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EXPERIMENTAL EXAMINATION OF SIMILARITY MEASURES AND PREPROCESSING METHODS USED FOR IMAGE REGISTRATION

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by

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I. Introduction and Summary

Machine processing of multispectral scanner imagery from aircraft and satellites for extraction of information about earth resources has evolved into a major field of research and application in recent years[1,2]. An integral part of this field is the utilization of multivariate statistical analysis methods based on coincident images from various spectral bands across the electromagnetic spectrum. Early multispectral scanner systems did not image all spectral bands in coincidence and an image registration problem existed which was studied by Anuta[3] in 1967 at LARS and others. The more recent availability of repetitive coverage by aircraft and satellite scanner systems has created interest in using the time dimension for analysis of the dynamics of scene objects. Machine (computer) analysis of the temporal dimension requires registration

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of images collected at different times under different sensor states. The techniques and software developed earlier at LARS have been applied successfully to the temporal registration problem; however, improved techniques are needed to improve the efficiency and accuracy of the process. Emmert[4] studied advanced methods for temporal registration; however, application of his results requires knowledge of temporal statistics which are not easily obtained. The study reported here was conducted to compare various image correlation algorithms which have been developed recently to define improved approaches for updating the existing LARS system. Particular emphasis was placed on the temporal registration of wheat land imagery in support of the LACIE (Large Area Crop Identification Experiment) program, however, the results have relevance for any registration problem.

The purpose of this study is thus the experimental examination of several approaches to the image correlation problem. The problem addressed is that of finding the relative translations of two sub-images which are not spatially distorted with respect to each other in any way. Emmert[4] studied the problem of detecting geometric distortion but work on this problem has been limited. The approaches first involve the type of similarity measure that is to be used in determining how well or where the images match. Secondly, different methods for preprocessing the images prior to use of the similarity measure and their effects on the ability to overlay two images are studied.

In order to find the correct registration position some sort of measure of the similarity of the two images to use must be chosen. The registration position is then that location at which the two images are most similar. For this study three measures are compared: the correlation coefficient, the correlation function, and the sum of the absolute values of the difference.

The correlation coefficient measures the linear relationship of two images on an absolute scale ($|\rho| \leq 1$). A value for $|\rho|$ close to one indicates that the two images are highly similar. This is the measure presently used in the registration system at LARS.

The correlation function is the sum of the cross product of the corresponding points in both images. This is an unnormalized measure, so that it denotes the registration merely by a maximum or minimum, the actual value having no bearing on the similarity.

The sum of the absolute difference method is like the correlation function in that it is an unnormalized measure. This is the measure used for a class of similarity detection algorithms developed at IBM[5].

Location of the correct registration position may be facilitated by preprocessing the images prior to use of a similarity measure to do the actual registration. Such preprocessing, if shown to give an improvement, can be viewed as part of an optimum type processor for the registration. For example, the entire registration procedure may be more reliable with a preprocessed data set, or one may gain a substantial savings in the time and number of operations required.

For this study five types of preprocessing were chosen: magnitude of the gradient (thresholded and not thresholded), local gradient*[6] (thresholded and not thresholded), and thresholding the image at its median.

The first part (Section II) of this study is concerned with the comparative performance between the similarity measures using the original data and data operated on by the several preprocessing techniques. For all cases it was found that the correlation coefficient gave the best performance (ability to locate acceptable registration positions). However, the absolute value of the difference also performed quite well for data preprocessed using the magnitude of the gradient, and gave almost the same results as the correlation coefficient for preprocessed data using the thresholded magnitude of the gradient. In view of the number of operations and time required for locating the registration positions, it may be advantageous to use the absolute value of the difference instead of the correlation coefficient for these latter two preprocessing techniques since the performance is about the same.

The relative reliability between the preprocessing methods for a given similarity measure is the object of the second part (Section III). The ability to locate acceptable registration

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*"local gradient" refers to an algorithm suggested by P. Beaudet at Computer Sciences Corporation.

positions for the preprocessing techniques using the correlation coefficient as the similarity measure are compared. The results can be divided up into three sections: overall results; high correlation results ($|\rho| \geq 0.5$ for the original data); and low correlation results ($|\rho| < 0.5$ for the original data).

Overall, the magnitude of the gradient performed the best (fewest number of false registration positions) while the thresholded gradient and thresholded local gradient also did quite well. In this case there did seem to be a slight improvement using the preprocessed data as opposed to the original data.

For the high correlation case all of the methods including the original data did exceedingly well with the magnitude of the gradient again performing the best. One may infer from these results that when the scenes are highly correlated, any choice will give about the same performance.

The most striking results were obtained for the low correlation comparisons. For this case there was a substantial increase in performance over the original data for the magnitude of the gradient, thresholded magnitude of the gradient, and thresholded local gradient. The magnitude of the gradient again gave the superior performance.

Based on these three sets of results it can be concluded that when the scenes are poorly correlated it is advantageous to employ preprocessing prior to the image registration operation. Of the preprocessing techniques examined above, the best choice seems to be the magnitude of the gradient since it had fewer indicated false positions than any of the other methods. It also requires a smaller number of operations for implementation than the other two methods that also did reasonably well (thresholded gradient and thresholded local gradient).

Several other points of interest were considered in this study. The experimental results obtained, support the concept that a correlation coefficient close to plus or minus one should give virtually no false registration positions. Also, when the magnitude of the correlation coefficient surface has a peak at a false registration location, it is possible to have a smaller peak at the correct shift position. This implies that search for a secondary peak may be useful in the false peak cases if some further method of distinguishing between the real and false peaks is available.

The correlation data also indicated that both positive and negative peak values can occur. This is anticipated in farmland since it is to be expected that over a span of time in crops or field conditions will occur that can produce negative type changes in the imagery. For example a crop such as winter wheat could be harvested in one field causing an increase in reflectance and an adjacent field could have grown a crop cover and have reduced its reflectance. The problem of searching simultaneously for a maximum and minimum of the correlation coefficient can be avoided by looking only at its absolute value. However, for the correlation function and absolute difference function there is no such way of avoiding this double search, which can be ambiguous since these have no absolute scale. One might consider it desirable to use a data set that inherently bypasses this problem. This is achieved via the magnitude of the gradient and local gradient preprocessing techniques. For these data sets it is necessary to record only one value for any similarity measure used.

One of the most important questions in this study was to determine the preferred processor combination of similarity measure and preprocessing technique to be used for registration. Basing the decision on the performance and number of operations involved, the choice appears to be the magnitude of the gradient in combination with the sum of the absolute value of the difference. In every case the magnitude of the gradient using the correlation coefficient performed better than all other methods. However, in view of the reduced number of operations involved, the absolute difference measure appears attractive since it still gave good performance in general.

II. Similarity Measures

A. Introduction

In considering approaches to the registration problem, a preliminary decision must be made as to what type of similarity measure is to be used to register the two images. These similarity measures may be divided into two general classes. The first provides a measure on an absolute scale. An example of this is the correlation coefficient, where the values range between plus and minus one. Not only is the scale limited for the correlation coefficient, but its value on that scale gives direct measures of how closely the images are linearly related.

The second class indicates the registration position by a maximum or minimum value at the registration location. Two examples of this are the correlation function and the sum of the absolute value of the difference between the two images. In these examples there is no absolute scale, so that the value of this maximum or minimum by itself will not give a good indication of how closely the two images match. The exception to this occurs in the absolute value of the difference case when the two images are identical. A value of zero then infers that the two images match perfectly. However, if one models the difference between the two images as additive noise, one may establish a confidence interval in the absolute value of the difference case by using the resulting minimum value in conjunction with the probability distribution of the noise[5].

The choice to be made with regard to the similarity measures is influenced by several factors. (1) How well do the different methods perform? Is there a way to theoretically predict this performance, and if so, what are the results? Also included in this question is whether there exists some sort of confidence measure so that one may evaluate the results quantitatively. (2) What operations are involved for each of the methods, and what are the comparative times needed? These details are presented in Tables 1 and 2. (3) If the methods have been determined to yield reasonable results with respect to the registration position obtained, then what are

TABLE 1. Equations for Correlation Coefficient, Correlation Function, and Absolute Value of Difference Function

A. Correlation Coefficient, ρ_{1k}:

$$\rho_{\ell k} = \frac{N^2\ \overline{xy}_{\ell k} - \bar{x}\ \bar{y}_{\ell k}}{\{(N^2\ \overline{x^2} - \bar{x}^2)(N^2\ \overline{y^2_{\ell k}} - \bar{y}^2_{\ell k})\}^{\frac{1}{2}}}$$; correlation coefficient at shift (ℓ, k)

where,

$$\overline{xy}_{\ell k} = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}\ y_{i+\ell,\ j+k}$$

$$\bar{x} = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}$$

$$\bar{y}_{\ell k} = \sum_{i=1}^{N} \sum_{j=1}^{N} y_{i+\ell,\ j+k}$$

$$\overline{x^2} = \sum_{i=1}^{N} \sum_{j=1}^{N} x^2_{ij}$$

$$\overline{y^2_{\ell k}} = \sum_{i=1}^{N} \sum_{j=1}^{N} y^2_{i+\ell,\ j+k}$$

B. Correlation Function, $r_{\ell k}$:

$$r_{\ell k} = \overline{xy}_{\ell k} = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}\ y_{i+\ell,\ j+k}$$; correlation function at shift (ℓ, k)

C. Absolute Value of Difference, $a_{\ell k}$:

$$a_{\ell k} = \sum_{i=1}^{N} \sum_{j=1}^{N} \left| x_{ij} - y_{i+\ell,\ j+k} \right|$$; absolute value of difference at shift (ℓ, k)

TABLE 2. Number of Operations Required for Methods Listed in Table 1.

A. Correlation Coefficient

Total No. of operations = No. of operations required for parts (1) thru (5) below

+ $3(M-N+1)^2$ integer multiplications

+ $3(M-N+1)^2$ interger adds

+ $(M-N\underline{-}1)^2$ square roots

+ $(M-N+1)^2$ real divides

(1) $\overline{xy}_{\ell k}$ for all shifts (ℓ,k):

There are two basic ways to compute this: (a) directly, (b) via a transform

(a) Direct calculation

No. of operations = $N^2(M-N+1)^2$ integer multiplies + $(N^2-1)(M-N+1)^2$ integer additions

(b) Via a transform

(i) Fast fourier transform

Let S = smallest power of 2 greater than or equal to M

No. of operations = $S^2(6 \log_2 S+1)$ complex multiples + $6 S^2 \log_2 S$ complex adds

(ii) Fast modulo number transform

Let S = smallest power of 2 greater than or equal to M

No. of operations = $S^2(6 \log_2 S+1)$ double precision multiplies + $6 S^2 \log_2 S$ double precision modulo additions.

(2) $\overline{x}$

No. of operations = N^2-1 integer adds

Table 2 continued

(3) $\bar{y}_{\ell k}$ for all shifts (ℓ,k)

This may be done directly, repeating all the (N^2-1) additions for each shift, or a savings may be realized by noting that it is not necessary to repeat all of these additions for each shift position.

(i) Direct method

No. of operations = $(N^2-1)(M-N+1)^2$ integer additions

(ii) First method reducing redundancy

No. of operations = $4M^2+N^2-4MN+2M-N$ integer additions or subtractions

(iii) Second method reducing redundancy

No. of operations = $6M^2+4M^2-9MN+4M-3N$ integer additions or subtractions

(4) $\overline{x^2}$

No. of operations = N^2 integer multiplications + (N^2-1) integer additions

(5) $\overline{y^2_{\ell k}}$ for all shifts (ℓ,k)

The same options apply for this as apply for $\bar{y}_{\ell k}$ in part (3) above.

(i) Direct method

No. of operations = $N^2(M-N+1)^2$ integer multiplications + $(N^2-1)(M-N+1)^2$ interger additions

(ii) First method reducing redundancy

No. of operations = M^2 integer multiplications + $4M^2+N^2-4MN+2M-N$ integer additions

(iii) Second method reducing redundancy

No. of operations = M^2 integer multiplications + $6M^2+4N^2-9MN+4M-3N$ integer additions

*It is possible to use modulo arithmetic rather than complex numbers for a fast convolution (3). A practical application of this transform for use on the IBM 360 computer was performed by M. Svedlow.

*Subprograms have been written by M. Svedlow to reduce the redundancy with the indicated number of operations.

Table 2 continued

B. Correlation Function

Total No. of operations = No. of operations required for part (1) of the correlation coefficient operations

C. Absolute Value of Difference

Total No. of operations = $N^2(M-N+1)^2$ integer subtractions
+ $N^2(M-N+1)^2$ sign checks
+ $(N^2-1)(M+N+1)^2$ integer additions

the tradeoffs between the methods and the time and operations involved? For example, if one method yields the correct registration position 95% of the time but requires twice the operational time as does a method which is able to find the correct location 75% of the time, which method is more desirable to use? One criterion that is essential for this decision is whether the occurrence of a false registration position is known to be false when it appears.

In order to obtain a feeling for the operations involved with the above mentioned similarity measures, refer to Tables 1 and 2. A smaller image, X, which is N by N points, is to be registered with a subimage of a larger image, Y, which is M by M points. The search area covers the Y image completely, that is, there are $(M-N+1)^2$ shift positions in the search.

B: Experimental Results

The basic concepts behind the use of similarity measures were presented in Section II-A. This section provides a discussion of the experimental results obtained with the previously mentioned similarity measures: the correlation coefficient, the correlation function, and the sum of the absolute value of the difference. The discussion is divided into two general parts for this section: results for the original data, and results for the preprocessed data.

PART 1: Results for Original Data

The results obtained using the original unenhanced data are presented and explained in this part. Two sites for which LANDSAT-1 data was available were chosen for this analysis. The first is in Central Missouri and the second is in Western Kansas. The area in Missouri was picked because a composite run of three frames of LANDSAT MSS data that had been overlayed previously by the registration system at LARS was available. The Kansas Wheatland area was chosen so that: (1) an analysis for seasonal change could be performed; (2) negative changes (negative correlation coefficient) could be examined by using inter-band correlations. The Kansas site is in Finney County and coincides with a LACIE test site. Tables 3 through 5 list the test site dates and general location. Specific test block coordinates are listed in Appendix I. Figure 1 contains an example image reproduction of the Kansas test site.

Table 3. Test Sites in Missouri

Approximate Location: Latitude 37°24' N

Longitude 88°45' W

LARS Run Number: 72033804

Date Data Taken	Designated Time In Following Tables	Corresponding Channels In Run 72033804
9/13/72	t_1	1 - 4
8/26/72	t_2	5 - 8
10/1/72	t_3	9 - 12

Data blocks of size 51 lines by 51 columns are used for comparisons. The locations of the test sites are given by the center of each block.

Spectral band 0.8 - 1.1 μm used for all results.

Table 4. Inter-Run Test Sites in Kansas

Approximate Location: Latitude 37°28' N

Longitude 100°31' W

LARS Run #	Date Data Taken	Designated Time In Following Tables
73046000	7/6/73	t_1
73064000	8/29/73	t_2
74024100	5/26/74	t_3
74024200	7/1/74	t_4

Additional Inter-Run Test Sites in Kansas

Approximate Location: Latitude 37°28'

Longitude 100°31'

LARS Run #	Date Data Taken	Designated Time In Following Tables
73046000	7/6/73	t_1
73064000	8/29/73	t_2
74024100	5/26/74	t_3
74024200	7/1/74	t_4

Table 4. Continued

Data blocks of size 51 lines by 51 columns are used for the comparisons. The locations of the test sites are given by the center of each block.

Spectral band 0.6 - 0.7 μm was used to generate all results.

Table 5. Inter-Band Test Sites in Kansas

Approximate Location: Latitude 37°28'

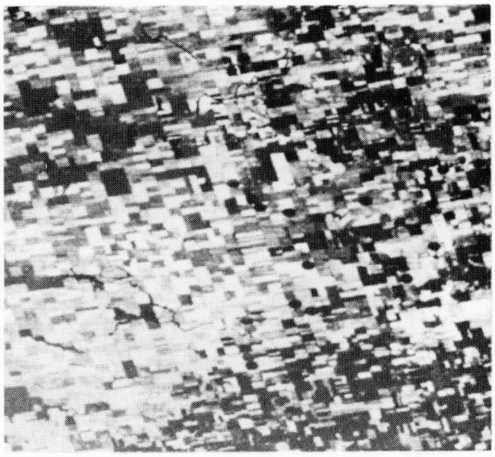
Longitude 100°31'

Spectral Bands Used: 0.6 - 0.7 μm

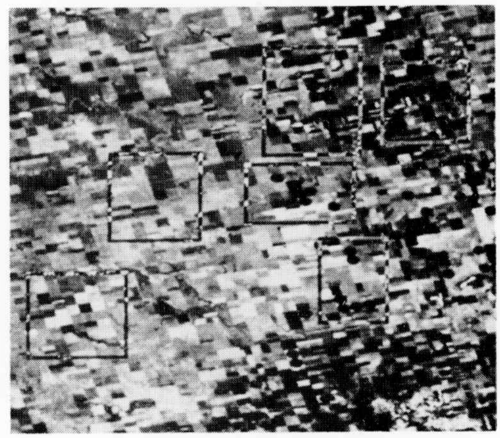
0.8 - 1.1 μm

LARS Run #	Date Data Taken	Designated Time In Following Tables
73046000	7/6/73	t_1
73064000	8/29/73	t_2

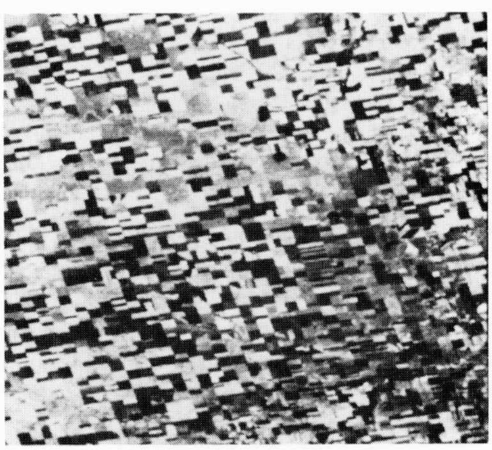
Data blocks of size 51 lines by 51 columns are used for the comparisons. The locations of the test sites are given by the center of each block.



