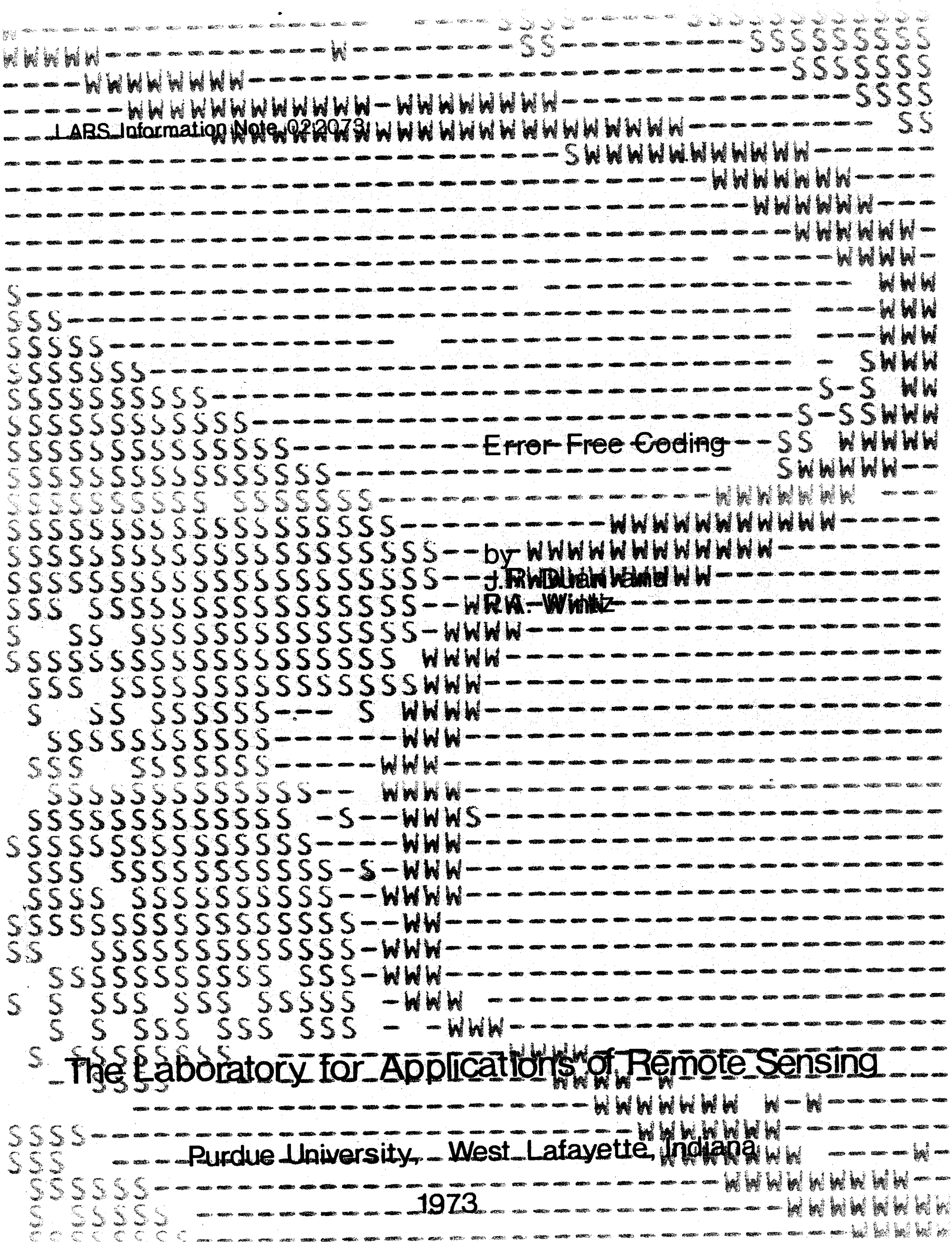

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Error Free Coding

by
J. R. Duan and
R. A. Wintz

The Laboratory for Applications of Remote Sensing

Purdue University, West Lafayette, Indiana

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ABSTRACT

An adaptive transform coding technique followed by a DPCM technique is employed to code multispectral data. A method of instantaneous expansion of quantization levels by reserving two codewords in the codebook to perform a folding over in quantization is implemented for data with incomplete knowledge of probability density function. Controlled redundancy is inserted periodically as fixed length codewords into the bit string of data packed with variable length codes to facilitate fast retrieval and to detect errors. Preliminary results of several sets of data from the ERTS-1 data frame and the ERIM[3] aircraft data frame showed that an error free reconstruction of the data can be achieved with four bits per picture element or less.

