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STATISTICAL SEPARABILITY OF
AGRICULTURAL COVER TYPES IN
SUBSETS OF ONE TO TWELVE
SPECTRAL CHANNELS

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STATISTICAL SEPARABILITY OF AGRICULTURAL COVER TYPES IN

SUBSETS OF ONE TO TWELVE SPECTRAL CHANNELS¹

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ABSTRACT

The purpose of this study was to determine the statistical separability of multi-spectral measurements from agricultural cover types: corn, soybeans, green forage (hay & pasture) and forest, in one to twelve spectral channels. Multispectral scanner data in twelve spectral channels in the wavelength range 0.4 to 11.7 μm , acquired on July 16 for three flightlines were analysed by applying automatic pattern recognition techniques. The same analysis was performed for the data acquired on August 12 over the same three flightlines to investigate the effect of time on statistical separability of agricultural cover types.

In the subsets of one to six spectral channels, the combination of wavelength regions (where V, N, M and T denote the visible, near infrared, middle infrared and thermal infrared wavelength regions, respectively): V, V M, V N M, V N M T, V V N M T, V V N M M T, respectively, were found to be the best choices for getting good overall statistical separability of the agricultural cover types for the data acquired on July 16 as well as August 12.

An effort was made to explain these results on the basis of spectral properties of agricultural cover types. The overall statistical separability of the agricultural cover types was found to be greater for the data of August 12 than the data of July 16 (Table II).

I - INTRODUCTION

The purpose of this study was to determine the statistical separability of multi-spectral measurements from agricultural cover types. The data was analysed in one to twelve spectral channels for selected flightlines of the 1971 Corn Blight Watch Experiment¹. The agricultural cover types selected were: corn, soybeans, green forage (hay & pasture), and forest. In particular, the objective of the study was to determine what combinations of one through eleven spectral channels out of 12 available spectral channels give greatest overall statistical separability of the agricultural cover types. In addition, the effect of time on the statistical separability of agricultural cover types was investigated.

II - LITERATURE REVIEW

The results of percent correct classification, obtained from the analysis of multispectral scanner (MSS) data by applying pattern recognition techniques, for a flightline divided into four classes (soybeans, corn, water and a mixture of stubble, diverted acres & pasture) were reported in the LARS⁺ annual report² (1970). These are summarized as follows.

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Performance	Percent Correct Classification Using the Following Wavelength Bands		
	3 Visible (ch 1,2,3) 1 Reflective IR (ch 4)	3 Visible (ch 1,2,3) 2 Reflective IR (ch 4,5)	2 Visible (ch 2,3) 2 Reflective IR (ch 4,5) 1 Thermal IR (ch 6)
Training fields (4303 sample points)	86.8	91.9	93.6
Test fields (7135 sample points)	82.8	83.9	86.4
channel 1 = 0.40 - 0.44 μm , channel 4 = 0.80 - 1.00 μm ,	channel 2 = 0.55 - 0.58 μm , channel 5 = 1.50 - 1.80 μm ,	channel 3 = 0.66 - 0.72 μm , channel 6 = 8.00 - 14.0 μm	
ch denotes spectral channel			

Vincent and Thomson^{3,4} (1972) have pointed out that the thermal-infrared wavelength region offers probably the best spectral information diagnostic of target composition for geological targets, because most rocks and minerals have major molecular absorption bands in the thermal infrared region. They used the data from the flight over a sand quarry near Mill Creek, Oklahoma in the wavelength ranges 8.2 to 10.9 μm and 9.4 to 12.1 μm , to produce an analog-processed image and a digital recognition map from which exposed quartz-sand and quartz-sandstone in the vicinity of the quarry were quite identifiable. In addition, the wavelength region from 11.5 to 12.5 μm is useful because the emittance of geological targets can be assumed to be constant in this wavelength region. They pointed out that medium-bandwidth (1 to 3 μm wide) filters seem to be a reasonable compromise between the dual requirements of 'availability of adequate energy' and 'sensitiveness to the positions of spectral features' in the thermal infrared wavelength band.

Bartolucci et.al.^{5,6} (1973) have analysed aircraft multispectral scanner data gathered at altitudes of 608 and 3040 m to determine the radiant temperatures of water. They found that 8 to 13.5 μm band yields more information on the thermal characteristics of water bodies, provides with better contrast between ground features, and gives radiant temperatures of water closer to its contact temperature than the 4.5 to 5.5 μm band. In addition, they found that the 9.3 to 11.7 μm band yields accuracies (contact temperature minus radiant temperature) similar to the 8.0 to 13.5 μm band, but for altitudes up to 1520 meters.