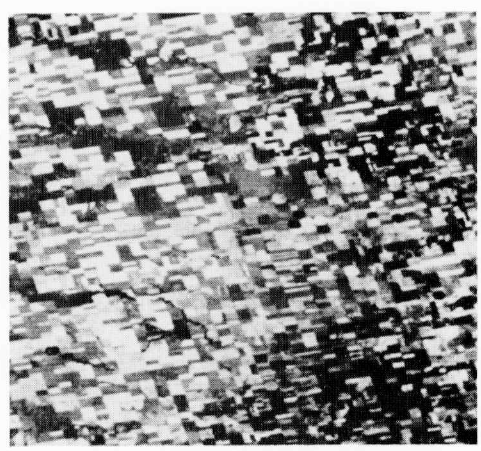
Band 6 (.8 - .9 μm)



Band 5 (.6 - .7 μm)



Band 5 (.6 - .7 μm)



Band 5 (.6 - .7 μm)

Figure 1. Example imagery of Kansas test site. Area is in in southwest part of the state.

The similarity measures (Table 1) were applied to twenty blocks of data from different times in the Missouri data. The output is the shift position for the maximum of the values from all shift positions in two dimensions. Each block was correlated for the three possible combinations of three times. Thus up to 60 correlation results were produced. Five test blocks were correlated in the Kansas test site for the temporal case for four times. Since there are six combinations of four times there were 5 x 6 = 30 correlations performed. Seven areas were correlated in the Kansas data for the interchannel case. Band 5 was correlated with band 7 for one time. The results of these correlations are presented in Appendix II. Fewer false peak positions were observed when the correlation coefficient is used as the similarity measure. A false position is an indicated registration position which is known to be wrong. For the Missouri data the scenes had been previously overlayed to within a few pixels, so that any indication contrary to this is designated as a false peak. The inter-run Kansas data was carefully checked visually and via the correlation coefficient, so that the indicated registration position must be within a few pixels. The inter-band Kansas data is inherently registered so that there should be no indicated relative shift.

It was also observed that even in the acceptable cases in the results do not agree exactly (a discrepancy of one shift position in either direction is observed in several cases). This does not lead to a problem, for the true registration location may lie between the pixels, so that the integer shift for either measure may be accepted. Refer to Section IV where this occurrence is examined experimentally using the correlation coefficient as the similarity measure.

Table 6 summarizes the results for all the correlations. These results support the proposition that the correlation coefficient is the most reliable of the similarity measures for the original data with an empirically correct performance of about 90%. The other two methods have a higher percentage of false position (false peak) occurrences than the correlation coefficient. One may even consider the correlation function results to be completely unacceptable.

PART 2: Results for Preprocessed Data

The experimental results of the similarity measure comparison for the original data were presented in Part 1. They indicated that the correlation coefficient performed in a manner superior to the other two methods. In this part, the same comparison is made for preprocessed data, and as will be seen, the same general trend is evident.

Five types of preprocessing were used for this study: magnitude of the gradient, local gradient (not thresholded), threshold at the median, thresholded magnitude of the gradient, and the local gradient (thresholded). Each of these preprocessing techniques is discussed in detail in Section 3.

Test sites were picked from the Kansas data (cf. Tables 4, 5). The test sites were chosen to provide both inter-run and inter-band analysis. Subimages 101 lines by 101 columns in size were

Table 6. Similarity Measure Comparisons:

Number Of Indicated Shift Errors (False Peaks) For Original Data

Missouri Data

Total # of comparisons for each method: 60

Similarity Measure	No. of Errors
ρ	0
Σxy	29
$\Sigma\|x-y\|$	13

Inter-Run Kansas Data

Total No. of comparisons for each method: 30

Similarity Measure	No. Of Errors
ρ	9
Σxy	27
$\Sigma\|x-y\|$	15

Inter-Band Kansas Data

Total No. of comparisons for each method: 7

Similarity Measure	No. Of Errors
ρ	2
Σxy	5
$\Sigma\|x-y\|$	2

Total No. of Errors For All Three Areas (Composite List)

Total No. of comparisons for each method: 97

Similarity Measure	No. Of Errors	% Correct
ρ	11	89%
Σxy	61	39%
$\Sigma\|x-y\|$	30	70%

preprocessed, and blocks 51 lines by 51 columns were registered using the three similarity measures.

The tables listing the similarity comparisons for the preprocessed data are quite lengthy and are included in Appendix III. They are summarized by tabulating the number of false peaks that were indicated in each case. Refer to Table 7 for this summary.

There are several conclusions one can make from Table 7: (1) In all cases the correlation coefficient yields the highest reliability, (2) The gradient yielded the highest performance percentage for the correlation coefficient and the absolute value of the difference. The thresholded gradient had the highest efficiency for the correlation function. (3) If one is concerned about the number of operations and time factor involved, it may be advantageous to use the correlation function or absolute value of the difference only if the percentage drop in reliability is not too great. For example, with the gradient of the data use of the absolute value of the difference instead of the correlation coefficient will provide a substantial savings in the computations required and still allow an operating efficiency over 90% (Remember that this percentage is only empirical, but it should give a fair indication of the general performance.).

A question arose in the introduction to similarity measures (Section II-A) concerning the ability to distinguish a false peak when it occurs. This is a vital question when it finally must be decided whether two subimages are in registration (A direct application of this decision is seen in Table 7).

Table 7. Similarity Measure Comparisons: Number Of Indicated Shift Errors For Preprocessed Data

Kansas: Inter-Run

Total No. Of Comparisons	66	66	66	30	30
Similarity Measure	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold = 11.0	Local Gradient Threshold = 14.0 $\sigma n^2 = 1.2$
ρ	0	14	23	3	2
Σxy	17	21	28	4	5
$\Sigma\|x-y\|$	5	31	25	4	6

Kansas: Inter-Band

Total No. Of Comparisons	7	7	7
Similarity Measure	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median
ρ	0	0	3
Σxy	0	0	4
$\Sigma\|x-y\|$	0	0	3

Table 7 continued

Similarity Measure Comparisons: Composite of Above Table

For All Kansas Correlations

Total No. Of Comparisons	73		73		73		30		30	
Similarity Measure	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold = 11.0		Local Gradient Threshold = 14.0 $\sigma n^2 = 1.2$	
	No. errors	% correct	No. errors	% correct	No. errors	% correct	No. errors	% correct	No. errors	% correct
ρ	0	100%	14	81%	26	64%	3	90%	2	93%
Σxy	17	77%	21	71%	32	56%	4	87%	5	83%
$\Sigma\|x-y\|$	5	93%	31	58%	28	62%	4	87%	6	80%

Since there is no a priori information as to the exact registration position one cannot determine directly whether the indicated location is false or not. However, this decision may be made in an indirect fashion.

One method can be used with an absolute type measure such as the correlation coefficient. A confidence interval may be set up so that values within certain ranges on this absolute scale will be a criterion for the acceptance or rejection of the tentative registration location (e.g., one might require that $|\rho| > 0.5$ for acceptance when this similarity measure is used).

A second method for making this decision may be used with any similarity measure. Several subimages of one frame are tentatively registered with the corresponding subimages of the second frame. Assuming that a high percentage of the subimage registrations are acceptable, the false peaks will be distinguished by inconsistancies in the relative translations between the corresponding subimages of the two frames. An acceptance interval outside of which the tentative registration positions are designated false can then be established. In such a situation the distinguishability is made easier if the deviations are large. This, if it is shown to be the case, is quite important because it infers that false peaks will be easily recognizable.

An example of the second method denotes the false peaks with the gradient data from Kansas using the absolute value of the difference as the similarity measure (cf. Appendix III.)

Table 8 contains the shift positions for the minimum of the absolute value of the difference listed according to the time pair for the last six areas in Kansas. The false shift positions are marked with an asterisk. Notice that these shift positions differ markedly from those that were accepted. In this case the false positions are easily seen. This result coupled with the high percentage performance of the gradient data with the absolute value of the difference measure indicates that this preprocessing and similarity measure combination may be a good method for registering subimages. A more general approach can be made using statistical decision theory whereby tradeoffs can be made between false peaks and missed peaks. In order to successfully apply this method more information on the statistical properties of the difference images will be required.

Table 8. Similarity Measure Comparisons: False Shift Position Recognition

Kansas: |Gradient|; min $\Sigma|x-y|$ measure

Shift for min $\Sigma|x-y|$

Time Pair	t_1 t_2		t_2 t_3		t_3 t_4	
Area	Δx	Δy	Δx	Δy	Δx	Δy
6	-1	-1	16	8*	16	15*
7	1	-1	-2	-1	-2	0
8	5	10*	3	10*	-15	-13*
9	1	0	-1	0	-2	-1
10	0	2	0	0	-1	-2
11	1	2	0	0	0	-2

*false shift positions

C: Conclusion

The similarity measure comparisons yielded the same general results for all the different types of data sets. The correlation coefficient performed with the fewest number of false registration positions in each of the cases. If an order is to be imposed on the remaining similarity measures tested, the absolute value of the difference seemed to perform slightly better than the correlation function.

In consideration of the time factor and number of operations involved, it appears likely that the correlation coefficient may not be the desired similarity measure when the reliability of another measure which requires fewer computations was reasonably high. This case is evident with the gradient data. The correct registration performance is still quite high when the absolute value of the difference is used instead of the correlation coefficient.

III. Preprocessing Methods

A: Introduction

In the search for an optimum processor for image registration it has been proposed that preprocessing of the data prior to the actual overlaying procedure may be a step towards the solution of this problem[6]. There are several underlying reasons for this suggestion. (1) With a given similarity measure, preprocessing may yield a greater reliability of the system's registration performance. (2) The time and operations required may be substantially reduced. An example of this is conversion of the original image into a binary image (data values of 0 and 1) so that logical operations may be used.

The study undertaken here is an experimental examination of several preprocessing techniques and their effects on image registration. Five methods were chosen: (1) magnitude of the gradient of the data, (2) magnitude of the gradient that has been thresholded, (3) Local gradient with no threshold, (4) Local gradient with a threshold, and (5) thresholding at the median of the data. Appendix IV contains the equations and number of operations required to preprocess a data set via each of these methods.

A comparison of the number of operations required for each of these preprocessing techniques shows that the fewest number are required by thresholding at the median, and the most by the local gradient.

When one considers the choice that must be made, there is one criterion that cannot be forgotten. The preprocessing technique chosen must be compatible with an automatic mode of operation in the registration system so that no human interaction is required. Such a requirement directly concerns the input parameters intrinsic in any of the preprocessing methods. Unless there is some way to choose the input parameters automatically, only those methods requiring no parameters can be considered for automatic operation.

The magnitude of the gradient and threshold at the median methods require no input parameters. The thresholded gradient image necessitates the choice of an appropriate threshold. It may be possible to do this automatically, but it was arbitrarily chosen for this study. The thresholded local gradient requires two input parameters, σ_n^2 and the threshold. The threshold and σ_n^2 must be chosen arbitrarily, but it is inferred[6] that once the threshold has been chosen it will be applicable to any image. The non-thresholded local gradient requires only the σ_n^2 parameter.

The remainder of this section presents the experimental results and their implications. The correlation coefficient was chosen as the similarity measure.

B: Experimental Results

Three general areas were picked for the experimental results: Hill County, Montana (Table 9) was chosen because a run of six LANDSAT MSS frames that were previously overlayed to within a few pixels was available (only five different dates were used because of faulty data in parts of some of the frames). Figure 2 contains example images from four times

Table 9. Test Sites in Hill County, Montana

LARS Run Number: 73124700

Date Data Taken	Designated Time In Following Tables	Corresponding Channels In Run 73124700
5/5/73	t_1	1 - 4
5/23/73	t_2	21 - 24
6/10/73	t_3	17 - 20
7/16/73	t_4	9 - 12
8/3/73	t_5	5 - 8

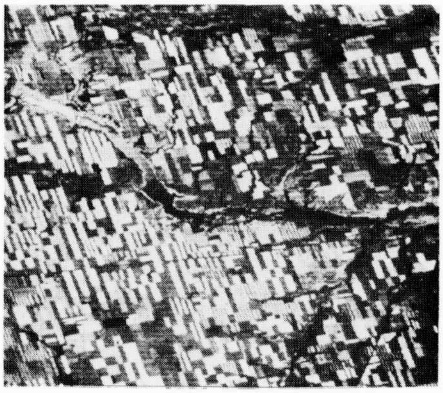
Test Sites

Line Center	Column Center	Area #
110	410	1
170	130	2
415	150	3

Spectral band 0.8 - 1.1 μm used to generate all results.

Data blocks of size 51 lines by 51 columns are used for the comparisons. The locations of the test sites are given by the center of each block.

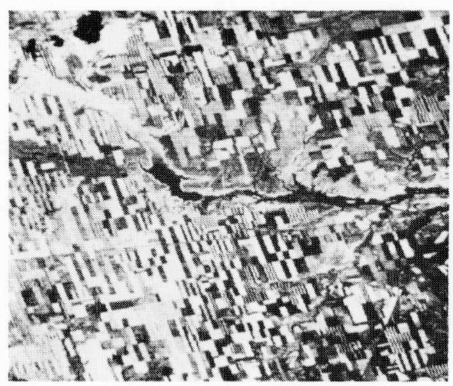
LARS Run 73124700
Line (1,470), Col (1,498)



(VI)
0.6 - 0.69 μm

(V)
0.6 - 0.69 μm

(IV)
0.6 - 0.69 μm



(III)
0.6 - 0.69 μm

Figure 2. Example imagery from Montana site. LARS Run 73124700, Lines 1,470; Cols 1,498.

from this site. A run consisting of three frames overlayed to within a few pixels was available for Tippecanoe County, Indiana (Table 10), so this was used. Figure 3 contains example imagery from the Indiana site. Test sites from Kansas also were picked. Figure 4 contains an example series of the various enhancements for one area in the Montana test site.

A series of tables in Appendix V contain two general sets of information. The first series presents a comparison of the indicated registration positions (integer values) for all of the preprocessing methods with a listing of the value of the correlation coefficient for the original data at the corresponding positions. The second set provides for a comparison of the relative values of the correlation coefficient for all of the techniques. All false registration position occurrences (cf. discussion in Section II-B Part 2) are noted by an asterisk or a blank in Appendix V.

For all of the following results the spectral band was used which performed the best for the original data with the correlation coefficient as the similarity measure.

The relative performance for finding the correct shift positions is difficult to see by looking at these Tables; however, the data can be analyzed more easily by summarizing the results in terms of the number of false shift position occurrences (a false shift position is one which disagrees by more than two shifts in any direction). This is done in Table 11 where the results for all values of the correlation coefficient are given.

Table 10. Test Sites in Tippecanoe County, Indiana

Approximate Location: Latitude 40°20' N

Longitude 86°21' W

LARS Run Number: 72053603

Date Data Taken	Designated Time In Following Tables	Corresponding Channels In Run 72053603
9/30/72	t_1	1 - 4
10/19/72	t_2	5 - 8
11/29/72	t_3	9 - 12

Data blocks 51 lines by 51 columns were used for correlations.

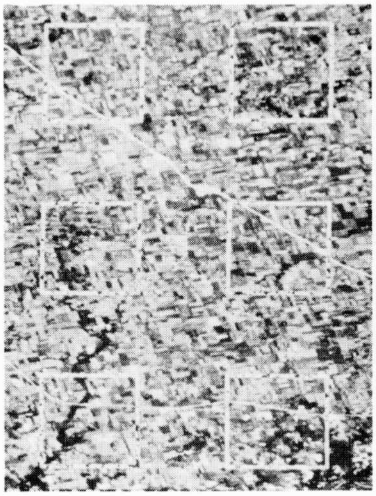
Spectral band 0.8 - 1.1 μm used to generate all results.

Test Sites

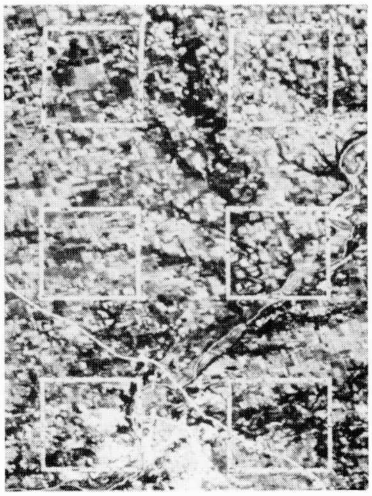
Line Center	Column Center	Area No.
200	200	1
200	400	2
200	600	3
200	800	4
400	200	5
400	400	6
400	600	7
400	800	8
600	200	9
600	400	10
600	600	11
600	800	12

Spectral band 0.8 - 1.1 μm was used to generate all of the results.

Data blocks of size 51 lines by 51 columns are used for the comparisons. The locations of the test sites are given by the center at each block.