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[2] J. R. Duan is a graduate student in Electrical Engineering and P. A. Wintz is Professor of Electrical Engineering, both at Purdue University.

[3] Earth Resources Institute of Michigan. Formerly: University of Michigan/WRL.

Introduction

In view of the massive amount of multispectral scanner data (LARS, 1973) that will be accumulated with the use of aircraft and satellites such as ERTS-1, it is desirable if one is able to remove the redundancy of the data through the knowledge of its statistical regularities so that the total number of bits recorded to enable reconstruction of the data can be reduced.

The ground station storage-space problem can thus be reduced and the quantity of storage tapes for distribution of these data can be reduced. This data compression technique is dictated either from an economic point of view or from necessity to make optimum use of the available storage. The problem can be visualized as follows: instead of trying to code the data point-by-point, one takes a subset of the data and codes the interrelationship between them in such a way that the data can be recovered with or without some tolerable errors. This can be done by taking advantage of their one-dimensional, two-dimensional or multi-dimensional correlations, in either the original data space or in a transformed space [Wintz, 1972] to ensure an efficient coding scheme. The objective of this study is to explore a class of informative-preserving techniques for minimizing the amount of "information" [Shannon, 1949] that has to be retained, coded and stored to represent the multispectral data. Extensive bibliographies on topics related to data compression techniques have been compiled by Wilkins and Wintz (1971), Pratt (1967), Rosenfeld (1968) and the University of Southern California (1972).

Much effort in the past, however, has been directed toward non-information preserving techniques (at least from the numerical error point of view) based on the model for the human visual system. The application for scientific measurements such as multispectral data is different. The aim is to extract and derive useful information from the gathered multispectral data; it is anticipated that new information can be obtained when new processing techniques are developed. Therefore, since it is difficult to predict the value of the collected data in the future, there is no way of knowing which part or parts of the data are redundant. The information preserving techniques used to pack these data become increasingly attractive. The requirements of these techniques allow handling of the data with a general purpose computer whereas the decoding and access time is kept simple and fast.

Mathematical Formulation of the Problem

It is convenient to think of this problem basically as a stochastic minimization problem. Because of the non-stationary nature of the data and the successive linear transformation and nonlinear quantization operations involved, a global solution is not readily available.

Consider a set of discrete data D which consists of countable subsets I_i, i=1,2,.... For any i, I_i is a vector subset, i.e., $I_i=\{X_1,X_2,..,X_n\}$ and the X_i, i=1,2,...,n are m x 1 random vectors collected from a data source. It is desired to find the best strategy to minimize the bits required to code the data D such that the data can be reconstructed within prescribed accuracy.

Let b denote the total bits required for a given piece of information I, where I ⊂ D. The transformation, the quantization schemes associated and the controlled redundancy introduced are to be found so that b is minimized.

$$b_{min}(Q_o,X,T)=\underset{\substack{T_\ell \varepsilon F\\ Q_o}}{\text{Min}}\ \underset{X\varepsilon I}{E}\ \{S\left(Q_o[T_\ell(X)]_k\right)+C(T_\ell,k)\}$$

subject to the constraint that

$$\{e[\hat{X}(T_\ell,Q_o,Q_1,k),X]\}<\varepsilon$$

where

$$\hat{X}=Q_1[T_\ell^{\,t}\{Q_o[T_\ell(X)]_k\}]$$

$\hat{X}$: is the reconstructed data vector

F : is the set of all available affine transformation

I : is any member in D

T_ℓ: is an m X m matrix in F

Q_o: is the nonlinear quantization operator with L_o quantization levels

Q_1 : is the nonlinear quantization operator with L_1 quantization levels

S : is the function giving the number of bits required to code k transformed variables

k : is the number of transformed variables kept, being determined through the constraint

T_ℓ^t: transpose of T_ℓ

C : is the function which gives the number of bits required for the controlled redundancy consisting of the specifications inserted in the bit string for fast retrieval and the bookkeeping information k.