Coggeshall and Hoffer⁷ (1973) have analysed the multispectral scanner data of a flightline having mostly forest. They have investigated the following in much detail. 1) determination of the optimum number of the 12 available multispectral scanner (MSS) wavelength bands to use for forest cover mapping with automatic data processing (ADP) techniques; 2) determination of the current capability to map basic forest cover using MSS data and ADP techniques; and 3) determination of the relative utility, to forest cover mapping, of the four spectral regions available in the twelve channel MSS data (i.e., visible, and near, middle and thermal infrared).

They concluded from the tests of classification accuracy of six cover types of interest (deciduous forest, coniferous forest, water, forage, corn and soybeans), that the use of five wavelength bands would fulfill the dual requirements of adequate accuracy and moderate computer time. Their results also indicated that the thermal infrared wavelength is desirable, but not necessary, for forest cover mapping, and that accurate classification of deciduous and coniferous forest cover can be achieved with the visible plus either the near or middle infrared spectral regions. However, the deletion of the thermal infrared region caused considerable confusion among the agricultural types. The authors felt a definite need for doing a somewhat similar analysis of large (much larger than analysed by Coggeshall and Hoffer) amounts of MSS data for agricultural cover types.

Recently, Kumar & Silva⁸ have investigated the statistical separability of the spectral classes of blighted corn in much detail, data quantity (168 fields having 18804 sample points in ten flightlines) and depth. They found that the greater the difference between the blight levels, the more statistically separable they usually were. In addition, they found that the spectral classes of corn (healthy & blighted) were most separable in the wavelength range 1.00 to 1.40 μm . Bauer¹ (1974) has discussed the results of 'wavelength band selection', obtained in the Corn Blight Watch Experiment.

III - METHOD OF ANALYSIS

Multispectral scanner data in twelve spectral channels in the wavelength range 0.4 to 11.7 μm , collected with an optical mechanical scanner at altitudes of 914 to 2133 meters (3000 to 7000 feet) over Western Indiana were analysed by applying automatic pattern recognition techniques. The wavelength bands of these twelve spectral channels are given in Table I. Only those corn fields having none or slight (blight level 1) blight rating¹⁰ were included in the analysis to eliminate the effects of blight severity, i.e., the corn fields having a rating of more than 1 were not included in the analysis. The authors have found from their experiments on blighted corn plants that blight level 1 is not likely to cause any significant changes in the reflection and emission from a plant canopy¹¹. The data of three selected flightlines, acquired on July 16, 1971, were analysed. Each of these three flightlines had fair or good amount of each of the four agricultural cover types: corn, soybeans, green forage and forest. In addition, none of these flightlines was predominated by forest. These three flightlines were selected carefully so that these combined could be considered to be representative of the four agricultural cover types in the Western Indiana. Thus, the characteristics of the flightlines were considerably different from the flightline analysed by Coggeshall and Hoffer⁷, which had mostly forest.

Black and white photography and gray scale printouts of the spectral channels of the flightlines were used to aid in locating the boundaries of the fields on the LARS (Laboratory for Applications of Remote Sensing, Purdue University) Digital Display*. Sufficient number of fields of each agricultural cover type were selected carefully so that they could be assumed to be representative of the flightline.

Using the same three flightlines and twelve spectral channels, an identical analysis was performed on the data acquired on August 12, 1971 to study the effect of time on the statistical separability of agricultural cover types. The multispectral scanner data was acquired on both dates (July 16 & August 12) between 10.30 a.m. and 12.05 p.m. (local solar time). In addition, these data were of good quality and free from problems like lack of sufficient ground observations, excessive cloud cover, etc. The analysis was done for the data acquired in the middle of July and the middle of August because corn and soybeans have reached their maximum vegetative growth by these times and one month of time is sufficient for significant changes to occur in the spectral properties of agricultural cover types. The authors wanted to avoid the analysis of data from late September afterwards because soybeans are harvested in September-October. The authors tried to keep all the variables other than time uniform in the two (July 16 and August 12) sets of data as far as possible. For example, an effort was made to select about the same field boundaries for the data of August 12 as for the data of July 16. A total (including data of July 16 and August 12) of 602 fields having 460360 sample points taken from three flightlines were analysed.