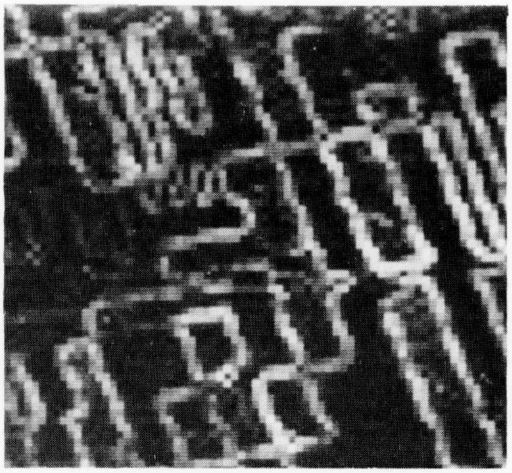


.8 - .9 μmeter band
Col 9,503
Line 100,700

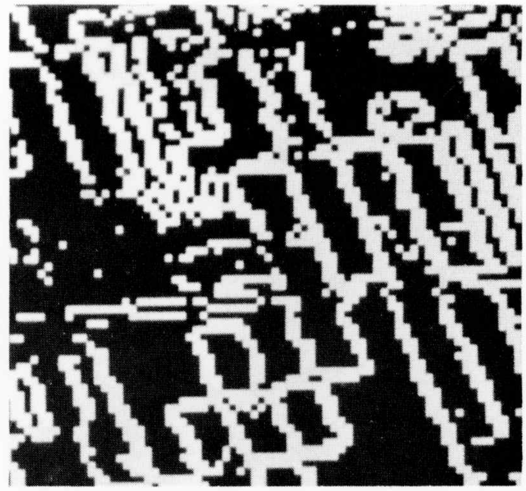


.8 - .9 μmeter band
Col 495,897
Line 100,700

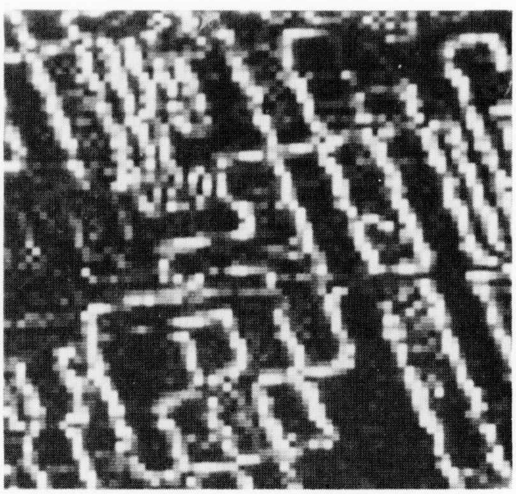
Figure 3. Examples of LANDSAT imagery for the Tippecanoe County, Indiana test site. LARS Run 72056303, data taken September 30, 1972.



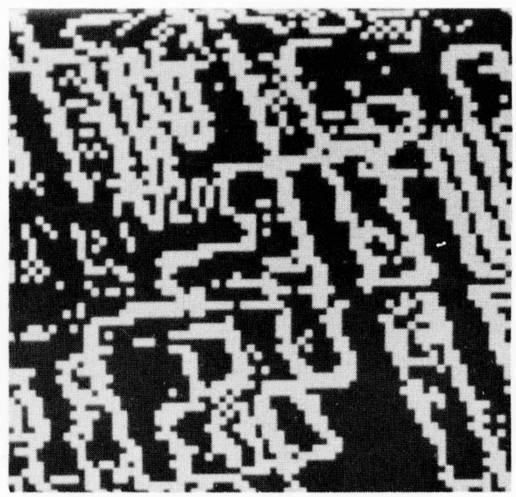
a. Magnitude of Gradient
Ch 24, .8 - 1.1 μm band.



b. Thresholded Magnitude of Gradient
Ch 18, .6 - .7 μm band.

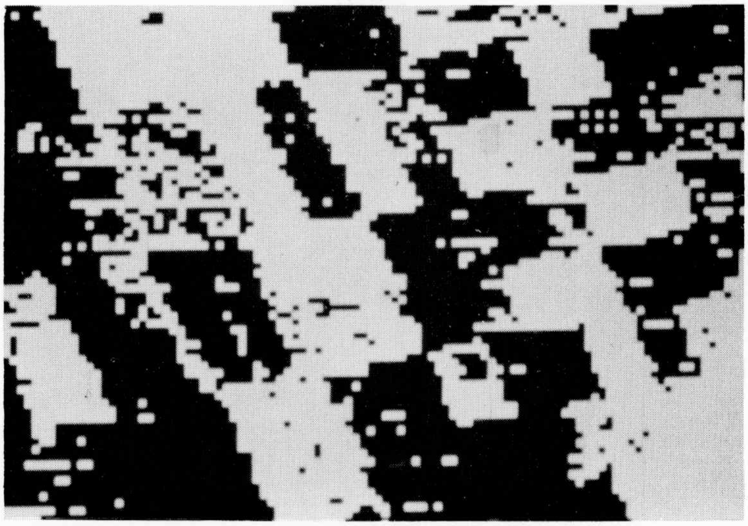


c. CSC Normalized Gradient
Ch 24, .8 - 1.1 μm band.



d. Thresholded CSC Gradient
Ch 24, T = 5.0, σn^2 = 1.2
.8 - 1.1 μm band.

Figure 4. Examples of enhancements for a Montana test subsite.
LARS Run 73124700, Lines 263, 337; Cols 138, 212.



e. Data Thresholded at the Median

Figure 4. continued

Table 11. Number of False Shift Position Occurrences Using the Correlation Coefficient as the Similarity Measure.

General Area	Designation
Hill County, Montana	A
Tippecanoe County, Indiana	B
Kansas: Inter-Run	C
Kansas: Inter-Band	D

(1): Number of False Shift Positions

(2): Percentage Correct

General Area	Number Correlations	Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold		Local Gradient Threshold $\sigma n^2 = 1.2$	
		(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
A	30*	0	100%	0	100%	1	97%	0	100%	0	100%	1	97%
B	36	0	100%	0	100%	0	100%	2	95%	0	100%	0	100%
C	66	20	68%	0	100%	17	74%	23	65%				
C	30									3	90%	3	90%
D	7	2	71%	0	100%	0	100%	3	58%				
Total	139	22	84%	0	100%	18	88%	28	80%				
	96+									3	97%	4	96%

* There are only 92 correlations for thresholded gradient.

\+ There are only correlations for thresholded gradient.

Considering all values of the correlation coefficient, the magnitude of the gradient performed the best. However, the percentage of accepted shift positions was almost equally as high for the thresholded gradient and the thresholded local gradient. Preprocessing of the data seemed to boost the ability to find acceptable registration positions over use of the original data for all but the thresholding at the median method.

Another view of the results may be made by dividing them into the cases where the magnitude of the correlation coefficient is either greater than or equal to 0.5, or less than 0.5 for the original data. This should give an indication of how well the preprocessing techniques compare for both the high and low correlation cases for the original data as opposed to the overall results in Table 12.

Table 13 presents the high correlation coefficient results. In this case each of the methods performed almost equally well with a very high percentage of acceptable registration locations. Notice that the results for the original data were extremely good (as opposed to Table 12) and that the magnitude of the gradient method again performed the best. This seems to imply that all methods work about the same when the original data is highly correlated ($|\rho| \geq 0.5$).

Now consider the low correlation coefficient case (Table 14). The percentage of acceptable registration positions for the gradient, thresholded gradient, and thresholded local gradient, shows a substantial improvement over the original data results.

Table 12. Number of False Shift Position Occurrences Using the Correlation Coefficient as the Similarity Measure.

General Area	Designation
Hill County, Montana	A
Tippecanoe County, Indiana	B
Kansas: Inter-Run	C
Kansas: Inter-Band	D

(1): Number of False Shift Positions

(2): Percentage Correct

$|\rho| < 0.5$ for Original Data

General Area	Number Correlations	Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold		Local Gradient Threshold $\sigma n^2 = 1.2$	
		(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
A	25*	0	100%	0	100%	1	96%	0	100%	0	100%	1	96%
B	25	0	100%	0	100%	0	100%	0	100%	0	100%	0	100%
C	25	0	100%	0	100%	4	84%	3	88%				
C	15									0	100%	1	93%
D	6	1	83%	0	100%	0	100%	2	67%				
TOTAL	81	1	99%	0	100%	5	94%	5	94%				
	65+									0	100%	2	97%

* There are only 24 correlations for thresholded gradient.

\+ There are only 64 correlations for thresholded gradient.

Table 13. Number of False Shift Position Occurrences Using the Correlation Coefficient as the Similarity Measure.

General Area	Designation
Hill County, Montana	A
Tippecanoe County, Indiana	B
Kansas: Inter-Run	C
Kansas: Inter-Band	D

(1): Number of False Shift Positions

(2): Percentage Correct

$|\rho| < 0.5$ for Original Data

General Area	Number Correlations	Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold		Local Gradient Threshold $\sigma n^2 = 1.2$	
		(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
A	5*	0	100%	0	100%	0	100%	0	100%	0	100%	0	100%
B	11	0	100%	0	100%	0	100%	2	80%	0	100%	0	100%
C	41	20	50%	0	100%	13	67%	20	50%				
C	15									3	80%	2	83%
D	1	1	0%	0	100%	0	100%	1	0%				
Total	58	21	62%	0	100%	13	78%	23	62%				
	31+									3	90%	2	93%

* There are only 2 correlations for thresholded gradient.

\+ There are only 28 correlations for thresholded gradient.

Table 14. Average of Magnitude of Correlation Coefficient for Each Preprocessing Method (False Registration Positions Excluded from Averaging)

General Area	Original Data	$\lvert \text{Gradient} \rvert$	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	$\lvert \text{Gradient} \rvert$ Threshold	Local Gradient Threshold $\sigma n^2 = 1.2$
Hill County	.713	.539	.341	.614	.391	.293
Tippecanoe County	.570	.423	.244	.348	.273	.181
Kansas: Inter-Run	.526	.398	.191	.460	.316	.171
Overall Average	.603	.453	.259	.474	.327	.215

The magnitude of the gradient again gave the superior results whereas the original data performed the worst. One may infer from the percentages obtained here (assuming that the number and range of correlation samples is a fair representation) that preprocessing may improve the ability to find acceptable registration positions when the magnitude of the correlation coefficient for the original data is small ($|\rho| < 0.5$).

It was mentioned earlier that a confidence measure for deciding which tentative registration positions are acceptable is necessary. For the results contained in the preceding tables relative shift position information was used to determine which shift positions to accept. However it should also be possible to use the value of the correlation coefficient to decide which positions are acceptable. Since a high value of the magnitude of the correlation coefficient (close to one) implies that the two data sets match fairly well linearly, one should expect few false registration positions for these cases. This is exactly what happened with the results for original data. Comparing Tables 12, 13, and 14 one will see that all but one of the false registration positions occurred when the magnitude of the correlation coefficient was less than 0.5 (the exception occurred for the Kansas inter-band data).

In the above analyses the indicated registration positions were determined by the location of the maximum of the absolute value of the correlation coefficient. As was found out, this indicated position is not always acceptable, the false positions occurring primarily in the low correlation cases for the original data (cf. Tables 12 and 13). One might expect that in these low correlation cases there has been much change between the images, and that the magnitude of the correlation coefficient plot as a function of the shift may have several peaks, one of which is at an acceptable location even though it is not the maximum. Therefore, in looking only for the maximum of the peaks, a false shift position will be recorded even when there does exist a (smaller) peak at an acceptable shift position.

This was examined experimentally with the false position occurrences for the Kansas inter-run data. For the original data, there was a smaller peak at an acceptable location in twelve out of twenty cases, which is roughly a 60% performance. This was also the case for the preprocessed data sets (magnitude of the gradient is excluded because there were no false locations). Such a result implies that the ability to find an acceptable registration position may be increased by looking not only for the maximum peak, but also for secondary peaks if the maximum peak proves to be false.

Another interesting comparison may be made concerning the average values of the magnitude of the correlation coefficients for the different preprocessing techniques and any relationship to performance. The results from Appendix V are averaged (false positions excluded) and presented in Table 14. There does seem to be a relationship between the preprocessing method and the average value of $|\rho|$. The original data consistently has the highest average, and the thresholded local gradient the lowest. The general order is: (1) original data, (2) threshold at the median, (3) magnitude of the gradient, (4) thresholded magnitude of the gradient, (5) local gradient (no threshold), and (6) thresholded local gradient. However, if one now refers to Tables 11, 12, and 13, one will find that there is no direct relationship between the relative values of the correlation coefficients for each of the preprocessing methods and their performance. The original data has the highest average coefficient value, but the magnitude of the gradient gave the highest performance.

C: Conclusions

The use of preprocessing as an initial step in the registration problem stems from the possibility that the reliability of the system may be increased and that the number of operations and time involved might be reduced. Five preprocessing techniques were experimentally compared along with the original data, using the correlation coefficient as the similarity measure. The five preprocessing methods used are: (1) magnitude of the gradient, (2) thresholded magnitude of the gradient, (3) local gradient (no threshold), (4) local gradient (thresholded), and (5) thresholding at the median.

In terms of the fewest number of false peak positions, the magnitude of the gradient performed the best overall. When one looks only at the cases where $|\rho| \geq 0.5$ for the original data, the magnitude of the gradient still performs the best, but there is really no distinguishability since all of the methods, including no preprocessing, worked almost equally well. For the low correlation cases ($|\rho| < 0.5$ for the original data), all of the preprocessing methods except thresholding at the median gave fewer false positions than the original data, with the magnitude of the gradient again doing the best. This implies that preprocessing may help in the low correlation cases.

It was found that the value of the correlation coefficient seemed to have a bearing on the decision for accepting tentative registration positions. In the cases where $|\rho| \geq 0.5$ for the original data, there were virtually no false shift positions (the exception being an inter-band correlation).

Considering the cases in which false tentative registration positions did occur using the maximum of $|\rho|$ as the position choice, it was found that a secondary peak in the correct location existed in about 50% of the cases. This implies that a search for a secondary peak in the false registration cases may improve the ability to find the correct position.

A last interesting comparison was made between the average value of the magnitude of the correlation coefficient and the preprocessing methods. The results indicated a general trend for all the data, but this ordering was not linked to the ability to find acceptable registration positions. The original data had the highest average for $|\rho|$, whereas the magnitude of the gradient had the highest performance.

IV: Interpolated Registration Positions

All of the previous results concerning the registration shift positions recorded those locations to the nearest integer shift. This was adequate in many instances, however it did lead to discrepancies in some cases. In such cases it may be advantageous to find the interpolated shift positions. Then one may find that within a certain tolerance, the discrepancies will disappear.

The primary concern of such differences in this study is the occurrences of one unit shift discrepancies for the acceptable registration positions of two images obtained for two different spectral bands or different preprocessing techniques. For example, one will find that all of the accepted registration shift positions of the different preprocessing techniques for the Kansas inter-run data do not necessarily agree completely among the different preprocessing methods. In many instances there is a discrepancy of one unit shift. If one views the magnitude of the correlation surface, a function of the shift, as a continuous function, then the peak of that function will not necessarily be at an integer shift position.

Interpolation of the correlation function about its peak is used to approximate the correct peak location for the continuous surface. The correlation function is modeled as a two-dimensional quadratic surface about its peak. This quadratic function is found from the integer shift position peak and its four surrounding points. Refer to Table 15 for a derivation of the equations used for this interpolation function.

Two sets of experimental results are used to illustrate the above assertion. As will be seen, they indicate that the

Table 15. Derivation of Interpolated Peak Position

$f(x,y) = a_1x^2 + a_2y^2 + a_3x + a_4y + a_5$, correlation function about its peak

$f(0,0)$ = integer shift position peak

$f(-1,0)$, $f(1,0)$, $f(0,-1)$, $f(0,1)$ = four surrounding points used for finding constants a_i ,$i=1,\cdot\cdot,5$.

x_{peak} = solution of $\frac{\partial f(x,y)}{\partial x} = 0$, x - coordinate of peak

y_{peak} = solution of $\frac{\partial f(x,y)}{\partial y} = 0$, y - coordinate of peak

Solution:

$$x_{peak} = \frac{f(1,0) - f(-1,0)}{2[2f(0,0) - f(1,0) - f(-1,0)]}$$

$$y_{peak} = \frac{f(0,1) - f(0,-1)}{2[2f(0,0) - f(0,1) - f(0,-1)]}$$

interpolated positions in the one unit integer shift difference cases are close enough (substantially less than one unit on the average) to warrant disregarding the discrepancies. The first set concerns the discrepancies when two different spectral bands are used to find the correct registration positions and the second involves the preprocessing methods. Hill County and Tippecanoe County data are used for this presentation. Appendix IV lists the integer and interpolated shift positions, and the magnitude of the interpolated discrepancy.

Integer shift discrepancies of one unit in either direction and the corresponding interpolated shifts are marked with an asterik. Blanks indicate false registration positions (shift difference greater than two). A box around an interpolated shift discrepancy indicates that the difference is larger than 0.5 units.

The reference chosen on which to base the discrepancy results is automatic for the spectral band comparison since there are only two bands. However, there are five preprocessing techniques plus the original data. For these results the original data has been picked as the reference.

The spectral band comparisons. Overall there are nineteen times that the one unit integer shift difference occurred, six for the Hill County data and thirteen for the Tippecanoe data. In all cases the interpolated difference is less than one unit, which implies that the for continuous correlation surface model, the actual peaks are closer than indicated by the one unit integer shift difference. Furthermore, in the majority of cases (fourteen) the interpolated peaks indicated that the discrepancies are substantially less than one unit; they are less than 0.5 units.

The comparisons yield the same general results for the different preprocessong methods. For all of these cases the interpolated difference is again less than one unit; nineteen out of fifty-five are less than 0.5 units.

From these results one may conclude that the integer shift registration positions are coarse enough approximations to the true registration positions to allow for the one unit discrepancies found.

V. Evaluation of Modified Local Gradient Algorithm

A: Introduction

A modified version of the Computer Sciences Corp. algorithm developed by M. L. Nack[7] was evaluated to compare it to the version analyzed in previous sections. In the program used for the evaluation the same basic operations have been performed, but with little emphasis on the maximum efficiency with respect to time required versus the number of operations performed. For instance, the algorithm was implemented using Fortran coding as opposed to assembly language.