The attempt here to solve this stochastic minimization problem is to follow a dynamic programming algorithm and decide at each step of the processing the optimum rules to follow [Duan, 1972]. At the end of the coding of a finite subset of data I , an evaluation phase of the whole process comes into play. The sensitivity of the variation of different parameters prescribed can be analyzed to improve further processing or reprocessing through iteration. The solution of absolute minimum of $b(Q_0,X,T)$ for a given piece of data, subject to all available variation of the parameters such as L_0, L_1, n, m, etc. and transforms T_ℓ in F, is not of great importance. A more stable solution of b_{min} for a larger set of data D with diverse properties is much more to be desired.

A simplified schematic is shown in Figure 1. Some aspects of the processing system are described in the following sections.

Data Partitioning

The first step is to partition the data into a convenient format. In order to take advantages of both the spectral domain structure as well as the two-dimensional spatial structure, a $n \times m \times l$ array of data is re-indexed to form a one-dimensional vector data [Wintz, 1972].

The inherent structure of the spectral data is not fully understood partly due to the nonstationary nature of the data. It is assumed, however, that they can be modeled as a stationary process within a limited period of time or a limited area in the spatial domain. Therefore, it is possible to calculate the statistical distribution of the data through a selected subset and then process the local data with an efficient algorithm.

Choice of Transformation Matrix

There is no restriction as to the type of transforms [Tasto and Wintz, 1971; Ready and Wintz, 1972] to be used, except, perhaps, a limitation on the total number of choices and their corresponding sizes. The common choices are the Fourier, Hadamard, Karhunen-Loeve, Haar and Slant transforms. For the initial stage of analysis, only the optimum transform is used. The optimum transform for minimizing the number of transform samples needed for best reconstruction in mean square error sense is the Karhunen-Loeve (K-L) transform. It also has some other optimum properties [Okamoto, 1968].

The K-L transformation matrix is often defined as being composed of eigenvectors of the covariance matrix of the data. However, if the mean of the data is not zero, the correlation matrix and covariance matrix are different and their corresponding normalized matrices are also different. The implication of these differences is that the principal components are not invariant if the data is manipulated by an affine or scale transformation. The performance of the four sets of basic functions thus generated is compared. It is found that sealing of the data results in a more stable probability density function, that is, the change of the dynamic ranges of the coefficients from one set of data to another does not jump violently. Also, the computational effort in subtracting the sample mean from the data can be skipped in the transformation of the data, and hence computational complexity is reduced in computing both the basis function and individual transformation of each block of data.

Quantization with Folding

In the effort to code the data without error, one is faced with the problem of quantizer saturation or, in other words, dynamic overload. The percentage of these overload occurrences can be made as low as desired by designing the quantizer to suit the data, once the probability density function of the data is completely known. In this case, only the sample probability density function of a selected subset is known. It is not efficient to have a quantizer with many bands having an extremely low probability of occurrence, and one can hardly change the quantizer effectively from time to time. Therefore, by reserving two codewords in the codebook to do folding when the dynamic range of the data undergoes some sudden changes out of the regular operating bounds, the saturation error can be completely eliminated. An example will serve to explain its implementation. Suppose the quantizer consists of 256 codewords corresponding to 254 different numbers. We will reserve two out of 256, say, the first and the 256th codewords, as coding folding information. Now if the number to be coded turns out to be 320, the data 320 is broken into two codewords, one is 256 and the other 320-256+1=65.

Adaptive Error Control System

After segmentation the multispectral data will be arranged in a vector form with every entry of the vector representing one picture element. Each pixel can take on any one of the intensity levels M. M is limited by the number of bits N assigned to each sample, $M=2^N$.

Let Y be the vector of the data in its transformed domain and X be the data vector.

$Y=[T_\ell]X$ where T_ℓ is the selected orthonormal transformation matrix.

From a quantized and truncated version of Y, e.g., $\hat{Y}=(y_1,y_2,\ldots,y_k,0,0,\ldots0)$, nonzero up to k terms. X is reconstructed.