Each field was treated as an independent unit and the fields of the same agricultural cover type were put in the same class. The sample points within each field were highly correlated. The LARSYS** statistics algorithm¹² was used to compute the mean vector and covariance matrix (mean and standard deviation) of the classes. In the LARSYS statistics algorithm, cluster algorithm, and feature selection algorithm, each sample point is treated independently in order to make the system convenient and flexible for usage. A key assumption made in these algorithms is that the distributions of the classes are Gaussian. Histograms of the spectral classes defined above were used to check unimodality of the statistical distributions in individual channels. The classes were redefined to eliminate distinct multiple modes. Divergence is defined for any two density functions. In the case of normal variables with unequal covariance matrices, divergence in n spectral channels C_1, C_2, \dots, C_n , is given¹³ by

$$D(i,j|C_1, C_2, \dots, C_n) = 1/2 \text{tr} [(\Sigma_i - \Sigma_j) (\Sigma_j^{-1} - \Sigma_i^{-1})] \quad (1)$$

$$+ 1/2 \text{tr} [(\Sigma_i^{-1} + \Sigma_j^{-1}) (U_i - U_j) (U_i - U_j)^T]$$

where

U and Σ represent the mean vector and covariance matrix respectively;
 $\text{tr} [A]$ (trace A) is the sum of the diagonal elements of A .

* The LARS Digital Display is a specially designed image display system linked to an IBM 360/Model 67 using a cathode ray tube as the pictorial medium for gray scale multispectral imagery.

** LARSYS is the earth resources data processing software system of the Laboratory for Applications of Remote Sensing, Purdue University, W. Lafayette, Indiana.

A modified form of the divergence D_T , referred to as "transformed divergence", has a behavior^{12,14} more like the probability of correct classification than the divergence, D .

$$D_T = 2 [1 - \exp(-D/8)] \quad (2)$$

Transformed divergence has been used throughout this study.

Although divergence only provides a measure of the distance between two class densities, its use has been extended to the multiclass case by taking the average over all pairs¹⁵. Let D_{Tij} denote the divergence between classes i and j of a certain flightline, then the average divergence over all class pairs of four classes (each agricultural cover was treated as a separate class) is given by

$$D_{TAVG} = \frac{1}{6} [D_{T12} + D_{T13} + D_{T14} + D_{T23} + D_{T24} + D_{T34}] \quad (3)$$

$$\text{Let } D_{TMIN} = \text{minimum of } \{D_{T12} + D_{T13} + D_{T14} + D_{T23} + D_{T24} + D_{T34}\} \quad (4)$$

Swain¹² (1972) has pointed out that one strategy is to select the subset of features for which the average divergence, D_{TAVG} , is maximum. While this strategy is certainly reasonable, there is no guarantee that it is optimal. Another strategy¹² is to maximize the minimum divergence, D_{TMIN} , i.e., to select the feature combination which provides the greatest separation between the hardest-to-separate pair of classes.

Let superscripts 1 and 2 with the symbol " D_T " denote the values of transformed divergence for the data acquired on July 16 and August 12 respectively. Let D_{TAVG1}^1 , D_{TAVG2}^1 , and D_{TAVG3}^1 denote the values of D_{TAVG} (see eq.(3)) of first, second, and third flightline respectively for the data acquired on July 16. Similarly, let D_{TMIN1}^1 , D_{TMIN2}^1 , and D_{TMIN3}^1 be the values of D_{TMIN} (see eq. (4)) in first, second and third flightline respectively for the data acquired on July 16.

$$\text{Let } \bar{D}_{TAVG}^1 = \frac{1}{3} [D_{TAVG1}^1 + D_{TAVG2}^1 + D_{TAVG3}^1] \quad (5)$$

$$\text{Similarly } \bar{D}_{TAVG}^2 = \frac{1}{3} [D_{TAVG1}^2 + D_{TAVG2}^2 + D_{TAVG3}^2] \quad (5A)$$

$$\text{Let } \bar{D}_{TMIN}^1 = \text{minimum of } \{D_{TMIN1}^1, D_{TMIN2}^1, D_{TMIN3}^1\} \quad (6)$$

$$\text{Similarly } \bar{D}_{TMIN}^2 = \text{minimum of } \{D_{TMIN1}^2, D_{TMIN2}^2, D_{TMIN3}^2\} \quad (6A)$$

The LARSYS feature selection processor was used to find \bar{D}_{TAVG}^1 and \bar{D}_{TMIN}^1 in all possible combinations of one, two, three, four, five, six, seven, eight, nine, ten, eleven and twelve spectral channels out of the available twelve spectral channels (i.e., the number of all possible combinations of r spectral channels out of n spectral channels = $q = \frac{n!}{r!(n-r)!}$). The total number of times each channel is selected in all possible combinations of r channels out of n available channels is given by

$$m = \frac{n!}{r!(n-r)!} \cdot \frac{r}{n} = \frac{(n-1)!}{(r-1)!(n-r)!} \quad (7)$$

Throughout this analysis, the combinations of one through twelve spectral channels were ranked so as to get the descending order of \bar{D}_{TAVG}^1 and \bar{D}_{TAVG}^2 for the data of July 16 and August 12 respectively, i.e., for the data of July 16, out of all possible combinations of r ($r = 1, 2, \dots, 12$) spectral channels out of twelve available channels, the channel combination which had the highest value of \bar{D}_{TAVG}^1 was ranked 'first'; the channel combination which had the second highest value of \bar{D}_{TAVG}^1 was ranked 'second' etc. From the values of the average transformed divergence, classification accuracy can be reasonably predicted from the results of Swain et al.^[14] (1973).