Two modifications were made on the original algorithm from the viewpoint of making it more compatible with the rest of the study. The overlay image was restricted to 51 lines by 51 columns (as opposed to 117 lines by 160 columns in the algorithm employed by Nack). Also the search area was restricted to a maximum shift of 16 from the tentative registration position (a 58 shift along the lines and 80 shift along the columns was used in the CSC algorithm). It was possible to restrict the search area because it was known beforehand that the images were registered to within a few pixels (refer to the discussions about how the test sites were chosen).

The second change concerns the rapid rejection algorithm. For the estimation of the pedestal mean and standard deviation every third sample from every third line for each image was used to compute the similarity measure at every third shift position. This procedure differed from that used by Nack only in the sample size picked and the location of the samples. However, the method employed here seemed to give a fairly close estimate of the true mean and standard deviation, so from the viewpoint of estimation, both of these estimation procedures should be considered equivalent.

The experimental analysis of this algorithm has been divided into two basic sections. The first part tested the algorithm without using the rapid rejection subroutine. In this section no estimation procedures were used, so that the similarity measure used by Nack (denote it by p_N) was calculated fully at all of the shift positions.

In the second part, the rapid rejection algorithm was employed with two different rejection level settings. The purpose of this line of analysis was to examine the effect of the estimation procedure on the registration performance compared with the performance when p_N was fully calculated at all shift positions.

B: Results without Rapid Rejection Algorithm

In discussing the results when the rapid rejection subroutine was not used, first begin with the input parameters to the algorithm. LANDSAT spectral bands 0.6-0.7μm and 0.8-1.1μm were used. The preprocessed images were thresholded at the top 15th percentile (Fifteen percent of the number of points in each spectral band of each image were set equal to one and the rest equal to zero. These fifteen percent of the data had the highest values.) The two spectral bands from each image then were combined by using a logical "and" operation. The nine highest values of p_N were used to test for a unique peak.

The experimental results seem to indicate that this processor worked quite well. The peak of the correlation surface was at an acceptable location 99% of the time (190 out of 192 attempts), so that there were only 2 cases in which a false peak was indicated. However, in 63 of these acceptable cases, the peak was designated as not unique by the uniqueness test. This might suggest a new criterion for the acceptability of the peak of the correlation surface as a registration location.

C: Rapid Rejection Algorithm Results

The input parameters listed above were the same when the rapid rejection subroutine was used. However, one additional parameter was required which is used for setting the rejection level. In the algorithm the rejection level is designated by the correlation surface pedestal mean plus a constant, K, times its standard deviation. The original algorithm employed at Computer Science Corporation uses a value of K = 3. (A value of K = 3 is claimed by Nack to allow about 5.6% of the values of p_N to exceed the rejection level. However, this is inconsistent with using a Gaussian model for the correlation surface. For the Gaussian model, K = 3 allows only about 0.1% of the values of p_N to exceed the rejection level. This discrepancy of the two percentages is seen in the discussion of the experimental results.) In this experimental study, values of K = 2 and K = 3 were used to see how they compared with each other and with the algorithm when the rapid rejection subroutine was not used.

When K was set equal to 3, the rejection level seemed to be set too high since fewer than 9 values of p_N were greater than this level in 84% of the registration attempts (167 out of 192). And in 33 of these cases (17% of the attempts), none of the values of p_N exceeded the rejection level (which automatically stopped the registration processor for these areas, so that no registration was performed).

For K = 2, the Gaussian model predicts that 2.3% or 25 values of p_N will exceed the rejection level for each registration attempt. This again fits the experimental results much better than Nack's model where $(100 \times (1/(2K^2)))\% = 12.5\%$ or 135 values should exceed the rejection level.

The next question is, what about false peak discernability? How does it compare with the algorithm when the rapid rejection subroutine is not used? There seemed to be a slight improvement (but one that really is not significant since there were only two false peaks to begin with). No false peaks were indicated for K = 3, and one false peak was indicated for K = 2. Probably the most significant result is that the performance was no worse.

Use of an estimation procedure for the rejection brings up another point that must be considered. The estimation of p_N allows the possibility that the actual maximum will be screened out before it is fully calculated. This did happen 5 times for K = 3 and 4 times for K = 2. However, in all of these cases, the location of this indicated maximum was no more than one shift from the true maximum which was found when the rejection subroutine was not used. Such a result suggests fully calculating the value of p_N in the neighborhood of the indicated maximum to make sure that this is the true maximum value.

D: Conclusions

The ability to find an acceptable peak without the rapid rejection subroutine seemed very good with about a 99% performance. This result is virtually the same as when the magnitude of the gradient of the images were registered using the correlation coefficient.

Use of the rapid rejection algorithm required an estimation procedure which in turn introduced some uncertainty as to whether the indicated peak was the true maximum or not. However, this seemed to be correctable by performing a local search around this indicated peak using all the data to calculate p_N. Also, use of a rejection level did not seem to really hinder the performance when the level was low enough (a level even lower than for $K = 2$ might even do better). Finally, as a conclusion to this evaluation, one must consider the time savings achieved by using the rapid rejection algorithm. For the sampling intervals used in these experiments, there was a time savings by a factor of about ten.

VI. Summary and Conclusions

This report documents the results of registration algorithm research conducted during 1974 and 1975 under NASA Contract NAS9-14016 administered from the Johnson Space Flight Center, Houston, Texas. A number of potentially useful image preprocessing and correlation algorithms were evaluated for effectiveness in temporal LANDSAT image registrations. Test sites were chosen which were typical of the high plains wheat growing areas of great interest to the NASA LACIE (Large Area Crop Inventory Experiment) program.

Six image preprocessing algorithms and three image correlation algorithms were implemented and tested on the example LANDSAT data to determine what the best approach is to finding translational misregistration between time separated data. The magnitude of the gradient enhancement method gave the best overall results for both highly correlated and relatively uncorrelated data. The sum of the absolute value of the difference is recommended as the best overall similarity measure although the correlation coefficient provides slightly better performance at much higher cost. The conclusions are discussed in detail in the summary in Section I and in the individual sections and will not be repeated in detail here.

Definitions of Notation Used

Reference image: This is the background image with which a smaller image, the overlay image, is registered. It is larger than the overlay image to allow for a search area in finding the correct registration position. The size of this image is determined by the size of the overlay image plus the number of shifts in the search for the registration location.

Overlay image: This is the image that is overlayed on a second image, the reference image, by moving the overlay image around until the registration position has been found. The size of this image is 51 lines by 51 columns for all of the results obtained in this study.

ρ: This denotes the correlation coefficient.

Max $|\rho|$: Value of ρ where the maximum of the absolute value of ρ occurs.

Max ΣXY: Maximum of the correlation function (sum of the cross product of two images).

Min $\Sigma|X-Y|$: Minimum of the sum of the absolute value of the difference between two images.

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APPENDIX I TEST SITE COORDINATES IN LARS DATA SETS

MISSOURI TEST SITE COORDINATES IN LARS RUN 72033804

Line Center	Column Center	Area No.
200	200	1
200	400	2
200	600	3
200	800	4
200	1000	5
400	200	6
400	400	7
400	600	8
400	800	9
400	1000	10
600	200	11
600	400	12
600	600	13
600	800	14
600	1000	15
800	200	16
800	400	17
800	600	18
800	800	19
800	1000	20

KANSAS INTER-RUN TEST SITE COORDINATES

LARS Run No.	73046000		73064000		74024100		74024200	
Area No.	Line Center	Column Center	Line Center	Column Center	Line Center	Column Center	Line Center	Column Center
1	111	445	115	389	294	392	218	336
2	116	570	121	514	300	518	223	462
3	211	435	216	380	396	384	318	327
4	111	1495	120	1440	298	1447	219	1398
5	352	210	358	154	538	157	459	98

APPENDIX II

SIMILARITY MEASURE COMPARISON DATA

A. SIMILARITY MEASURE COMPARISONS: MISSOURI

Area No.	Time Pair Compared	Shift for max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.68307	0	1				
	t_1t_3	.66614	0	1			0	1
	t_2t_3	.40775	0	0				
2	t_1t_2	.62720	-1	2			-1	1
	t_1t_3	.76410	0	1			0	1
	t_2t_3	.58512	1	-1				
3	t_1t_2	.80910	-1	1			-1	2
	t_1t_3	.82562	-1	1			-1	1
	t_2t_3	.70994	1	-1			0	-1
4	t_1t_2	.49857	-1	2				
	t_1t_3	.60655	0	0			0	0
	t_2t_3	.37789	0	-1				
5	t_1t_2	.68176	0	1				
	t_1t_3	.56564	0	0			-1	1
	t_2t_3	.21258	0	0				
6	t_1t_2	.78118	-1	1	-1	1	-1	1
	t_1t_3	.75100	0	1	0	1	-1	1
	t_2t_3	.52125	0	0	0	0		

INTER-BAND TEST SITE LOCATIONS IN KANSAS

Area No.	LARS Run No.	Line Center	Column Center
1	73064000	115	389
2	73064000	121	514
3	73064000	216	380
4	73064000	282	434
5	73064000	100	440
6	73064000	245	1750
7	73046000	211	435

ADDITIONAL TEST SITES FOR KANSAS

LARS Run No.	73046000		73064000		74024100		74024200	
Area No.	Line Center	Column Center	Line Center	Column Center	Line Center	Column Center	Line Center	Column Center
6	100	170	105	114	284	118	207	61
7	100	310	105	254	284	258	207	201
8	250	170	255	114	434	118	357	61
9	250	310	255	254	434	258	357	201
10	400	360	405	304	584	308	507	251
11	400	510	405	454	584	458	507	401

A. SIMILARITY MEASURE COMPARISONS: MISSOURI (Continued)

Area No.	Time Pair Compared	Shift for max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x−y|	
			Δx	Δy	Δx	Δy	Δx	Δy
7	t_1t_2	.73226	-1	1			-1	1
	t_1t_3	.64765	0	0			0	0
	t_2t_3	.44625	0	0	0	0		
8	t_1t_2	.94351	-1	0	-1	1	-1	0
	t_1t_3	.94904	0	0	0	0	0	0
	t_2t_3	.92100	1	0	0	0	1	0
9	t_1t_2	.75044	-1	1	0	1	0	1
	t_1t_3	.67579	0	0	0	0	0	0
	t_2t_3	.39060	0	-1	1	-1		
10	t_1t_2	.65386	-1	1			-1	1
	t_1t_3	.60805	0	0			-1	0
	t_2t_3	.32139	1	-1				
11	t_1t_2	.86717	0	1	0	1	0	1
	t_1t_3	.64806	0	1	0	0	0	1
	t_2t_3	.48953	0	0	0	-1		
12	t_1t_2	.63014	0	1			0	1
	t_1t_3	.62631	0	0			0	0
	t_2t_3	.27317	0	-1			1	1
13	t_1t_2	.78658	-1	1			-1	1
	t_1t_3	.64183	0	0			1	0
	t_2t_3	.40879	1	-1				

A. SIMILARITY MEASURE COMPARISONS: MISSOURI (End)

Area No.	Time Pair Compared	Shift for max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
14	t_1t_2	.92710	0	1	0	1	0	1
	t_1t_3	.85639	0	0	0	0	-1	-1
	t_2t_3	.79213	0	-1	0	0	-1	-2
15	t_1t_2	.97032	0	0	1	0	0	0
	t_1t_3	.83383	0	0	1	-1	0	0
	t_2t_3	.78673	0	0	1	-3	-1	0
16	t_1t_2	.84869	0	0	0	0	0	0
	t_1t_3	.72280	0	0	0	0	0	0
	t_2t_3	.59826	0	0	0	0	0	0
17	t_1t_2	.81050	0	0	0	0	0	0
	t_1t_3	.70230	1	0			1	0
	t_2t_3	.56917	0	0			1	0
18	t_1t_2	.76580	0	1	0	1	0	1
	t_1t_3	.72029	1	0	1	0	1	0
	t_2t_3	.57711	1	-1	1	0	1	-1
19	t_1t_2	.88327	0	1	0	1	0	1
	t_1t_3	.72891	0	0	0	-1	0	0
	t_2t_3	.65717	0	-1	0	-1	0	-1
20	t_1t_2	.84307	0	1	1	1	0	1
	t_1t_3	.70848	1	0	1	-1	1	0
	t_2t_3	.50547	1	-1			0	-1

Notes:

1. Lower indexed time is reference image.
2. Blank denotes false registration position; $|\Delta x|$ or $|\Delta y| \geq 5$

B. SIMILARITY MEASURE COMPARISONS: KANSAS, INTER-RUN

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|		
			Δx	Δy	Δx	Δy	Δx	Δy	
1	t_1t_2	.721	0	0			0	0	
	t_1t_3	.252							
	t_1t_4	.706	0	0			0	0	
	t_2t_3	-.395	-1	-1			-1	-1	(max)*
	t_2t_4	.755	-1	-1			0	0	
	t_3t_4	-.558	0	0			0	0	(max)*
2	t_1t_2	.769	0	0	0	0	0	0	
	t_1t_3	.224							
	t_1t_4	.721	0	0			0	0	
	t_2t_3	-.258							
	t_2t_4	.664	0	0			0	0	
	t_3t_4	-.406	0	0	0	0			
3	t_1t_2	.522	0	0					
	t_1t_3	-.384							
	t_1t_4	.400	0	0			0	0	
	t_2t_3	-.347							
	t_2t_4	.587	0	0			-1	-1	
	t_3t_4	-.418	0	0					
4	t_1t_2	.581	0	0	0	0	0	0	
	t_1t_3	-.218							
	t_1t_4	.544	0	0			0	0	
	t_2t_3	-.287							
	t_2t_4	.709	0	0					
	t_3t_4	-.510	1	0					

B. SIMILARITY MEASURE COMPARISONS: KANSAS, INTER-RUN (Cont.)

Area No.	Time Pair Compared	$\max\lvert\rho\rvert$	Shift for $\max\lvert\rho\rvert$		Shift for $\max\Sigma xy$		Shift for $\min\Sigma\lvert x-y\rvert$		
			Δx	Δy	Δx	Δy	Δx	Δy	
5	t_1t_2	.399	0	0			1	0	
	t_1t_3	.284							
	t_1t_4	.343							
	t_2t_3	-.359	0	-1			0	-1	(max)*
	t_2t_4	.503	0	0			-1	1	
	t_3t_4	-.673	0	0					

*Note: This is the shift position of the maximum. The reason for this (instead of noting the minimum) is that the data is negatively correlated (i.e., $\rho < o$).

Lower indexed time is overlay image.

Blanks denote false registration positions.

C. SIMILARITY MEASURE COMPARISONS: KANSAS, INTER-BAND

Band 0.6-0.7μm is overlay image

Band 0.8-1.1μm is reference image

Area No.	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
		Δx	Δy	Δx	Δy	Δx	Δy
1	-.79194	0	0	0	0(min)*	0	0(max)*
2	-.67758	0	0	0	0(min)*	0	0(max)*
3	-.50780	0	0			0	0(max)*
4	-.68329	0	0			0	0(max)*
5	-.55866	0	0	3	- 1(min)*	0	0(max)*
6	-.52217	2	4				
7	.35043	10	- 2				

*Note: The minimum (maximum) is chosen for Σxy (Σ|x-y|) instead of vice versa because the data is negatively correlated (ρ < o).

Blanks indicate false registration positions.

Values other than 0 for Δx and Δy also indicate false registration positions.

APPENDIX III DETAILED RESULTS OF SIMILARITY MEASURE COMPARISONS FOR PREPROCESSED DATA

A. SIMILARITY MEASURE COMPARISONS: |GRADIENT| DATA INTER-RUN; KANSAS.

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.399	1	0	1	0	1	0
	t_1t_3	.209	1	0			1	0
	t_1t_4	.405	0	0			0	0
	t_2t_3	.324	0	-1	0	-1	0	-1
	t_2t_4	.549	-1	-1	-1	-1	0	-1
	t_3t_4	.488	0	0	0	0	0	0
2	t_1t_2	.590	0	0	0	0	0	0
	t_1t_3	.195	0	0			-1	0
	t_1t_4	.500	0	0	0	0	0	0
	t_2t_3	.224	0	-1			-1	-1
	t_2t_4	.482	0	0	0	0	0	0
	t_3t_4	.366	0	1	0	1	0	0
3	t_1t_2	.427	0	0	0	0	0	0
	t_1t_3	.374	-1	-1	-1	-1	-1	-1
	t_1t_4	.376	0	0			0	0
	t_2t_3	.472	0	-1	0	-1	0	-1
	t_2t_4	.499	0	-1	0	-1	0	-1
	t_3t_4	.350	0	1	0	1	0	1

A. SIMILARITY MEASURE COMPARISONS:
|GRADIENT| DATA INTER-RUN; KANSAS. (Cont.)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
4	t_1t_2	.572	0	0	0	0	0	0
	t_1t_3	.355	-1	-1	-1	-1	-1	-1
	t_1t_4	.526	0	0	0	0	0	0
	t_2t_3	.309	0	0	0	0	0	0
	t_2t_4	.619	0	0	0	0	0	0
	t_3t_4	.451	0	0	0	0	0	0
5	t_1t_2	.408	0	0	0	0	0	0
	t_1t_3	.198	0	-1			0	-1
	t_1t_4	.402	0	0	0	0	0	0
	t_2t_3	.410	0	-1	0	-1	0	-1
	t_2t_4	.487	0	0	0	0	0	0
	t_3t_4	.383	0	0	0	0	0	0
6	t_1t_2	.21387	-1	-1			-1	-1
	t_1t_3	.48072	-2	-2			-2	-2
	t_1t_4	.51340	-4	-1	-4	-1	-4	-1
	t_2t_3	.44984	-2	-1				
	t_2t_4	.44610	-4	1	-4	1		
	t_3t_4	.50909	-2	1	-2	1	-2	1
7	t_1t_2	.52489	1	-1	1	-1	1	-1
	t_1t_3	.34400	-1	-2	-1	-2	-1	-2
	t_1t_4	.33869	-2	-1			-2	-1
	t_2t_3	.39083	-2	-1	-2	0	-2	-1
	t_2t_4	.45229	-2	0	-2	0	-2	0
	t_3t_4	.60310	0	1	0	1	-1	1

A. SIMILARITY MEASURE COMPARISONS:
|GRADIENT| DATA INTER-RUN; KANSAS. (Cont.)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
8	t_1t_2	.18197	0	0				
	t_1t_3	.33492	-1	-1	-1	-1	-1	-1
	t_1t_4	.33117	-3	-1	-3	-1	-3	-1
	t_2t_3	.26479	-1	0	-1	0		
	t_2t_4	.29613	-4	0				
	t_3t_4	.54530	-2	0	-2	0	-2	0
9	t_1t_2	.33027	1	0	1	0	1	0
	t_1t_3	.57084	0	0	0	0	0	0
	t_1t_4	.46140	-1	0	-1	0	-1	0
	t_2t_3	.48354	-1	0	-1	0	-1	0
	t_2t_4	.41106	-2	-1	-2	-1	-2	-1
	t_3t_4	.52964	-1	-1	-1	-1	-1	-1
10	t_1t_2	.45778	0	2	0	2	0	2
	t_1t_3	.23210	0	2			0	2
	t_1t_4	.29890	-1	0			-1	0
	t_2t_3	.26618	0	0	0	0	0	0
	t_2t_4	.38466	-1	-2			-1	-2
	t_3t_4	.41544	-1	-2			-1	-2
11	t_1t_2	.32969	1	2	1	2	1	2
	t_1t_3	.29432	1	3	1	3	1	3
	t_1t_4	.41131	1	0	1	0	1	0
	t_2t_3	.28905	0	0	0	0	0	0
	t_2t_4	.29072	0	-2	0	-2	0	-2
	t_3t_4	.23325	0	-3			0	-3

A. SIMILARITY MEASURE COMPARISONS:
|GRADIENT| DATA INTER-RUN; KANSAS. (End)

Notes: Blanks indicate false shift positions.

