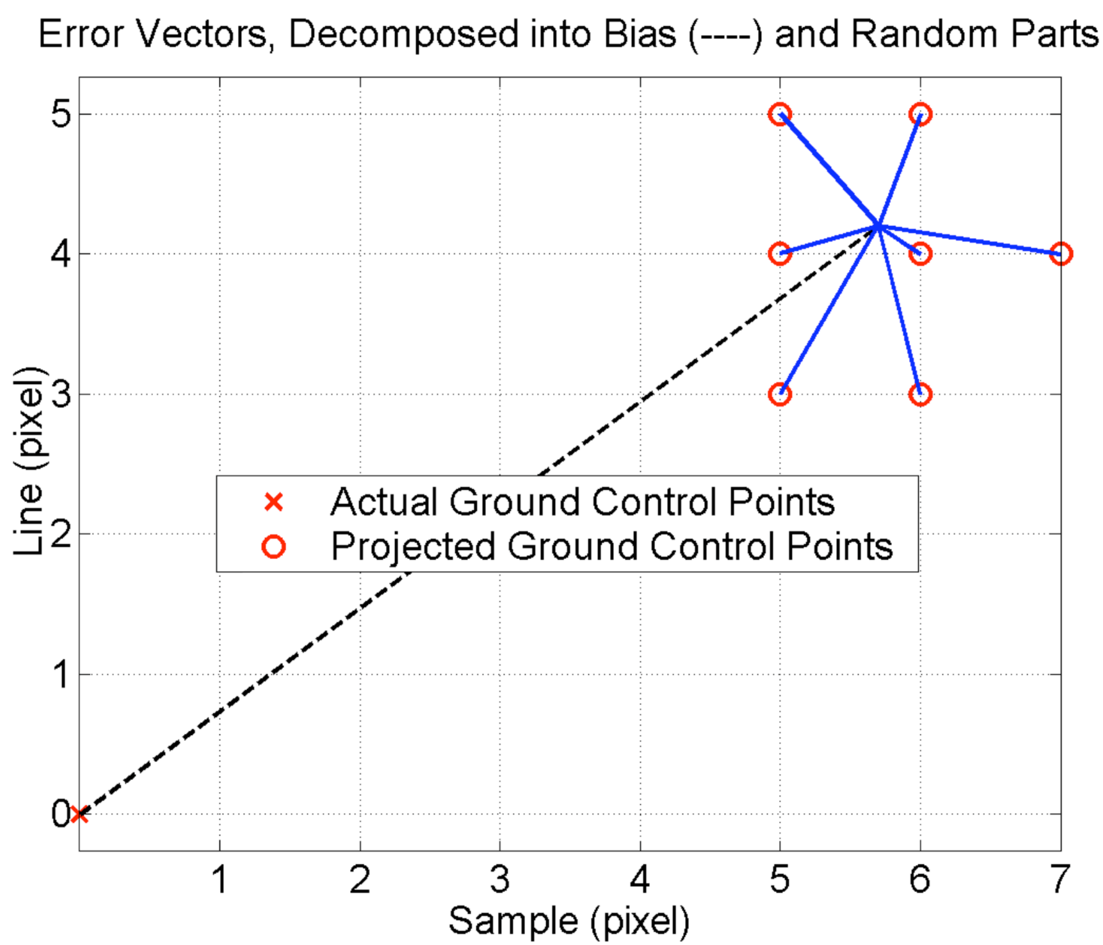


Error Vectors of Purdue Imagery (Left)

(Left and Right refer to a stereo pair)



Error Vectors of Purdue Imagery (Left)



Summary of Results for 70 Images, for 2 sensors, each with 2 classes, bias and random errors corresponding to 40% confidence region

Satellite[1]	**Tilt Angle**[2]	e_b[11] (Pixel)	e_r[12] (Pixel)
IKONOS	Small	4.15	0.95
IKONOS	Large	6.49	0.96
QuickBird	Small	11.21	1.27
QuickBird	Large	17.57	0.96

Conclusions

- The results are surprisingly good considering that we are doing "direct geopositioning", i.e. RPC's come from the satellite navigation data (i.e. no control points used in projection – only for checking)
- Furthermore, by far, most of the error is in the common bias term, which means if you introduce one high quality control point, and augment the RPC's with shift terms, you are down in the 1-2 pixel error range
- We have recommended a method of error propagation using these eb/er terms in image space, current Eb/Er terms from the NITF standard have ambiguous definition and are not applied uniformly.