To fulfill one of the main objectives of the study-- evaluation of different combinations of wavelength regions, the following criterion is proposed.

Each of 12 available channels of the multispectral scanner can be placed in one of the four wavelength regions--visible, near infrared, middle infrared and thermal infrared, as shown in Table I. Thus, any combination of spectral channels can be called as the corresponding combination of the wavelength regions. For example, channel combination 1, 8, 10 and 12 is called combination of visible, near infrared, middle infrared and thermal infrared wavelength regions, and is denoted by V N M T. Consider first s combinations of r spectral channels out of n available channels.

Number of times a particular combination of wavelength regions occurs in first s combinations of spectral channels, the combinations of spectral channels being ranked in the descending order of \bar{D}_{TAVG}^1 (for data of July 16) and \bar{D}_{TAVG}^2 (for data August 12).

Let p = $\frac{\text{total number of times the same combination of wavelength regions occurs when considering all possible (i.e., } \frac{n!}{r!(n-r)!} \text{) combinations of spectral channels.}}{\text{total number of times the same combination of wavelength regions occurs when considering all possible (i.e., } \frac{n!}{r!(n-r)!} \text{) combinations of spectral channels.}}$ (8)

Let the value of p lie in the very approximate range $\frac{0.15 n!}{r!(n-r)!}$ to $\frac{0.25 n!}{r!(n-r)!}$. The numbers 0.15 and 0.25 donot have any significance and are used to denote very approximate limits of p. Value of 'p' for a certain combination of wavelength regions denotes the probability of selection of that wavelength region in first s combinations of spectral channels, the probability of selection of all possible combinations of spectral channels comprising that wavelength combination being taken as one. Thus p, can be considered to be a criterion for evaluating the ability of combinations of wavelength regions for providing statistical separability of the agricultural cover types. The greater the value of p for a given combination of wavelength regions, the more statistically separable the agricultural cover types are likely to be in that combination of wavelength regions.

IV - RESULTS AND DISCUSSION

The overall separability of green forage from the other agricultural cover types was found to be considerably lower than the corresponding separability of corn, soybeans and forest because the standard deviation of the mean response of green forage was largest among the agricultural cover types. This is because there was much natural variability in the spectral characteristics of hay as well as pasture. The overall separability of forest from other agricultural cover types was found to be considerably higher than the corresponding separability of corn, soybeans and green forage.

Tables II (A) & II (B) give the values of \bar{D}_{TAVG}^1 and \bar{D}_{TAVG}^2 respectively, for first five combinations of one to eleven spectral channels. The values of divergence shown in the Tables II (A) & II (B) are smaller than these have generally been found by the authors in the past during the analysis of similar multispectral scanner data (for example, see ref. 8). This is primarily because the standard deviation of the mean response of the agricultural classes in the spectral channels were relatively large and as a result, there was considerable overlap among the values of their mean response. It is believed that the divergence values shown in this table would have been considerably higher if the agricultural classes were further subdivided into spectrally distinct classes using LARSYS cluster algorithm in the 12 available channels; and the weights between the spectrally distinct classes within each agricultural class were taken equal to zero in calculating the divergence values using the LARSYS feature selection algorithm (i.e., the feature selection algorithm disregards the divergence between the spectrally distinct classes within each agricultural class). This is especially true for extremely low values of \bar{D}_{TMIN}^1 (see Table V). For the data of July 16, minimum of the values of the pairwise divergences, \bar{D}_{TMIN}^1 , was mostly found to be between the classes green forage and corn. Green forage and corn were extremely hard to separate because of considerable overlap in the values of their mean response due to extremely large standard deviation of the green forage. \bar{D}_{TMIN}^1 and \bar{D}_{TAVG}^1 could have been much higher if the green forage had been further subdivided into spectrally distinct classes using LARSYS cluster algorithm, as explained above. In addition, it was harder to separate corn from soybeans for the data of July 16 than for the data of August 12. Table V shows that there are large differences in the highest and the lowest

values of \bar{D}_{TAVG} as well as \bar{D}_{TMIN} , obtained from the divergence values in all possible combinations of spectral channels. This indicates the importance of selecting the proper combination of the spectral channels in order to get good separability of the agricultural cover types.