Lower indexed time is overlay image.

B. SIMILARITY MEASURE COMPARISONS: CSC GRADIENT (NO THRESHOLD) (NOISE FACTOR: 1.2) INTER-RUN; KANSAS.

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.121	3	0				
	t_1t_3	.097						
	t_1t_4	.124	-1	0	-1	0		
	t_2t_3	.141	2	-1	2	-1	2	-1
	t_2t_4	.264	0	-1	0	-1	0	-1
	t_3t_4	.230	0	0	0	0	-1	0
2	t_1t_2	.293	0	0	0	0	0	0
	t_1t_3	.079						
	t_1t_4	.130	0	0	0	0	0	0
	t_2t_3	.087	1	-1	1	-1		
	t_2t_4	.220	0	0	0	0	0	0
	t_3t_4	.121	0	0	0	0		
3	t_1t_2	.111						
	t_1t_3	.167	-1	-1	-1	-1	-1	-1
	t_1t_4	.135	0	0	0	0		
	t_2t_3	.248	0	-1	0	-1		
	t_2t_4	.198	-1	-1	-1	-1		
	t_3t_4	.114	1	1				
4	t_1t_2	.252	0	0	0	0	0	0
	t_1t_3	.157	-1	-1	-1	-1		
	t_1t_4	.297	0	0	0	0	0	0
	t_2t_3	.121	0	0	0	0		
	t_2t_4	.396	0	0	0	0	0	0
	t_3t_4	.172	0	0	0	0	0	0

B. SIMILARITY MEASURE COMPARISONS: CSC GRADIENT (NO THRESHOLD) (NOISE FACTOR: 1.2) INTER-RUN; KANSAS. (Cont.)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
5	t_1t_2	.259	0	0	0	0	0	0
	t_1t_3	.119					1	-1
	t_1t_4	.164	0	0			0	0
	t_2t_3	.173	0	-2			0	-1
	t_2t_4	.227	0	0	0	0	0	0
	t_3t_4	.258	0	0	0	0	0	0
6	t_1t_2	.09156					-1	-1
	t_1t_3	.16153	-3	-2	-4	-2		
	t_1t_4	.19956	-3	-1	-3	-1		
	t_2t_3	.20611	-2	-1	-2	-1		
	t_2t_4	.23204	-4	1	-4	1		
	t_3t_4	.20960	-2	2	-2	2		
7	t_1t_2	.20916	0	-1	0	-1	1	-1
	t_1t_3	.15987	-1	-1			-1	-1
	t_1t_4	.15262						
	t_2t_3	.19848	-2	-1	-2	-1	-2	-1
	t_2t_4	.20525	-2	0	-2	0		
	t_3t_4	.30233	-1	1	-1	1	-1	1
8	t_1t_2	.08085						
	t_1t_3	.11939	-1	-1	-1	-1		
	t_1t_4	.14743						
	t_2t_3	.11477	-1	0	-1	0		
	t_2t_4	.13789						
	t_3t_4	.25111	-2	0	-2	0	-2	0

B. SIMILARITY MEASURE COMPARISONS: CSC GRADIENT (NO THRESHOLD) (NOISE FACTOR: 1.2) INTER-RUN; KANSAS. (End)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
9	t_1t_2	.11428	3	0			1	0
	t_1t_3	.20666	0	0	0	0	0	0
	t_1t_4	.13428	0	0	0	0	0	0
	t_2t_3	.19180	-1	0	-1	0		
	t_2t_4	.12358	-2	-1	-2	-1		
	t_3t_4	.18459	-1	-1	-1	-1	-1	-1
10	t_1t_2	.25041	0	2	0	2	0	2
	t_1t_3	.13205						
	t_1t_4	.10843					-1	0
	t_2t_3	.10323			-4	1		
	t_2t_4	.12813	-1	-2			-1	-2
	t_3t_4	.20679	-1	-2	-1	-2	-1	-2
11	t_1t_2	.10274						
	t_1t_3	.10506	0	3			0	3
	t_1t_4	.13314						
	t_2t_3	.14824	2	0	2	0	2	0
	t_2t_4	.15167	0	-2	0	-2	1	-2
	t_3t_4	.10379	1	0	1	0	1	-3

Notes: Lower indexed time is overlay image.

Blanks denote false registration positions.

C. SIMILARITY MEASURE COMPARISONS: THRESHOLD AT MEDIAN INTER-RUN; KANSAS.

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.516	0	0	0	0	0	0
	t_1t_3	.162						
	t_1t_4	.589	0	0	-1	0	0	0
	t_2t_3	-.402	0	-1	0	-1(min)*	0	-1(max)*
	t_2t_4	.696	0	-1	-1	-1	0	-1
	t_3t_4	-.465	0	0	0	0(min)*	0	0(max)*
2	t_1t_2	.580	0	0	0	0	0	0
	t_1t_3	.174						
	t_1t_4	.518	0	0	0	0	0	0
	t_2t_3	-.234	1	-1	1	0(min)*	1	-1(max)*
	t_2t_4	.574	0	0	0	0	0	0
	t_3t_4	-.258	0	0	0	0(min)*	0	0(max)*
3	t_1t_2	.353	1	0			0	0
	t_1t_3	-.315						
	t_1t_4	.245						
	t_2t_3	-.270						
	t_2t_4	.375	-1	0				
	t_3t_4	-.356	-1	-1			-1	-1(max)*
4	t_1t_2	.676	0	0	0	0	0	0
	t_1t_3	.171	-1	-1				
	t_1t_4	.810	0	0	0	0	0	0
	t_2t_3	-.230						
	t_2t_4	.717	0	0	0	0	0	0
	t_3t_4	-.237						

C. SIMILARITY MEASURE COMPARISONS:
THRESHOLD AT MEDIAN INTER-RUN; KANSAS. (Cont.)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ| Δx	Δy	Shift for maxΣxy Δx	Δy	Shift for minΣ|x-y| Δx	Δy
5	t_1t_2	.279			3	1	3	1
	t_1t_3	.468	1	-1	1	1	1	-1
	t_1t_4	-.328	0	-1	0	1(min)*	0	-1(max)*
	t_2t_3	.216						
	t_2t_4	.318						
	t_3t_4	-.633	0	0	-1	0(min)*	0	0(max)*
6	t_1t_2	.26834						
	t_1t_3	.51290	-2	-2	-3	-2	-2	-2
	t_1t_4	.33371						
	t_2t_3	.28927						
	t_2t_4	.51306	-3	1	-3	1	-3	1
	t_3t_4	.33337						
7	t_1t_2	.49217	1	-1	1	-1	1	-1
	t_1t_3	.32395	-2	-1	-2	-2	-2	-1
	t_1t_4	.28603						
	t_2t_3	-.29396			-2	-1		
	t_2t_4	.45339	-2	1	-2	1	-2	1
	t_3t_4	.30229						
8	t_1t_2	.19890						
	t_1t_3	.29055	-1	2			-1	2
	t_1t_4	.32827						
	t_2t_3	-.31523	1	0	2	1(min)*		
	t_2t_4	.33828	-4	0	-4	0	-4	0
	t_3t_4	-.60046	-2	0	-2	0(min)*	-2	0

C. SIMILARITY MEASURE COMPARISONS:
THRESHOLD AT MEDIAN INTER-RUN; KANSAS. (End)

Area No.	Time Pair Compared	$\max\|\rho\|$	Shift for $\max\|\rho\|$		Shift for $\max\Sigma xy$		Shift for $\min\Sigma\|x-y\|$	
			Δx	Δy	Δx	Δy	Δx	Δy
9	t_1t_2	-.25875	0	0			0	0(max)*
	t_1t_3	.65760	0	0	0	0	0	0
	t_1t_4	-.48894	-1	0	-1	0(min)*	-1	0(max)*
	t_2t_3	-.49230	-1	0	0	0(min)*	-1	0(max)*
	t_2t_4	.44258	-2	-1	-1	0	-2	-1
	t_3t_4	-.70742	-1	-1	-1	-1(min)*	-1	-1(max)*
10	t_1t_2	.22317						
	t_1t_3	.43762	0	2	0	2	0	2
	t_1t_4	.24510						
	t_2t_3	-.26112						
	t_2t_4	.41741	-1	-1	-1	-2	-1	-1
	t_3t_4	-.42389	0	-2			0	-2(max)*
11	t_1t_2	.52422	1	2	1	2	1	2
	t_1t_3	-.22204	1	3	2	3(min)*	1	3(max)*
	t_1t_4	.52730	1	0	1	0	1	0
	t_2t_3	-.27267	1	0	1	0(min)*	1	0(max)*
	t_2t_4	.39674	0	-2	0	-2	0	-2
	t_3t_4	-.20400						

Note: The minimum (maximum) is chosen for Σxy ($\Sigma|x-y|$) instead of vice versa because the data is negatively correlated (i.e., $\rho < 0$).

Smaller indexed time is overlay image.

Blanks denote false registration positions.

D. SIMILARITY MEASURE COMPARISONS:
|GRADIENT| (THRESHOLD = 11.0)
INTER-RUN; KANSAS

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.318	1	0	1	0	1	0
	t_1t_3	.212	0	0	0	0	0	0
	t_1t_4	.319	0	0	0	0	0	0
	t_2t_3	.278	0	-1	0	-1	0	-1
	t_2t_4	.436	0	-1	0	-1	0	-1
	t_3t_4	.442	0	0	0	0	0	0
2	t_1t_2	.428	0	0	0	0	0	0
	t_1t_3	.139						
	t_1t_4	.372	0	0	0	0	0	0
	t_2t_3	.174	-1	-1	-1	-1	-1	-1
	t_2t_4	.368	0	0	0	0	0	0
	t_3t_4	.308	0	1	0	1	0	1
3	t_1t_2	.294	0	0	0	0	0	0
	t_1t_3	.293	-1	-1	-1	-1	-1	-1
	t_1t_4	.251					0	0
	t_2t_3	.322	0	-1	0	-1	0	-1
	t_2t_4	.336	0	-1	0	-1	0	-1
	t_3t_4	.271	0	1	0	1		
4	t_1t_2	.412	0	0	0	0	0	0
	t_1t_3	.261	0	-1	0	-1	0	-1
	t_1t_4	.393	0	0	0	0	0	0
	t_2t_3	.199	0	0	0	0		
	t_2t_4	.448	0	0	0	0	0	0
	t_3t_4	.347	0	0	0	0	0	0

D. SIMILARITY MEASURE COMPARISONS:
|GRADIENT| (THRESHOLD = 11.0)
INTER-RUN; KANSAS [cf. Table 4] (End)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
5	t_1t_2	.209	0	0			0	0
	t_1t_3	.204						
	t_1t_4	.299	0	0	0	0	0	0
	t_2t_3	.252	0	-1	0	-1	0	-1
	t_2t_4	.266	0	0	0	0	0	0
	t_3t_4	.278	0	0	0	0	0	0

Lower indexed time is overlay image.

Blanks denote false registration positions.

E. SIMILARITY MEASURE COMPARISONS:
CSC GRADIENT (THRESHOLD = 14.0) (NOISE FACTOR = 1.2)
INTER-RUN; KANSAS.

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.086	3	0			3	0
	t_1t_3	.077	0	0			0	0
	t_1t_4	.113	0	-1	0	-1	0	-1
	t_2t_3	.126	0	-1	0	-1	1	-1
	t_2t_4	.246	0	-1	0	-1	0	-1
	t_3t_4	.189	-1	0	-1	0	0	0
2	t_1t_2	.227	0	0	0	0	0	0
	t_1t_3							
	t_1t_4	.148	0	0	0	0	0	0
	t_2t_3							
	t_2t_4	.222	0	0	0	0	0	0
	t_3t_4	.112	0	0	0	0	0	0
3	t_1t_2	.151	0	0	0	0	0	0
	t_1t_3	.191	-1	-1	-1	-1	-1	-1
	t_1t_4	.118	0	0				
	t_2t_3	.128	0	-1	0	-1		
	t_2t_4	.168	0	-1	0	-1	0	-1
	t_3t_4	.125	0	1	0	1		
4	t_1t_2	.246	0	0	0	0	0	0
	t_1t_3	.195	-1	-1	-1	-1	-1	-1
	t_1t_4	.234	0	0	0	0	0	0
	t_2t_3	.136	0	0	-1	-1		
	t_2t_4	.224	0	0	0	0	0	0
	t_3t_4	.234	0	0	0	0	0	0

E. SIMILARITY MEASURE COMPARISONS:
CSC GRADIENT (THRESHOLD = 14.0) (NOISE FACTOR = 1.2)
INTER-RUN; KANSAS. (End)

Area No.	Time Pair Compared	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
			Δx	Δy	Δx	Δy	Δx	Δy
5	t_1t_2	.158	0	0	0	0	0	0
	t_1t_3	.138	0	-1	0	-1	0	-1
	t_1t_4	.130	0	0	0	0	0	0
	t_2t_3	.186	0	-1	0	-1	0	-1
	t_2t_4	.222	0	0	0	0	0	0
	t_3t_4	.179	0	0	0	0	0	0

Lower indexed time is overlay image.

Blanks denote false registration positions.

F. SIMILARITY MEASURE COMPARISONS:
|GRADIENT| INTER-BAND; KANSAS.

Area No.	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
		Δx	Δy	Δx	Δy	Δx	Δy
1	.71788	0	0	0	0	0	0
2	.56882	0	0	0	0	0	0
3	.64330	0	0	0	0	0	0
4	.57730	0	0	0	0	0	0
5	.63140	0	0	0	0	0	0
6	.52579	0	0	0	0	0	0
7	.41216	0	0	0	0	0	0

0.6-0.7μm band is overlay image.

G. SIMILARITY MEASURE COMPARISONS: CSC GRADIENT (NO THRESHOLD) (NOISE FACTOR = 1.2) INTER-BAND; KANSAS.

Area No.	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
		Δx	Δy	Δx	Δy	Δx	Δy
1	.46262	0	0	0	0	0	0
2	.36623	0	0	0	0	0	0
3	.45519	0	0	0	0	0	0
4	.44706	0	0	0	0	0	0
5	.45613	0	0	0	0	0	0
6	.29682	0	0	0	0	0	0
7	.23567	0	0	0	0	0	0

0.6-0.7μm band is overlay image.

H. SIMILARITY MEASURE COMPARISONS: THRESHOLD AT MEDIAN INTER-BAND; KANSAS

Area No.	max|ρ|	Shift for max|ρ|		Shift for maxΣxy		Shift for minΣ|x-y|	
		Δx	Δy	Δx	Δy	Δx	Δy
1	-.77746	0	0	0	0(min)*	0	0(max)*
2	-.70507	0	0	0	0(min)*	0	0(max)*
3	.51139	0	0	0	0	0	0
4	-.27823						
5	-.24162						
6	-.42308	0	0			0	0(max)*
7	-.50440						

*The minimum (maximum) is chosen for Σxy (Σ|x-y|) instead of vice versa because the data is negatively correlated ($\rho < 0$).

0.6-0.7μm band is overlay image.

Blanks denote false registration positions.

APPENDIX IV EQUATIONS AND OPERATIONS REQUIRED FOR PREPROCESSING OPERATIONS

A. Magnitude of the Gradient of an Image

Equation per point:

$$|\text{Gradient}|_{i,j} = \tfrac{1}{2}\sqrt{(f_{i+1,j} - f_{i-1,j})^2 + (f_{i,j+1} - f_{i,j-1})^2}$$

$f_{i,j}$ = value of image at location (i,j)

Number of Operations per point:

3 integer subtractions or additions

\+ 2 integer multiplications

\+ 1 square root

\+ 1 divide

Note 1: Division by 2 may be omitted since it is the same for all points.

Note 2: For a thresholded image there will be a comparison operation for each point also.