Let $\hat{x}$ be the reconstructed data, we have $\hat{x} = [T_\ell]^t\hat{Y}$.

The number of terms kept in Y, e.g. k, is chosen according to how it is desired to control the difference function $e(x,\hat{x})$. Therefore, the number of bits assigned to each block is dependent on the convergence rate of the block data under the prescribed difference criterion. This is necessary to ensure an efficient coding in the DPCM stage. It is found by the appropriate choice of the difference bound $E|e(x,\hat{x})|<\varepsilon$ and by selecting the probability that the reconstructed data be inside the bounds, i.e., $|e(x,\hat{x})|<\varepsilon$. In this way the data can be coded with bits per picture element less than the entropy defined by the first order probability density of its gray level distributions.

The typical tendency of bit requirement for transform coding and DPCM is plotted in Figure 2. Some recent results from ERTS-1 data are listed in Table 1. The results indicate that error free reconstruction with a little over 3 bits per picture element is likely for the whole frame of ERTS-1 scanner data.

ADAPTIVE TRANSFORM CODING	DPCM	SUM	SIZE	PICTURE RUN NO.
0.397	2.508	2.905*	400x400	72044400
0.392	2.484	2.876	400x400	72044400
0.225	2.992	3.217	400x400	72044400
0.228	2.904	3.132	400x400	72044400
0.211	2.515	2.726	400x400	72001405
0.235	2.821	3.056	400x400	72001405

Table 1: Bit Requirements for Coding ERTS-1 Data

*(Not Including the Controlled Redundancy for Fast Retrieval)

The Packed Data Format

The layout of packed data format with insertions of controlled redundancy is as follows:

SPEC DATA .... SPEC DATA ....

255 BYTES 255 BYTES

The specification (also referred to as controlled redundancy) is inserted every n bytes (in this case n = 256) of data (packed with variable length codes) to initiate the start of the next block of data. The specification byte serves two purposes:

(a) It tells the exact position of the starting byte for the next block of data so that if one does not want to start decoding from the very first line and first sample of the data, as many data samples as desired can be skipped.

(b) The specification bytes serve as a check in case any bit error has occurred between two specification bytes and the decoding can start anew for the next block of data with a maximum loss of only a block of data.

Without the specification bytes any bit error will destroy the whole set of data packed from then on.

Results and Conclusions

(1) It is found that one can take advantage of both the transform techniques and DPCM by first coding the data through a few terms of the transformed coefficients so that the data can be reconstructed with controlled error (or rather controlled difference between the original and reconstructed versions) and then coding the element difference with the variable-length Shannon-Fanno-Huffman Code.

(2) From this study, it is found desirable to scale the data either by its sample standard deviation or by its modified root mean square value of the data before subjecting the data to transformation. The computational time can be saved because only about 20~50% at the most of the transformed coefficients need be calculated.

(3) Given an incomplete knowledge of the true probability density function of the data, the concept of folding can greatly simplify the design of the quantizer with only a slight complication for the decoder. The idea of folding takes care of instantaneous increase of the dynamic range encountered.

(4) The preliminary results for the aircraft and ERTS-1 data exemplify that a reduction of 2:1 with no error reconstruction can be achieved on the average.

(5) The software for decoding the bit string packed by the adaptive transform and DPCM technique is under development. About 1% of the controlled specification is being implemented for detection of error and fast data retrieval.

REFERENCES

Duan, J. R. 1972. Information Preserving Adaptive Techniques for Data Compression. Ph. D. Thesis Proposal: School of Electrical Engineering, Purdue University, West Lafayette, Indiana.

Laboratory for Applications of Remote Sensing (LARS), 1973. The Multispectral Scanner. Focus, No. 1. Purdue University, Lafayette, Indiana.

Okamoto, M. 1968. Optimality of Principal Components. In Multivariate Analysis II, ed. P. R. Krishnaiah.

Pratt, W. K. 1967. A Bibliography on Television Bandwidth Reduction Studies. IEEE Transactions on Information Theory, IT - 13: 114-15.