As one would expect, the greatest separability of the agricultural cover types is obtained by using all the 12 available channels (Table II). However, an increase in the number of channels used in a classification algorithm, requires a disproportionate increase in computer time⁷. It seems from the values of average transformed divergence (Table II) that the subset of five or six spectral channels for the data of Aug. 12 and the subset of five spectral channels for the data of July 16 may fulfill the dual requirements of adequate classification accuracy and moderate computer time⁷. However, this conclusion is very preliminary because no classification of the area or cost benefit analysis was done.

Tables III (A) and III (B) give the frequency each spectral channel was selected in combinations of three to six spectral channels for the data acquired on July 16 and August 12, respectively. These show that channel numbers 1, 2, 6 and 7 are more valuable for the data of July 16 than the data of August 12; whereas, channel numbers 3, 4, 5, 8 and 11 are more valuable for the data acquired on August 12. Channel numbers 9 and 12 seem to be a little bit more valuable for the data of July 16. Channel number 10 seems to be about equally valuable for the data of July 16 and August 12.

Table IV gives the 'p' value (eq. (8)) for many combinations of three to six wavelength regions. The conclusions suggested from the data analysed are as follows (Tables II & IV). Where no date of acquiring the data is mentioned, the conclusions are applicable to the data acquired on July 16 as well as August 12.

For getting greatest overall statistical separability of agricultural cover types: channel 7 (0.61 - 0.70 μm , red channel) is found to be the best channel (Table II). If a scanner is to have only one channel, it should be in the visible wavelength region, probably in the red. It should be pointed out that the predominant pigments of the plant leaf absorb in the vicinity of 0.44 μm but only chlorophyll¹⁶ absorbs in the red in the vicinity of 0.64 μm . The reason of channel 7 being the best channel may be that there are significant differences in the chlorophyll content of different agricultural cover types which give rise to differences in their mean response in channel 7, and hence a relatively large value of average transformed divergence between them. Additional reason for channel 7 being the best channel may be that the red wavelength region is extremely favorable for qualitative and quantitative description of the soils¹⁷. Also, thermal channel is found to be the second best channel (Table II) for the data acquired on August 12. However, channel 6 (0.58 - 0.65 μm) is found to be the second best channel for the data acquired on July 16.

In the subset of two channels, one channel in the visible and the other in the middle infrared are found to be the best choice. Because of the presence of water absorption bands in the middle infrared wavelength region, surface geometry of the target and the moisture content of its top (of the order of micrometers) layers determine its reflectance in the middle infrared wavelength region. The middle infrared wavelength region is valuable because there are significant differences in the geometry (of the surface) and/or moisture content of top layers of the agricultural cover types.

In the subset of three channels, one channel in the visible, one in the middle infrared and thermal channel are found to be the best choice (Table IV). The thermal channel is valuable because it is the only channel measuring emission from the natural targets. In the subset of four channels, one channel in each of the visible, the near infrared, the middle infrared and the thermal infrared are found to be the best choice. It indicates that each wavelength region is valuable in its own way. This is because there are differences in the spectral characteristics of the agricultural cover types in each of wavelength regions. The near infrared wavelength region is useful because substantial contrasts between the agricultural covers and soils occur in this region. Thus, the near infrared region is especially useful when there are substantial differences in the percentage ground covers of the agricultural cover types.

In the subset of five channels, 2 channels in the visible and one channel in each of the near infrared, the middle infrared and the thermal infrared are found to be the best choice (Table IV). In the subset of six channels, two channels in each of the visible and the middle infrared, one channel in each of the near infrared and the thermal infrared are found to be the best choice. Also two channels in each of the visible and the near infrared, and one channel in each of the middle infrared and the thermal infrared are found to be the second best choice.

It was found that in the subset of six channels, the thermal channel was selected in each of the first 184 combinations and 127 combinations for the data acquired on July 16 and August 12 respectively. The thermal channel contains information about the radiant temperatures of the targets in the wavelength region 9.3 to 11.7 μm . There are found to be significant differences in the radiant temperatures of the agricultural cover types. The radiant temperature of a plant can be found by doing an energy balance on it and it depends upon factors such as: radiation incident on the plant, plant geometry and size, spectral properties of the plant (including soil), percent ground cover, convection coefficient and transpiration rate of the plant, etc.¹⁸. There are found to be differences in the values of the above variables for different agricultural cover types which give rise to differences in their radiant temperatures. Thus, irrespective of what other five channels are used, adding thermal channel to it usually increased the overall separability of the agricultural cover types significantly.