B. Local Gradient of Image

Equation per point:

$$(\text{Local Gradient})_{ij} = \frac{|\text{Gradient}|^2_{i,j}}{\text{variance}_{i,j} + \sigma_n^2}$$

$$|\text{Gradient}|^2_{i,j} = (f_{i+1,j} - f_{i-1,j})^2 + (f_{i,j+1} - f_{i,j-1})^2$$

f_{ij} = value of image at location (i,j)

σ_n^2 = arbitrary constant term; prevents denominator from becoming zero.

$\text{variance}_{i,j}$ = variance of the five data points $(f_{i+1,j}, f_{i-1,j}, f_{i,j+1}, f_{i,j-1}, f_{ij})$ from a locally flat plane; the locally flat plane is obtained from a least squares fit through these five points.

$$\text{variance}_{i,j} = \tfrac{1}{2}[(\hat{f}^2_{ij} + \hat{f}^2_{i,j-1} + \hat{f}^2_{i,j+1} + \hat{f}^2_{i+1,j} + \hat{f}^2_{i-1,j}]$$

where, $\hat{f}_{ij} = (f_{ij} - \text{plane}_{ij})$

*The variance is normalized by a factor of 2 rather than 5, because there are only 2 degrees of freedom.

Number of Operations per point:

6 integer additions or subtractions

+ 13 real additions or subtractions

+ 9 real multiplications

Note: This is using the following equation for the $\text{variance}_{i,j}$.

$$\text{variance}_{ij} = \tfrac{1}{2}[(f_{i,j-1} - C_1+C_2)^2 + (f_{i,j} - C_1)^2 + (f_{i,j+1} - C_1-C_2)^2 + (f_{i-1,j} - C_1-C_3)^2 + (f_{i+1,j} - C\ -C_3)^2]$$

$$C_1 = \frac{1}{5}\ [f_{i,j} + f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}]$$

$$C_2 = \frac{1}{2}\ [f_{i,j+1} - f_{i,j-1}]$$

$$C_3 = \frac{1}{2}[f_{i-1,j} - f_{i+1,j}]$$

Note: For a thresholded image there will be a comparison operation for each point also.

C. Threshold at Median

Number of Operations for Image =

1 addition for each point needed to estimate the median

+ 1 threshold comparison for each image point

APPENDIX V

A. PREPROCESSING COMPARISONS: INDICATED SHIFT POSITIONS

HILL COUNTY, MONTANA

SPECTRAL BAND 0.8-1.1 μm USED

LOWER INDEXED TIME IS REFERENCE IMAGE.

Area No.	Time Pair Compared	Original Data ρ	Shift for max |ρ|: Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.62652	0	0	0	0	0	0	0	0	0	0	0	0
	t_1t_3	.46497	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_1t_4	.32173	-1	-1	0	-1	0	-1	-1	-1	0	0	0	-1
	t_1t_5	.39931	0	0	0	0	0	0	0	0	0	0	0	0
	t_2t_3	.64986	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_2t_4	.48653	-1	-1	0	-1	0	-1	-1	-1	-1	-1	0	-1
	t_2t_5	.63528	0	0	0	0	0	0	0	0	0	0	0	0
	t_3t_4	.82356	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
	t_3t_5	.82029	1	0	2	0	1	0	2	0	2	0	2	0
	t_4t_5	.80111	1	1	1	1	0	1	1	1	1	1	0	1

Area No.	Time Pair Compared	Original Data ρ	Shift for max |ρ|: Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
2	t_1t_2	.97525	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_1t_3	.95145	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_1t_4	.92858	-2	0	-2	0	-2	0	-2	0	-2	0	-2	0
	t_1t_5	.76302	-2	0	-2	0	-4	-2*	-2	0	-2	-1	-2	0
	t_2t_3	.96125	0	0	0	0	0	0	0	0	0	0	0	0
	t_2t_4	.94251	-1	0	-1	0	0	0	-1	0	0	0	-1	0
	t_2t_5	.79110	-1	-1	-1	-1	-1	-1	-1	0	-1	0	-1	-1
	t_3t_4	.96326	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_3t_5	.80669	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_4t_5	.79886	0	0	0	0	-1	0	0	0	0	0	-1	-1
3	t_1t_2	.72789	-1	0	-1	0	-1	0	-1	-1	generated		-1	0
	t_1t_3	.43157	-1	-1	-1	0	-1	0	-1	-1	data		-1	0
	t_1t_4	.51138	-1	-1	-1	-1	-1	-1	-1	-1	not			
	t_1t_5	.66034	-2	0	-2	0	-2	0	-2	0	usable		-2	0
	t_2t_3	.58504	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_2t_4	.65331	-1	0	0	0	-1	0	-1	0	0	0	-1	0
	t_2t_5	.78552	-2	0	-2	0	-2	0	-2	0	-2	0	-2	0
	t_3t_4	.73584	0	0	0	0	0	0	0	0	0	0	0	0
	t_3t_5	.61763	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0
	t_4t_5	.77009	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0

B. PREPROCESSING COMPARISONS: INDICATED SHIFT POSITIONS

TIPPECANOE COUNTY, INDIANA

SPECTRAL BAND 0.8-1.1 μm USED

Lower indexed time is reference image

Shift for max $|\rho|$

Area No.	Time Pair Compared	Original Data ρ	Original Data Δx	Original Data Δy	|Gradient| Δx	|Gradient| Δy	Local Gradient No Threshold $\sigma n^2 = 1.2$ Δx	Local Gradient No Threshold $\sigma n^2 = 1.2$ Δy	Threshold at Median Δx	Threshold at Median Δy	|Gradient| Threshold=6.0 Δx	|Gradient| Threshold=6.0 Δy	Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$ Δx	Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$ Δy
1	t_1t_2	.58280	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
	t_1t_3	.43552	3	-1	3	-1	3	-1	4	-1	3	-1	3	-1
	t_2t_3	.65037	2	0	2	0	2	0	2	0	2	0	2	0
2	t_1t_2	.65879	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
	t_1t_3	.44449	2	0	2	0	2	0	2	0	2	0	2	0
	t_2t_3	.59280	1	1	1	1	1	1	1	1	1	1	1	1
3	t_1t_2	.59152	1	0	1	0	1	0	1	0	1	0	1	0
	t_1t_3	.30043	1	0	1	0	1	0			1	0	2	0
	t_2t_3	.60230	0	1	0	1	0	1	0	1	0	1	0	1
4	t_1t_2	.60441	1	0	1	0	0	0	0	0	0	0	1	0
	t_1t_3	.28573	1	1	1	1	0	1	1	1	1	1	0	1
	t_2t_3	.52106	0	1	0	1	0	1	0	1	0	1	1	1

'ea No.	Time Pair Compared	Original Data ρ	Shift for max |ρ|											
			Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Local Gradient Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
5	t_1t_2	.67739	0	-1	0	-1	0	-1	0	-1	1	-1	1	-1
	t_1t_3	.57390	1	-2	1	-2	1	-2	1	-2	1	-2	1	-2
	t_2t_3	.69159	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1
6	t_1t_2	.41030	1	0	1	0	0	-2	-2	0*	0	-1	0	-1
	t_1t_3	.46791	1	-1	1	-1	0	-1	1	-1	1	-1	1	-1
	t_2t_3	.56415	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1
7	t_1t_2	.71035	0	0	0	0	1	0	0	0	0	0	0	0
	t_1t_3	.57191	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1
	t_2t_3	.77022	0	0	0	0	0	0	0	0	0	0	0	0
8	t_1t_2	.69335	0	0	0	0	0	0	0	0	0	0	0	0
	t_1t_3	.47342	0	1	0	1	0	1	-1	1	0	1	0	1
	t_2t_3	.61384	0	1	-1	0	0	1	0	0	-1	1	-1	1
9	t_1t_2	.57186	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1
	t_1t_3	.48221	0	-1	0	-2	0	-1	0	-1	0	-2	0	-2
	t_2t_3	.72323	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1

Area No.	Time Pair Compared	Original Data ρ	Shift for max |ρ|											
			Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
10	t_1t_2	.46346	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1
	t_1t_3	.32387	-1	-1	-1	-2	-1	-2	-1	-1	-1	-2	0	-2
	t_2t_3	.70464	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1
11	t_1t_2	.80057	0	0	0	0	0	0	0	0	0	0	0	0
	t_1t_3	.60337	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
	t_2t_3	.72622	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
12	t_1t_2	.63470	0	0	0	0	0	0	0	0	0	0	0	0
	t_1t_3	.37653	-1	0	-1	0	-1	0	-2	-1	-1	0	-1	0
	t_2t_3	.60818	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0

C. PREPROCESSING COMPARISONS: INDICATED SHIFT POSITIONS

KANSAS; INTER-RUN DATA

SPECTRAL BAND 0.6-0.7 μm USED

Lower indexed time is overlay image.

Blanks indicate false shift position.

Area No.	Time Pair Compared	Original Data ρ	Shift for max |ρ|: Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
1	t_1t_2	.721	0	0	1	0	3	0*	0	0	1	0	3	0*
	t_1t_3	.252			1	0					0	0	0	0
	t_1t_4	.706	0	0	0	0	-1	0	0	0	0	0	0	-1
	t_2t_3	-.395	-1	-1	0	-1	2	-1	0	-1	0	-1	0	-1
	t_2t_4	.755	-1	-1	-1	-1	0	-1	0	-1	0	-1	0	-1
	t_3t_4	-.558	0	0	0	0	0	0	0	0	0	0	-1	0
2	t_1t_2	.769	0	0	0	0	0	0	0	0	0	0	0	0
	t_1t_3	.224			0	0								
	t_1t_4	.721	0	0	0	0	0	0	0	0	0	0	0	0
	t_2t_3	-.258			0	-1	1	-1	1	-1	-1	-1		
	t_2t_4	.664	0	0	0	0	0	0	0	0	0	0	0	0
	t_3t_4	-.406	0	0	0	1	0	0	0	0	0	1	0	0

Area No.	Time Pair Compared	Original Data ρ	Shift for max |ρ|: Original Data		|Gradient|		Local Gradient Threshold: No Threshold $\sigma n^2 = 1.2$		Local Gradient Threshold: at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
3	t_1t_2	.522	0	0	0	0			1	0	0	0	0	0
	t_1t_3	-.384			-1	-1	-1	-1			-1	-1	-1	-1
	t_1t_4	.400	0	0	0	0	0	0					0	0
	t_2t_3	-.347			0	-1	0	-1			0	-1	0	-1
	t_2t_4	.587	0	0	0	-1	-1	-1	-1	0	0	-1	0	-1
	t_3t_4	-.418	0	0	0	1	1	1	-1	-1	0	1	0	1
4	t_1t_2	.581	0	0	0	0	0	0	0	0	0	0	0	0
	t_1t_3	-.218			-1	-1	-1	-1	-1	-1	0	-1	-1	-1
	t_1t_4	.544	0	0	0	0	0	0	0	0	0	0	0	0
	t_2t_3	-.287			0	0	0	0			0	0	0	0
	t_2t_4	.709	0	0	0	0	0	0	0	0	0	0	0	0
	t_3t_4	-.510	1	0	0	0	0	0			0	0	0	0
5	t_1t_2	.399	0	0	0	0	0	0			0	0	0	0
	t_1t_3	.284	1	-2	0	-1			1	-1			0	-1
	t_1t_4	.343			0	0	0	0	0	-1	0	0	0	0
	t_2t_3	-.359	0	-1	0	-1	0	-2			0	-1	0	-1
	t_2t_4	.503	0	0	0	0	0	0			0	0	0	0
	t_3t_4	-.673	0	0	0	0	0	0	0	0	0	0	0	0

Note: For Areas 6 thru 11 there are no results for the Thresholded Gradient and Thresholded CSC Gradient.

Shift for max |ρ|

rea No.	Time Pair Compared	Original Data ρ	Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
6	t_1t_2	.28973			-1	-1								
	t_1t_3	.51226	-3	-2	-2	-2	-3	-2	-2	-2				
	t_1t_4	.39710			-4	-1	-3	-1						
	t_2t_3	-.38098	-2	-1	-2	-1	-2	-1						
	t_2t_4	.64242	-4	1	-4	1	-4	1	-3	1				
	t_3t_4	.44618			-2	1	-2	2						
7	t_1t_2	.57726	0	-1	1	-1	0	-1	1	-1				
	t_1t_3	.36512	-1	-1	-1	-2	-1	-1	-2	-1				
	t_1t_4	.27805			-2	-1								
	t_2t_3	-.31689			-2	-1	-2	-1						
	t_2t_4	.45752	-2	0	-2	0	-2	0	-2	1				
	t_3t_4	-.44679	-1	1	0	1	-1	1						

D. PREPROCESSING COMPARISONS: INDICATED SHIFT POSITIONS

KANSAS, INTER-BAND DATA

No Data for Thresholded Gradient and
Thresholded Local Gradient was Generated

Area No.	Original Data ρ	Original Data		|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median	
		Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
1	-.79194	0	0	0	0	0	0	0	0
2	-.67758	0	0	0	0	0	0	0	0
3	-.50780	0	0	0	0	0	0	0	0
4	-.68329	0	0	0	0	0	0		
5	-.55866	0	0	0	0	0	0		
6	-.52217			0	0	0	0	0	0
7	-.35043			0	0	0	0		

Spectral band 0.6-0.7μm is overlay image.

Blanks indicate false peaks.

E. PREPROCESSING COMPARISONS: MAX $|\rho|$ FOR INDICATED REGISTRATION POSITIONS

HILL COUNTY, MONTANA

SPECTRAL BAND 0.8-1.1 μm USED

Lower indexed time is reference image.

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=6.0	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$
1	t_1t_2	.62652	.57693	.39641	.58225	.27242	.27203
	t_1t_3	.46497	.47076	.25353	.42671	.18942	.14338
	t_1t_4	.32173	.37390	.18464	.40631	.14968	.12993
	t_1t_5	.39931	.37647	.21628	.40131	.15809	.16420
	t_2t_3	.64986	.56259	.33847	.50833	.39892	.23680
	t_2t_4	.48653	.38376	.19981	.48410	.17473	.17499
	t_2t_5	.63528	.51227	.33279	.49969	.39984	.25527
	t_3t_4	.82356	.64802	.42400	.75732	.42925	.37087
	t_3t_5	.82029	.62683	.41171	.79519	.52048	.41261
	t_4t_5	.80111	.60703	.44092	.77165	.42889	.42032

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=6.0	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$
2	t_1t_2	.97525	.89437	.51140	.69799	.68406	.52654
	t_1t_3	.95145	.76605	.44084	.62302	.57641	.31270
	t_1t_4	.92858	.71803	.37932	.53586	.59977	.32658
	t_1t_5	.76302	.31742	.18585*	.60313	.21762	.24137
	t_2t_3	.96125	.83548	.47028	.65433	.60377	.42057
	t_2t_4	.94251	.74843	.39652	.58192	.52573	.36872
	t_2t_5	.79110	.38775	.21597	.60496	.30232	.27779
	t_3t_4	.96326	.84429	.58172	.61596	.64869	.38095
	t_3t_5	.80669	.43379	.25919	.61888	.25272	.28449
	t_4t_5	.79886	.42109	.23938	.60414	.25484	.29078
3	t_1t_2	.72789	.39085	.34971	.64358	generated	.24593
	t_1t_3	.43157	.27914	.21420	.48752	data	.15457
	t_1t_4	.51138	.28887	.17255	.50743	not	.12540*
	t_1t_5	.66034	.38545	.26036	.62555	usable	.22469
	t_2t_3	.58504	.51351	.31492	.69366	.35581	.28698
	t_2t_4	.65331	.50881	.34152	.68569	.24277	.24853
	t_2t_5	.78552	.61690	.36484	.78870	.39381	.33317
	t_3t_4	.73584	.55347	.40231	.75711	.44888	.36202
	t_3t_5	.61763	.46311	.29773	.68444	.42807	.33759
	t_4t_5	.77009	.66112	.46445	.73409	.48869	.46301

			Shift for max $\|\rho\|$											
ea No.	Time Pair Compared	Original Data ρ	Original Data		$\|$Gradient$\|$		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		$\|$Gradient$\|$ Threshold=6.0		Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
8	t_1t_2	.37212			0	0								
	t_1t_3	.49577	-2	0	-1	-1	-1	-1	-1	2				
	t_1t_4	.38605			-3	-1								
	t_2t_3	-.32943	-2	0	-1	0	-1	0	1	0				
	t_2t_4	.41326	-4	0	-4	0			-4	0				
	t_3t_4	-.76839	-2	0	-2	0	-2	0	-2	0				
9	t_1t_2	-.29084	0	0	1	0			0	0				
	t_1t_3	.78652	0	0	0	0	0	0	0	0				
	t_1t_4	-.52178	-1	0	-1	0	0	0	-1	0				
	t_2t_3	-.48838	-1	0	-1	0	-1	0	-1	0				
	t_2t_4	.44741	-2	-1	-2	-1	-2	-1	-2	-1				
	t_3t_4	-.70526	-1	-1	-1	-1	-1	-1	-1	-1				
10	t_1t_2	.50776	1	2	0	2	0	2						
	t_1t_3	-.31161			0	2			0	2				
	t_1t_4	.29226			-1	0								
	t_2t_3	-.26887			0	0								
	t_2t_4	.48579	-1	-1	-1	-2	-1	-2	-1	-1				
	t_3t_4	-.39042	-1	-2	-1	-2	-1	-2	0	-2				

Shift for max |ρ|

Area No.	Time Pair Compared	Original Data ρ	Original Data Δx	Original Data Δy	|Gradient| Δx	|Gradient| Δy	Local Gradient No Threshold $\sigma n^2 = 1.2$ Δx	Local Gradient No Threshold $\sigma n^2 = 1.2$ Δy	Threshold at Median Δx	Threshold at Median Δy	|Gradient| Threshold=6.0 Δx	|Gradient| Threshold=6.0 Δy	Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$ Δx	Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$ Δy
11	t_1t_2	.56279	1	2	1	2			1	2				
	t_1t_3	-.21496			1	3	0	3	1	3				
	t_1t_4	.59746	1	0	1	0			1	0				
	t_2t_3	-.32602	1	0	0	0	2	0	1	0				
	t_2t_4	.49942	1	-2	0	-2	0	-2	0	-2				
	t_3t_4	-.23296			0	-3	1	0*						

F. PREPROCESSING COMPARISONS: MAX $|\rho|$ FOR INDICATED REGISTRATION POSITIONS

TIPPECANOE COUNTY, INDIANA

SPECTRAL BAND 0.8-1.1μm USED

Lower indexed time is reference image.