Ready, P. J. and Wintz, P. A. 1972. Multispectral Data Compression through Transform Coding and Block Quantization. Ph. D. Thesis: School of Electrical Engineering, Purdue University, West Lafayette, Indiana. (Available as: Technical Report TR-EE 72-2 and LARS Information Note 050572.)

Rosenfeld, A. 1968. Bandwidth Reduction Bibliography. IEEE Transactions on Information Theory, IT-14: 601-02.

Shannon, C. E. 1949. A Mathematical Theory of Communication. Urbana: University of Illinois Press.

Tasto, M., and Wintz, P. A. 1971. Image Coding by Adaptive and Block Quantization. IEEE Transactions of Communications Technology Com-19:957-71.

University of Southern California. 1972. Bibliography on Digital Image Processing and Related Topics. Electronic Sciences Laboratory, USCEE Report 410.

Wilkins, L. C., and Wintz, P. A. 1971. Bibliography on Data Compression, Picture Properties, and Picture Coding. IEEE Transactions on Information Theory IT-17:180-97.

Wintz, P. A. 1972. Transform Picture Coding. Proceedings of the IEEE. 60-7:809-820

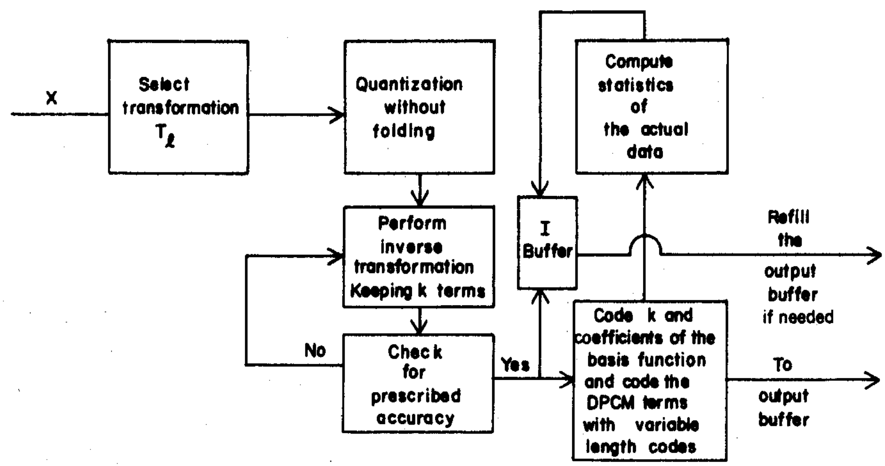

Figure 1. Simplified Block Diagram Showing the Adaptive Error Control System

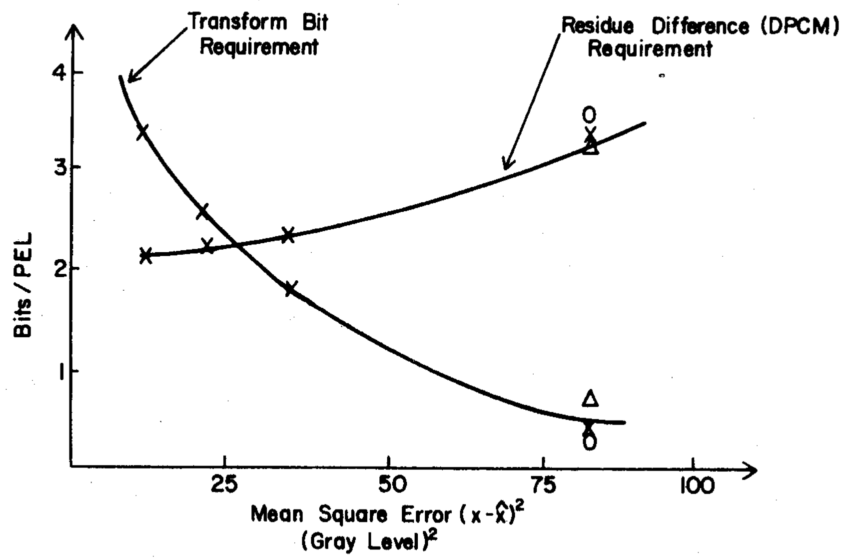

Figure 2. Typical Bit Requirements for the Two Stages of Processing