It can be seen from Tables II to V that there are considerable differences in the results obtained from the analysis of data of July 16 and August 12. However, it is encouraging to find that the best combinations (best combination defined as the one with highest value of p) of two to six wavelength regions are the same for the data of July 16 as well as August 12. It means that although the analysis was done for the data acquired at two (July 16 and August 12) times, the results obtained from this analysis may be applicable to the data acquired at some other times of the year.

In conclusion, it should be said that determining which combinations of one, two, ..., eleven spectral channels out of twelve available spectral channels give greatest overall statistical separability of the agricultural cover types is a complex problem because statistical separability in any given combination of spectral channels depends upon many variables, such as quality of data in the spectral channels, quality and quantity of the ground truth available, time of acquiring the data, human decisions (number of fields, field boundaries to be selected, etc.) involved, environmental variables, plant variables, soil variables, etc.¹⁸. However, determining which combinations of one through six wavelength regions give greatest overall statistical separability of the agricultural cover types is a relatively less complex problem (Table IV). It should be pointed out, although the flightlines analysed had considerably different characteristics than the flightline analysed by Coggeshall and Hoffer⁷ (Secs. II & III), many of the conclusions presented in this paper are the same as obtained by them. It means that although the analysis was done for three flightlines, the results obtained from this analysis may well be applicable to other flightlines having considerably different characteristics than these three flightlines. The overall statistical separability of the agricultural cover types was found to be greater for the data of August 12 than the data of July 16 (Table II).

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TABLE I Wavelength Bands of the Spectral Channels

Channel No.	Wavelength Band (Micrometers)	Wavelength Region
1	0.46 - 0.49	visible
2	0.48 - 0.51	visible
3	0.50 - 0.54	visible
4	0.52 - 0.57	visible
5	0.54 - 0.60	visible
6	0.58 - 0.65	visible
7	0.61 - 0.70	visible
8	0.72 - 0.92	near infrared
9	1.00 - 1.40	near infrared
10	1.50 - 1.80	middle infrared
11	2.00 - 2.60	middle infrared
12	9.30 - 11.70	thermal infrared

TABLE II Transformed Divergence for Subset of One to Twelve Spectral Channels

(A) Data Acquired: July 16, 1971

Note: The combination of channels were selected in the descending order of \bar{D}_{TAVG}^1 (see eqs. (5) & (6))

Channels Selected	\bar{D}_{TMIN}^1	\bar{D}_{TAVG}^1	Channels Selected	\bar{D}_{TMIN}^1	\bar{D}_{TAVG}^1	Channels Not Selected	\bar{D}_{TMIN}^1	\bar{D}_{TAVG}^1	Channels Not Selected	\bar{D}_{TMIN}^1	\bar{D}_{TAVG}^1
Subset of One Channel			Subset of Four Channels			Subset of Seven Channels			Subset of Ten Channels		
7	43	1112	4 7 11 12	584	1732	1 3 5 6 10	867	1843	3 5	1057	1887
6	59	986	4 7 10 12	555	1728	1 3 5 6 9	882	1842	1 3	1016	1885
11	52	950	5 7 11 12	526	1711	1 3 5 8 10	870	1841	1 5	1018	1883
2	35	894	7 9 11 12	573	1711	1 3 5 6 8	843	1840	3 10	1039	1881
1	33	855	4 6 11 12	598	1710	2 3 5 6 9	884	1839	3 9	1051	1881
Subset of Two Channels			Subset of Five Channels			Subset of Eight Channels			Subset of Eleven Channels		
7 10	254	1448	2 4 7 11 12	759	1785	1 3 5 10	938	1861	3	1079	1894
7 11	284	1447	1 4 7 11 12	748	1784	1 3 5 9	955	1860	5	1075	1892
7 12	339	1420	2 4 7 10 12	733	1783	1 3 5 6	910	1860	1	1049	1891
6 11	339	1380	1 4 7 10 12	711	1781	1 3 5 8	912	1858	4	1066	1889
6 10	293	1367	2 4 6 11 12	790	1776	3 5 6 9	965	1857	2	1068	1887
Subset of Three Channels			Subset of Six Channels			Subset of Nine Channels			Subset of Twelve Channels		
7 11 12	434	1626	2 4 7 9 11 12	798	1821	1 3 5	980	1876	All 12		1898
7 10 12	414	1618	2 4 7 8 11 12	831	1819	3 5 10	1017	1873	Channels		
7 9-12	422	1593	1 4 7 8 11 12	836	1817	3 5 9	1030	1873			
6 11 12	501	1584	2 4 7 10 11 12	803	1817	3 5 6	986	1872			
4 7 10	398	1577	1 4 7 9 11 12	787	1815	1 3 10	974	1872			

TABLE II (CONTINUED)