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=6.0	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$
1	t_1t_2	.58280	.43705	.19638	.26934	.28030	.14527
	t_1t_3	.43552	.35652	.15422	.16488	.19116	.12307
	t_2t_3	.65037	.58510	.34592	.41420	.35913	.28596
2	t_1t_2	.65879	.46885	.25216	.42012	.34860	.24486
	t_1t_3	.44449	.29876	.16234	.27126	.18446	.09468
	t_2t_3	.59280	.39473	.21984	.35701	.26223	.14039
3	t_1t_2	.59152	.41344	.17319	.46502	.23004	.13465
	t_1t_3	.30043	.30577	.14357	.15890*	.17835	.10817
	t_2t_3	.60230	.52788	.29299	.32838	.36792	.20024
4	t_1t_2	.60441	.41586	.35296	.43717	.27371	.16090
	t_1t_3	.28573	.23118	.22713	.13939	.13960	.06205
	t_2t_3	.52106	.46817	.34071	.25870	.28803	.18301

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=6.0	Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$
5	t_1t_2	.67739	.52008	.27477	.46094	.39122	.25418
	t_1t_3	.57390	.39174	.19489	.40401	.25102	.21851
	t_2t_3	.69159	.52546	.38309	.36952	.36686	.25705
6	t_1t_2	.41030	.44513	.18406	.12054*	.21942	.10475
	t_1t_3	.46791	.33288	.15585	.28570	.16708	.13834
	t_2t_3	.56415	.42137	.24834	.35397	.21534	.16464
7	t_1t_2	.71035	.48067	.21982	.45196	.34428	.18476
	t_1t_3	.57191	.40248	.26546	.31140	.25107	.18469
	t_2t_3	.77022	.49589	.28434	.57154	.31337	.19344
8	t_1t_2	.69335	.50790	.30632	.53544	.35612	.28359
	t_1t_3	.47342	.30151	.19205	.22947	.18954	.13953
	t_2t_3	.61384	.43284	.27789	.36514	.28172	.16700
9	t_1t_2	.57186	.33712	.19284	.31283	.25575	.17469
	t_1t_3	.48221	.30743	.21744	.24260	.17074	.15970
	t_2t_3	.72323	.61500	.44026	.41899	.41876	.35014
10	t_1t_2	.46346	.39579	.19292	.32860	.23808	.14914
	t_1t_3	.32387	.24139	.17758	.21828	.15193	.07571
	t_2t_3	.70464	.55325	.32144	.42113	.38742	.25253

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=6.0	Local Gradient Threshold = 11.0 $\sigma n^2 = 1.2$
11	t_1t_2	.80057	.66052	.37472	.48954	.48449	.32981
	t_1t_3	.60337	.42387	.22073	.27456	.2521	.15934
	t_2t_3	.72622	.53379	.25998	.45155	.37218	.23655
12	t_1t_2	.63470	.39447	.20407	.49031	.27887	.15032
	t_1t_3	.37653	.22031	.13188	.20731	.11542	.11790
	t_2t_3	.60818	.37998	.20736	.35320	.24878	.17519

G. PREPROCESSING COMPARISONS: MAX $|\rho|$

FOR INDICATED REGISTRATION POSITIONS

KANSAS; INTER-RUN DATA

SPECTRAL BAND 0.6-0.7μm USED

Lower indexed time is overlay image.

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=11.0	Local Gradient Threshold=14.0 $\sigma n^2 = 1.2$
1	t_1t_2	.721	.39854	.12096*	.51557	.31768	.08595*
	t_1t_3	.252*	.20874	.09726*	.16194*	.21179	.07720
	t_1t_4	.706	.40493	.12438	.55885	.31949	.11332
	t_2t_3	-.395	.32374	.14077	-.40199	.27753	.12622
	t_2t_4	.755	.54852	.26399	.69573	.43585	.24635
	t_3t_4	-.558	.48807	.22991	-.46454	.44201	.18862
2	t_1t_2	.769	.58967	.29297	.57962	.42826	.22654
	t_1t_3	.224*	.19543	.07937*	-.18994*	.13893*	.08393*
	t_1t_4	.721	.50045	.13019	.51756	.37207	.14774
	t_2t_3	-.258*	.22436	.08691	-.23391	.17352	.07658*
	t_2t_4	.664	.48200	.21993	.57359	.36781	.22210
	t_3t_4	-.406	.36599	.12055	-.25788	.30757	.11171

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=11.0	Local Gradient Threshold=14.0 $\sigma n^2 = 1.2$
3	t_1t_2	.522	.42663	.11116*	.35315	.29367	.15083
	t_1t_3	-.384*	.37395*	.16753	-.31456*	.29285	.19086
	t_1t_4	.400	.37557	.13451	.24505*	.25108*	.11759
	t_2t_3	-.347*	.47235*	.24843	-.26977*	.32151	.12824
	t_2t_4	.587	.49875	.19751	.37547	.33620	.16800
	t_3t_4	-.418	.35028	.11350	-.35623	.27074	.12477
4	t_1t_2	.581	.57249	.25195	.67596	.41180	.24573
	t_1t_3	-.218*	.35491*	.15686	.17068	.26134	.19492
	t_1t_4	.544	.52627	.29741	.80994	.39320	.23361
	t_2t_3	-.287	.30869*	.12124	-.22905*	.19946	.13604
	t_2t_4	.709	.61946	.39622	.71707	.44845	.22432
	t_3t_4	-.510	.45109	.17222	-.25697*	.34729	.23512
5	t_1t_2	.399	.40807	.25943	.27942*	.20872	.15804
	t_1t_3	.284*	.19782*	.11893*	.46785	.20395*	.13829
	t_1t_4	.343*	.40244*	.16407	-.32828	.29931	.12966
	t_2t_3	-.359	.40971	.17263	-.35948*	.25193	.18613
	t_2t_4	.503	.48695	.22666	.31848*	.26592	.22233
	t_3t_4	-.673	.38288	.25771	-.63264	.27764	.17917

Note: For areas 6 thru 11 there are no results for the thresholded gradient and thresholded CSC gradient.

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=11.0	Local Gradient Threshold=14.0 $\sigma n^2 = 1.2$
6	t_1t_2	.28973*	.21387	.09156*	.26834*		
	t_1t_3	.51226	.48072	.16153	.51290		
	t_1t_4	.39710*	.51340	.19956	.33371*		
	t_2t_3	-.38098	.44984	.20611	.28927*		
	t_2t_4	.64242	.44610	.23204	.51306		
	t_3t_4	.44618*	.50909	.20960	.33337*		
7	t_1t_2	.57726	.52489	.20916	.49217		
	t_1t_3	.36512	.34400	.15987	.32395		
	t_1t_4	.27805*	.33869	.15262*	.28603*		
	t_2t_3	-.31689*	.39083	.19848	-.29396*		
	t_2t_4	.45752	.45229	.20525	.45339		
	t_3t_4	-.44679	.60310	.30233	.30229*		
8	t_1t_2	.37212*	.18197	.08085*	.19890*		
	t_1t_3	.49577	.33492	.11939	.29055		
	t_1t_4	.38605*	.33117	.14743*	.32827*		
	t_2t_3	-.32943	.26479	.11477	-.31523		
	t_2t_4	.41326	.29613	.13789*	.33828		
	t_3t_4	-.76839	.54530	.25111	-.60046		

Area No.	Time Pair Compared	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median	|Gradient| Threshold=11.0	Local Gradient Threshold=14.0 $\sigma n^2 = 1.2$
9	t_1t_2	-.29084	.33027	.11428*	-.25875		
	t_1t_3	.78652	.57084	.20666	.65760		
	t_1t_4	-.52178	.46140	.13428	-.48894		
	t_2t_3	-.48838	.48354	.19180	-.49230		
	t_2t_4	.44741	.41106	.12358	.44258		
	t_3t_4	-.70526	.52964	.18459	-.70742		
10	t_1t_2	.50776	.45778	.25041	.22317*		
	t_1t_3	-.31161*	.23210	.13205*	.43762		
	t_1t_4	.29226*	.29890	.10843*	.24510*		
	t_2t_3	-.26887*	.26618	.10323*	-.26112*		
	t_2t_4	.48579	.38466	.12813	.41741		
	t_3t_4	-.39042	.41544	.20679	-.42389		
11	t_1t_2	.56279	.32969	.10274*	.52422		
	t_1t_3	-.21496	.29432	.10506	-.22204		
	t_1t_4	.59746	.41131	.13314*	.52730		
	t_2t_3	-.32602	.28905	.14824	-.27267		
	t_2t_4	.49942	.29072	.15167	.39674		
	t_3t_4	-.23296*	.23325	.10379	-.20400*		

H. PREPROCESSING COMPARISONS: MAX $|\rho|$

FOR INDICATED REGISTRATION POSITIONS

KANSAS; INTER-BAND DATA

Area No.	Original Data	|Gradient|	Local Gradient No Threshold $\sigma n^2 = 1.2$	Threshold at Median
1	-.79194	.71788	.46262	-.77746
2	-.67758	.56882	.36623	-.70507
3	-.50780	.64330	.45519	.51139
4	-.68329	.57730	.44706	-.27823*
5	-.55866	.63140	.45613	-.24162*
6	-.52217*	.52579	.29682	-.42308
7	.35043*	.41216	.23567	-.50440*

Spectral Band 0.6-0.7μm is overlay image.

APPENDIX VI

A. SHIFT POSITION DISCREPANCY STUDY: SPECTRAL BANDS

HILL COUNTY, MONTANA

Area No.	Times Compared	0.7-0.8µm Integer Shift x	0.7-0.8µm Integer Shift y	0.7-0.8µm Interpolated Shift x	0.7-0.8µm Interpolated Shift y	Interpolated Discrepancy Between Bands |Δx|	Interpolated Discrepancy Between Bands |Δy|	0.8-1.1µm Interpolated Shift x	0.8-1.1µm Interpolated Shift y	0.8-1.1µm Integer Shift x	0.8-1.1µm Integer Shift y
1	t_1t_2	0	0	0.00,	0.08	0.07	0.05	-0.07,	0.03	0	0
	t_1t_3	-1	0	-1.36,	0.09	0.08	0.02	-1.44,	0.11	-1	0
	t_1t_4	-1	-1	-0.78,	-0.76	0.12	0.02	-0.90,	-0.78	-1	-1
	t_1t_5	0	0	-0.19,	0.04	0.07	0.00	-0.12,	0.04	0	0
	t_2t_3	-1	0	-1.31,	-0.06	0.05	0.05	-1.26,	0.01	-1	0
	t_2t_4	-1	-1	-0.62,	-1.01	0.00	0.08	-0.62,	-0.93	-1	-1
	t_2t_5	0	0	0.01,	-0.11	0.07	0.04	0.08,	-0.07	0	0
	t_3t_4	1	-1	0.86,	-0.91	0.02	0.01	0.84,	-0.90	1	-1
	t_3t_5	1	0	1.34,	-0.12	0.13	0.01	1.47,	-0.11	1	0
	t_4t_5	1	1	0.51,	0.87	0.07	0.02	0.58,	0.89	1	1

Area No.	Times Compared	0.7-0.8μm Integer Shift x	0.7-0.8μm Integer Shift y	0.7-0.8μm Interpolated Shift x	0.7-0.8μm Interpolated Shift y	Interpolated Discrepancy Between Bands |Δx|	Interpolated Discrepancy Between Bands |Δy|	0.8-1.1μm Interpolated Shift x	0.8-1.1μm Interpolated Shift y	0.8-1.1μm Integer Shift x	0.8-1.1μm Integer Shift y
2	t_1t_2	-1	0	-1.21,	0.20	0.04	0.01	-1.17,	0.19	-1	0
	t_1t_3	-1	0	-0.73,	-0.31	0.12	0.15	-0.85,	-0.17	-1	0
	t_1t_4	-2	0	-1.76,	-0.24	0.01	0.02	-1.75,	-0.26	-2	0
	t_1t_5	-2	-1*	-2.21,	-0.60	0.34	0.10	-1.87,	-0.50	-2	0*
	t_2t_3	1*	0	0.55,	-0.08	0.17	0.26	0.38,	-0.34	0*	0
	t_2t_4	-1	0	-0.53,	-0.39	0.00	0.02	-0.53,	-0.41	-1	0
	t_2t_5	-1	-1	-1.10,	-0.74	0.11	0.09	-1.21,	-0.65	-1	-1
	t_3t_4	-1	0	-1.10,	-0.20	0.14	0.17	-0.96,	0.03	-1	0
	t_3t_5	-1	0	-1.03,	0.01	0.09	0.12	-0.94,	-0.11	-1	0
	t_4t_5	0	0	-0.17,	-0.07	0.10	0.05	-0.07,	-0.12	0	0
3	t_1t_2	-1	-1*	-0.61,	-0.73	0.07	0.37	-0.54,	-0.36	-1	0*
	t_1t_3	-1	0*	-0.68,	-0.36	0.53	0.20	-1.21,	-0.56	-1	-1*
	t_1t_4	-1	-1	-0.98,	-0.54	0.21	0.23	-1.19,	-0.77	-1	-1*
	t_1t_5	-2	0	-2.09,	-0.10	0.05	0.13	-2.04,	-0.23	-2	0
	t_2t_3	-1	0	-0.67,	0.12	0.03	0.02	-0.70,	0.10	-1	0
	t_2t_4	0*	0	-0.38,	0.22	0.24	0.26	-0.62,	-0.04	-1*	0
	t_2t_5	-2	0	-1.70,	0.30	0.03	0.01	-1.67,	0.31	-2	0
	t_3t_4	0	0	0.02,	0.10	0.08	0.14	0.10,	-0.04	0	0
	t_3t_5	-1	1*	-1.00,	0.64	0.03	0.31	-1.03,	0.33	-1	0*
	t_4t_5	-1	0	-1.25,	0.29	0.15	0.05	-1.10,	0.24	-1	0

B. SHIFT POSITION DISCREPANCY STUDY: SPECTRAL BANDS
TIPPECANOE COUNTY, INDIANA

Area No.	Times Compared	0.6-0.7µm Integer Shift x	0.6-0.7µm Integer Shift y	0.6-0.7µm Interpolated Shift x	0.6-0.7µm Interpolated Shift y	Interpolated Discrepancy Between Bands $\lvert\Delta x\rvert$	Interpolated Discrepancy Between Bands $\lvert\Delta y\rvert$	0.8-1.1µm Interpolated Shift x	0.8-1.1µm Interpolated Shift y	0.8-1.1µm Integer Shift x	0.8-1.1µm Integer Shift y
1	t_1t_2	1	-1	1.01,	-1.08	0.05	0.03	0.96,	-1.05	1	-1
	t_1t_3							3.07,	-1.11	3	-1
	t_2t_3	2	0	1.89	0.12	0.10	0.02	1.99,	0.10	2	0
2	t_1t_2	1	-1	0.94,	-1.02	0.09	0.04	1.03,	-0.98	1	-1
	t_1t_3	2	0	2.01,	-0.48	0.00	0.16	2.01,	-0.32	2	0
	t_2t_3	1	1	0.95,	0.51	0.12	0.01	1.07,	0.52	1	1
3	t_1t_2	1	0	0.75,	-0.36	0.21	0.10	0.96,	-0.26	1	0
	t_1t_3	2*	0	1.97,	0.20	0.66	0.11	1.31,	0.33	1*	0
	t_2t_3	1*	1	0.65,	0.80	0.32	0.03	0.33,	0.77	0*	1
4	t_1t_2	1	0	0.71,	0.29	0.09	0.15	0.62,	0.14	1	0
	t_1t_3	2*	2*	1.66,	1.74	0.72	0.77	0.94,	0.97	1*	1*
	t_2t_3	1*	1	0.69,	1.08	0.42	0.20	0.27,	0.88	0*	1
5	t_1t_2	0	-1	0.30,	-0.92	0.04	0.08	0.34,	-1.00	0	-1
	t_1t_3							0.91,	-1.84	1	-2
	t_2t_3							0.37,	-0.87	0	-1

Area No.	Times Compared	0.6-0.7µm Integer Shift x	0.6-0.7µm Integer Shift y	0.6-0.7µm Interpolated Shift x	0.6-0.7µm Interpolated Shift y	Interpolated Discrepancy Between Bands |Δx|	Interpolated Discrepancy Between Bands |Δy|	0.8-1.1µm Interpolated Shift x	0.8-1.1µm Interpolated Shift y	0.8-1.1µm Integer Shift x	0.8-1.1µm Integer Shift y
6	t_1t_2	1	0	0.62,	-0.09	0.04	0.16	0.66,	-0.25	1	0
	t_1t_3							0.59,	-0.89	1	-1
	t_2t_3	0	-1	-0.36,	-0.77	0.21	0.00	-0.15,	-0.77	0	-1
7	t_1t_2	0	0	0.47,	-0.38	0.31	0.17	0.16,	-0.21	0	0
	t_1t_3	0	-1	-0.18,	-0.84	0.18	0.05	0.00,	-0.79	0	-1
	t_2t_3	0	0	-0.32,	-0.42	0.10	0.01	-0.22,	-0.43	0	0
8	t_1t_2	0	0	0.17,	0.05	0.09	0.00	0.26,	0.05	0	0
	t_1t_3	0	1	-0.17,	-0.71	0.17	0.07	0.00,	0.78	0	1
	t_2t_3	-1*	0*	-0.70,	0.31	0.30	0.27	-0.40,	0.58	0*	1*
9	t_1t_2	0	0*	-0.16,	-0.46	0.02	0.19	-0.14,	-0.65	0	-1*
	t_1t_3	0	-2*	0.40,	-1.57	0.37	0.11	0.03,	-1.46	0	-1*
	t_2t_3	-1*	-1	-0.71,	-1.01	0.80	0.05	0.09,	-0.96	0*	-1
10	t_1t_2	0	-1	-0.23,	-0.79	0.22	0.03	-0.01,	-0.82	0	-1
	t_1t_3	0*	-1	-0.41,	-1.14	0.24	0.19	-0.65,	-1.33	-1*	-1
	t_2t_3	-1	-1	-0.75,	-0.75	0.08	0.01	-0.67,	-0.74	-1	-1

Area No.	Times Compared	0.6-0.7μm Integer Shift x	0.6-0.7μm Integer Shift y	0.6-0.7μm Interpolated Shift x	0.6-0.7μm Interpolated Shift y	Interpolated Discrepancy Between Bands $\lvert\Delta x\rvert$	Interpolated Discrepancy Between Bands $\lvert\Delta y\rvert$	0.8-1.1μm Interpolated Shift x	0.8-1.1μm Interpolated Shift y	0.8-1.1μm Integer Shift x	0.8-1.1μm Integer Shift y
11	t_1t_2	0	0	-0.15,	0.06	0.04	0.01	-0.11,	-0.07	0	0
	t_1t_3	0*	0*	-0.40,	-0.23	0.35	0.41	-0.75,	-0.64	-1*	-1*
	t_2t_3	-1	-1	-0.55,	-0.63	0.07	0.05	-0.62,	-0.58	-1	-1
12	t_1t_2	0	0	-0.14,	0.37	0.05	0.14	-0.19,	0.23	0	0
	t_1t_3	-1	0	-1.13,	0.21	0.22	0.33	-1.35,	-0.12	-1	0
	t_2t_3	-1	0	-1.03,	-0.16	0.00	0.15	-1.03,	-0.31	-1	0

Blanks indicate false shift positions.