(B) Data Acquired: August 12, 1971

Note: The combinations of channels were selected in the descending order of \bar{D}_{TAVG}^2 (see eqs. (5) & (6))

Channels Selected	\bar{D}_{TMIN}^2	\bar{D}_{TAVG}^2	Channels Selected	\bar{D}_{TMIN}^2	\bar{D}_{TAVG}^2	Channels Not Selected	\bar{D}_{TMIN}^2	\bar{D}_{TAVG}^2	Channels Not Selected	\bar{D}_{TMIN}^2	\bar{D}_{TAVG}^2
Subset of One Channel											
7	138	1141	4 8 11 12	1390	1832	1 3 5 6 9	1643	1931	1 6	1696	1951
12	71	1109	3 8 11 12	1301	1829	1 2 3 5 6	1647	1930	3 6	1691	1950
11	153	1004	4 7 11 12	1391	1828	1 3 5 7 9	1636	1929	2 6	1691	1949
10	33	996	2 4 8 11	1405	1827	1 3 5 6 7	1661	1928	1 3	1691	1949
6	163	925	4 8 10 12	1420	1826	1 2 - 6 9	1587	1927	5 6	1694	1949
Subset of Two Channels											
7 11	444	1573	2 4 8 11 12	1531	1884	1 3 5 6	1680	1942	6	1702	1953
9 12	429	1555	4 7 8 11 12	1484	1884	2 3 5 6	1673	1939	1	1703	1952
3 8	433	1528	4 8 10 11 12	1485	1883	1 3 5 7	1672	1939	3	1697	1952
2 8	349	1519	4 7 10 11 12	1483	1881	1 2 5 6	1667	1938	5	1700	1952
11 12	691	1506	4 9 10 11 12	1503	1880	2 3 5 7	1661	1937	2	1696	1951
Subset of Three Channels											
3 8 11	1015	1733	2 4 8 10 11 12	1603	1914	1 3 6	1685	1947	All avail-	1708	1955
3 8 10	1083	1728	4 7 8 10 11 12	1566	1914	1 5 6	1688	1946	able twel-		
4 8 11	1166	1726	4 6 8 10 11 12	1570	1911	3 5 6	1686	1946	ve channels		
4 9 12	986	1720	4 7 9 10 11 12	1556	1910	2 5 6	1684	1945	used		
4 10 12	1248	1715	2 4 9 10 11 12	1592	1909	1 3 5	1684	1945			
Subset of Four Channels											
Subset of Five Channels											
Subset of Six Channels											
Subset of Seven Channels											
Subset of Eight Channels											
Subset of Nine Channels											
Subset of Ten Channels											
Subset of Eleven Channels											
Subset of Twelve Channels											

\bar{D}_{TAVG}^1 and \bar{D}_{TAVG}^2 denote the average transformed divergence, averaged over agricultural class pairs, for the data acquired on July 16 and August 12 respectively (see eq. (5)).
 \bar{D}_{TMIN}^1 and \bar{D}_{TMIN}^2 denote the minimum values of the pairwise divergence between the agricultural classes for the data acquired on July 16 and August 12 respectively (see eq. (6)).
 The wavelength band of each channel is given in Table I.

TABLE III Frequency of Selection of Each Channel

(A) Data Acquired: July 16, 1971

Number of Times the Channel is Selected Out of First

Channel Number	Forty combinations of three channels	Hundred combinations of four channels	150 combinations of five channels	200 combinations of six channels
1	2	18	38	61
2	6	26	51	84
3	4	17	31	59
4	6	32	58	93
5	3	14	31	65
6	10	28	59	91
7	24	49	84	143
8	4	27	44	74
9	10	33	58	92
10	13	41	72	111
11	16	46	82	131
12	20	78	145	196

(B) Data Acquired: August 12, 1971

1	1	9	20	49
2	6	20	45	72
3	6	21	41	68
4	11	46	82	117
5	7	24	38	70
6	5	13	33	57
7	9	26	57	89
8	18	48	80	125
9	5	34	56	100
10	11	37	79	124
11	17	51	100	152
12	24	69	119	117

Note: The combinations of channels were selected in the descending order of \bar{D}_{TAVG}^1 and \bar{D}_{TAVG}^2 for the data acquired on July 16 and August 12 respectively (see eq. (5)).

TABLE IV Frequency of Selection of Various Combinations of Wavelength Regions

(A) Data Acquired: July 16, 1971

Note: The combinations of channels were selected in the descending order of \bar{D}_{TAVG}^{-1} (see eqs.(5) & (6)).

Subset of 3 channels	p value	Subset of 5 channels	p value
V M T	0.71	V V N M T	0.74
V N T	0.50	V V V M T	0.61
V V M	0.31	V V M M T	0.52
V N M	0.25	V N M M T	0.52
V V T	0.19	V N N M T	0.42
		V V V N T	0.23
		V V N M M	0.05
		V V V N M	0.02

Subset of 4 channels	p value	Subset of 6 channels	p value
V N M T	0.89	V V N M M T	0.73
V V M T	0.76	V V N N M T	0.59
V M M T	0.57	V V V V M T	0.57
V V N T	0.38	V V V N M T	0.56
V N N T	0.28	V V V M M T	0.51
V V N M	0.20	V V V N N T	0.08
V V V M	0.07	V V V N M M	0.06

(B) Data Acquired: August 12, 1971

Note: The combinations of channels were selected in the descending order of \bar{D}_{TAVG}^{-2} (see eqs. (5) & (6)).

Subset of 3 channels	p value	Subset of 5 channels	p value
V M T	0.93	V V N M T	0.78
V N T	0.71	V N N M T	0.64
V N M	0.43	V N M M T	0.57
V V M	0.07	V V M M T	0.43
V V N	0.05	V V V M T	0.30
		V V N M M	0.31
		V N N M M	0.14
		V V N N M	0.09
		V V V N M	0.09
		V V V N T	0.04
		V V N M T	0.03

Subset of 4 channels	p value	Subset of 6 channels	p value
V N M T	1.00	V V N M M T	0.88
V N N T	0.57	V V N N M T	0.71
V V M T	0.48	V N N M M T	0.57
V V N T	0.38	V V V N M T	0.56
V N M M	0.36	V V V M M T	0.46
V M M T	0.29	V V V N M M	0.24
V V N M	0.27	V V N N M M	0.19
V N N M	0.07	V V V N N T	0.11
V V M M	0.05	V V V V M T	0.10
		V V V N N M	0.06

V, N, M and T denote visible, near infrared, middle infrared and thermal infrared wavelength regions respectively (see Table I). p value is calculated using eq.(8) from about first 1/5th of the all possible combinations of r (r= 3,4,5 and 6) spectral channels out of available 12 spectral channels.

TABLE V - Range of Values \bar{D}_{TMIN} and \bar{D}_{TAVG}

Note: All possible combinations of spectral channels were considered for each of the divergence values given in this table.

(A) Data Acquired: July 16, 1971

	Subset of One channel	Subset of Two channels	Subset of Three channels	Subset of Four channels	Subset of Five channels	Subset of Six channels	Subset of Seven channels	Subset of Eight channels	Subset of Nine channels	Subset of Ten channels	Subset of Eleven channels
Highest \bar{D}_{TAVG}^{-1}	1112	1448	1626	1732	1785	1821	1843	1861	1876	1887	1894
Lowest \bar{D}_{TAVG}^{-1}	214	622	1009	1126	1332	1559	1604	1655	1702	1756	1788
Highest \bar{D}_{TMIN}^{-1}	67	333	495	628	748	820	888	920	962	987	1009
Lowest \bar{D}_{TMIN}^{-1}	12	24	54	196	416	549	608	672	788	874	957

Using all available twelve channels: $\bar{D}_{TAVG}^{-1} = 1898$, $\bar{D}_{TMIN}^{-1} = 1101$

(B) Data Acquired: August 12, 1971

	Subset of One channel	Subset of Two channels	Subset of Three channels	Subset of Four channels	Subset of Five channels	Subset of Six channels	Subset of Seven channels	Subset of Eight channels	Subset of Nine channels	Subset of Ten channels	Subset of Eleven channels
Highest \bar{D}_{TAVG}^{-2}	1141	1573	1733	1832	1884	1914	1931	1942	1947	1951	1953
Lowest \bar{D}_{TAVG}^{-2}	739	970	1096	1257	1422	1579	1697	1778	1866	1909	1936
Highest \bar{D}_{TMIN}^{-2}	219	691	1260	1428	1536	1603	1647	1680	1688	1696	1703
Lowest \bar{D}_{TMIN}^{-2}	33	202	323	418	603	793	943	1099	1390	1537	1616

Using all available twelve channels: $\bar{D}_{TAVG}^{-2} = 1955$, $\bar{D}_{TMIN}^{-2} = 1708$

\bar{D}_{TAVG}^{-1} and \bar{D}_{TAVG}^{-2} denote the average transformed divergence averaged over agricultural class pairs, for the data acquired on July 16 and August 12 respectively (see eq. (5)).

\bar{D}_{TMIN}^{-1} and \bar{D}_{TMIN}^{-2} denote the minimum values of the pairwise divergence between the agricultural classes for the data acquired on July 16 and August 12 respectively (see eq. (6)).