C. INTERPOLATED SHIFT POSITIONS: PREPROCESSED DATA

HILL COUNTY, MONTANA

SPECTRAL BAND 0.8-1.1μm USED TO GENERATE THESE RESULTS

Area No.	Times Compared	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		x	y	x	y	x	y	x	y	x	y
1	t_1t_2	0.00	0.04	0.18	0.02	-0.01	0.22	0.24	0.11	0.03	0.01
	t_1t_3	-1.28	0.07	-1.04	0.02	-1.26	0.13	-1.30	0.28	-1.27	0.04
	t_1t_4	-0.27*	-0.78	-0.17*	-0.93	-0.68	-0.95	-0.04*	-0.48*	-0.18*	-0.92
	t_1t_5	0.17	-0.07	0.18	-0.09	0.00	-0.01	0.13	0.16	0.20	-0.09
	t_2t_3	-1.23	0.05	-1.31	0.04	-1.19	0.04	-1.14	0.00	-1.28	0.00
	t_2t_4	-0.33*	-0.85	-0.16*	-0.95	-0.70	-0.94	-0.59	-0.81	-0.15*	-0.96
	t_2t_5	0.18	-0.14	0.01	-0.13	0.25	-0.02	0.28	-0.19	0.31	-0.15
	t_3t_4	0.92	-0.91	0.94	-0.97	0.88	-0.91	0.98	-0.91	1.00	-0.96
	t_3t_5	1.59*	-0.10	1.49	-0.16	1.68	-0.01	1.65*	0.01	1.76*	0.00
	t_4t_5	0.59	0.90	0.37*	0.84	0.71	0.95	0.62	0.98	0.37*	0.79

Area No.	Times Compared	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		x	y	x	y	x	y	x	y	x	y
2	t_1t_2	-1.17	0.15	-1.28	0.15	-1.06	0.10	-1.12	0.11	-1.05	0.09
	t_1t_3	-0.91	-0.09	-0.95	-0.08	-0.85	-0.14	-0.94	-0.04	-0.91	-0.17
	t_1t_4	-1.74	-0.28	-1.81	-0.34	-1.59	-0.44	-1.82	-0.16	-1.77	-0.44
	t_1t_5	-1.91	-0.37			-1.94	-0.32	-2.16	-0.54*	-1.88	-0.36
	t_2t_3	0.31	-0.27	0.29	-0.31	0.12	-0.08	0.19	-0.17	0.15	-0.20
	t_2t_4	-0.51	-0.43	-0.50*	0.29	-0.61	-0.37	-0.50*	0.19	-0.60	-0.32
	t_2t_5	-1.32	-0.53	-1.00	-0.89	-0.85	-0.29*	-0.72	-0.39*	-0.99	-0.56
	t_3t_4	-0.93	-0.04	-0.94	-0.01	-0.74	-0.09	-0.89	-0.02	-0.86	-0.07
	t_3t_5	-0.93	-0.10	-0.81	-0.29	-0.91	-0.06	-0.89	-0.17	-0.94	-0.09
	t_4t_5	0.13	-0.15	0.60*	0.13	-0.10	-0.04	0.11	-0.24	-0.90*	-0.90*
3	t_1t_2	-0.58	-0.47	-0.77	-0.34	-0.60	-0.89*	generated		-0.81	-0.20
	t_1t_3	-0.91	-0.35*	-0.90	0.00*	-1.33	-0.58	data		-0.82	-0.05*
	t_1t_4	-0.85	-0.52	-0.80	-0.61*	-1.19	-0.76	not			
	t_1t_5	-2.06	-0.06	-2.05	-0.05	-2.03	-0.23	usable		-2.04	0.02
	t_2t_3	-0.71	-0.05	-0.62	-0.18	-0.91	0.22	-0.58*	-0.07	-0.75	-0.07
	t_2t_4	-0.45*	0.08	-0.61	-0.11	-0.76	0.02	-0.44	0.00	-0.53	-0.19
	t_2t_5	-1.74	0.16	-1.74	0.04	-1.73	0.32	-1.64	0.12	-1.71	-0.03
	t_3t_4	0.09	-0.01	0.02	0.04	0.14	-0.08	-0.02	-0.01	0.06	-0.02
	t_3t_5	-1.10	0.41	-1.07	0.20	-0.94	0.25	-1.09	0.37	-1.06	0.21
	t_4t_5	-1.19	0.30	-1.08	0.17	-1.01	0.20	-1.18	0.26	-1.11	0.18

Blanks indicate false shift positions.

D. SHIFT POSITION DISCREPANCY: PREPROCESSED DATA

HILL COUNTY, MONTANA

|interpolated shift difference| = |original data - preprocessed data|

SPECTRAL BAND 0.8-1.1μm USED TO GENERATE THESE RESULTS

Area No.	Times Compared	Gradient		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		$\lvert \text{Gradient} \rvert$ Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		$\lvert \Delta x \rvert$	$\lvert \Delta y \rvert$	$\lvert \Delta x \rvert$	$\lvert \Delta y \rvert$	$\lvert \Delta x \rvert$	$\lvert \Delta y \rvert$	$\lvert \Delta x \rvert$	$\lvert \Delta y \rvert$	$\lvert \Delta x \rvert$	$\lvert \Delta y \rvert$
1	t_1t_2	0.07	0.01	0.25	0.01	0.06	0.18	0.31	0.08	0.04	0.02
	t_1t_3	0.16	0.04	0.40	0.09	0.18	0.02	0.14	0.17	0.17	0.07
	t_1t_4	0.63+	0.00	0.73+	0.15	0.22	0.17	0.86+	0.30+	0.72+	0.14
	t_1t_5	0.05	0.11	0.30	0.13	0.12	0.03	0.02	0.12	0.32	0.13
	t_2t_3	0.03	0.04	0.05	0.03	0.07	0.03	0.12	0.01	0.02	0.01
	t_2t_4	0.29+	0.08	0.46+	0.02	0.08	0.01	0.03	0.12	0.37+	0.03
	t_2t_5	0.10	0.07	0.07	0.06	0.17	0.05	0.20	0.12	0.23	0.08
	t_3t_4	0.08	0.01	0.10	0.07	0.04	0.01	0.14	0.01	0.16	0.06
	t_3t_5	0.12+	0.01	0.02+	0.05	0.21+	0.10	0.18+	0.12	0.29+	0.11
	t_4t_5	0.01	0.01	0.21+	0.05	0.13	0.06	0.04	0.09	0.21+	0.10

+ one unit shift discrepancy with original data

Blanks indicate false shift positions.

Area No.	Times Compared	|Gradient| Δx	|Gradient| Δy	Local Gradient No Threshold $\sigma n^2 = 1.2$ Δx	Local Gradient No Threshold $\sigma n^2 = 1.2$ Δy	Threshold at Median Δx	Threshold at Median Δy	|Gradient| Threshold=6.0 Δx	|Gradient| Threshold=6.0 Δy	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$ Δx	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$ Δy
2	t_1t_2	0.00	0.04	0.11	0.04	0.11	0.09	0.05	0.08	0.12	0.10
	t_1t_3	0.06	0.08	0.10	0.09	0.00	0.03	0.09	0.13	0.06	0.00
	t_1t_4	0.01	0.02	0.06	0.08	0.16	0.18	0.07	0.10	0.02	0.18
	t_1t_5	0.04	0.13			0.04	0.18	0.29	0.04+	0.01	0.14
	t_2t_3	0.07	0.07	0.09	0.03	0.26	0.26	0.19	0.17	0.23	0.14
	t_2t_4	0.02	0.02	0.03+	$\boxed{0.70}$	0.08	0.04	0.03+	$\boxed{0.60}$	0.07	0.09
	t_2t_5	0.11	0.12	0.21	0.24	0.36	0.36+	0.49	0.26+	0.22	0.09
	t_3t_4	0.03	0.07	0.02	0.04	0.22	0.12	0.07	0.05	0.10	0.10
	t_3t_5	0.01	0.01	0.13	0.18	0.03	0.05	0.05	0.06	0.00	0.02
	t_4t_5	0.20	0.03	$\boxed{0.67}$+	0.25	0.03	0.08	0.18	0.12	$\boxed{0.83}$+	$\boxed{0.78}$+
3	t_1t_2	0.04	0.11	0.23	0.02	0.06	$\boxed{0.53}$+			0.27	0.16
	t_1t_3	0.30	0.21+	0.31	$\boxed{0.56}$+	0.12	0.02			0.39	$\boxed{0.51}$+
	t_1t_4	0.34	0.22	0.39	0.16	0.00	0.01				
	t_1t_5	0.02	0.17	0.01	0.18	0.01	0.00			0.00	0.25
	t_2t_3	0.01+	0.15	0.08	0.28	0.21	0.12	0.12+	0.17	0.05	0.17
	t_2t_4	0.17+	0.12	0.01	0.07	0.14	0.06	0.18	0.04	0.09	0.15
	t_2t_5	0.07	0.15	0.07	0.27	0.06	0.01	0.03	0.19	0.04	0.34
	t_3t_4	0.01	0.03	0.08	0.08	0.04	0.04	0.12	0.03	0.04	0.02
	t_3t_5	0.07	0.08	0.04	0.13	0.09	0.08	0.06	0.04	0.03	0.12
	t_4t_5	0.09	0.06	0.02	0.07	0.09	0.04	0.08	0.02	0.01	0.06

E. INTERPOLATED SHIFT POSITIONS: PREPROCESSED DATA

TIPPECANOE COUNTY, INDIANA

SPECTRAL BAND 0.8-1.1μm USED TO GENERATE THESE RESULTS

Area No.	Times Compared	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		x	y	x	y	x	y	x	y	x	y
1	t_1t_2	1.03	-0.94	0.98	-1.10	1.04	-1.04	1.11	-0.94	1.21	-0.89
	t_1t_3	2.95	-0.92	2.96	-1.12	3.55*	-1.46	3.13	-0.69	3.04	-1.06
	t_2t_3	1.92	0.11	2.17	0.00	1.99	0.08	1.98	0.08	1.90	-0.03
2	t_1t_2	1.03	-0.98	0.94	-1.01	1.17	-1.02	1.03	-0.96	1.05	-0.97
	t_1t_3	2.06	-0.28	1.85	-0.11	2.00	-0.35	1.96	-0.33	1.78	-0.38
	t_2t_3	1.13	0.55	1.02	0.67	1.10	0.53	1.06	0.67	1.13	0.65
3	t_1t_2	1.08	-0.14	0.87	0.04	1.04	-0.35	0.96	-0.15	0.84	-0.20
	t_1t_3	1.49	0.34	1.30	0.05			1.26	0.24	1.75*	0.05
	t_2t_3	0.36	0.81	0.23	0.94	0.09	0.79	0.16	0.90	0.48	1.00
4	t_1t_2	0.53	0.12	0.18*	0.01	0.40*	0.06	0.49	0.00	0.51	0.09
	t_1t_3	0.57	1.06	0.08*	0.93	0.93	0.84	0.61	0.95	0.23*	0.83
	t_2t_3	0.22	0.92	0.13	0.90	0.40	0.80	0.47	1.00	0.59*	1.03

Area No.	Times Compared	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		x	y	x	y	x	y	x	y	x	y
5	t_1t_2	0.44	-1.04	0.41	-1.03	0.42	-1.03	0.55	-0.94	0.54*	-0.97
	t_1t_3	1.05	-1.93	1.27	-1.92	1.06	-1.90	1.11	-1.91	1.07	-1.92
	t_2t_3	0.39	-0.98	0.21	-0.94	0.38	-0.83	0.37	-0.97	0.24	-0.92
6	t_1t_2	0.79	-0.30					0.47	-0.59	-0.05*	-0.59*
	t_1t_3	0.58	-0.87	0.44*	-1.48	0.66	-0.87	0.62	-0.90	0.60	-0.89
	t_2t_3	-0.05	-0.80	-0.19	-0.79	0.16	-0.82	-0.14	-0.84	-0.16	-0.79
7	t_1t_2	0.19	-0.17	0.61*	-0.03	0.29	-0.34	0.11	-0.23	0.26	-0.16
	t_1t_3	-0.03	-0.80	0.03	-0.83	0.03	-0.76	0.06	-0.90	-0.05	-0.90
	t_2t_3	-0.24	-0.39	-0.17	-0.19	-0.18	-0.28	-0.14	-0.38	-0.04	-0.12
8	t_1t_2	0.25	-0.02	0.19	0.03	0.15	0.03	0.17	-0.04	0.21	-0.04
	t_1t_3	0.01	0.72	0.42	0.89	-0.59*	1.08	-0.09	0.81	0.01	0.83
	t_2t_3	-0.82*	0.45*	-0.16	0.84	-0.25	0.50*	-0.56*	0.63	-0.77*	0.63
9	t_1t_2	-0.30	-0.75	-0.23	-0.73	-0.20	-0.59	-0.30	-0.73	-0.27	-0.64
	t_1t_3	-0.15	-1.59*	-0.06	-1.43	-0.46	-1.30	-0.20	-1.68*	0.06	-1.58*
	t_2t_3	0.04	-0.94	0.06	-0.95	-0.15	-1.01	0.03	-0.95	0.09	-0.98
10	t_1t_2	-0.12	-0.85	-0.03	-0.94	-0.15	-0.72	-0.13	-0.74	-0.18	-0.88
	t_1t_3	-0.79	-1.56*	-0.83	-1.86*	-1.08	-1.28	-0.57	-1.60*	-0.45	-1.52*
	t_2t_3	-0.77	-0.79	-0.85	-0.90	-0.45*	-0.87	-0.69	-0.82	-0.78	-0.83

Area No.	Times Compared	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		x	y	x	y	x	y	x	y	x	y
11	t_1t_2	-0.14	-0.05	0.05	-0.02	-0.02	-0.02	-0.12	0.02	-0.08	-0.03
	t_1t_3	-0.92	-0.72	-0.74	-0.95	-0.73	-0.58	-1.00	-0.74	-0.73	-0.94
	t_2t_3	-0.71	-0.65	-0.83	-0.53	-0.65	-0.67	-0.76	-0.68	-0.73	-0.85
12	t_1t_2	-0.15	0.21	-0.22	0.18	-0.16	0.15	-0.17	0.15	-0.19	0.29
	t_1t_3	-1.10	0.10	-1.16	-0.01	-1.78*	-0.61*	-1.36	0.11	-1.02	-0.03
	t_2t_3	-0.96	-0.05	-0.99	0.02	-1.18	-0.35	-1.04	-0.04	-0.98	-0.08

F. SHIFT POSITION DISCREPANCY: PREPROCESSED DATA

TIPPECANOE COUNTY, INDIANA

SPECTRAL BAND 0.8-1.1μm USED TO GENERATE THESE RESULTS

|interpolated shift difference| = |original data - preprocessed data|

Area No.	Times Compared	|Gradient| |Δx|	|Gradient| |Δy|	Local Gradient No Threshold $\sigma n^2 = 1.2$ |Δx|	Local Gradient No Threshold $\sigma n^2 = 1.2$ |Δy|	Threshold at Median |Δx|	Threshold at Median |Δy|	|Gradient| Threshold=6.0 |Δx|	|Gradient| Threshold=6.0 |Δy|	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$ |Δx|	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$ |Δy|
1	t_1t_2	0.07	0.11	0.02	0.05	0.08	0.01	0.15	0.11	0.25	0.16
	t_1t_3	0.12	0.19	0.11	0.01	0.48*	0.35	0.06	0.52	0.03	0.05
	t_2t_3	0.07	0.01	0.18	0.10	0.00	0.02	0.01	0.02	0.09	0.13
2	t_1t_2	0.00	0.00	0.09	0.03	0.14	0.04	0.00	0.02	0.02	0.01
	t_1t_3	0.05	0.04	0.16	0.21	0.01	0.03	0.05	0.01	0.23	0.06
	t_2t_3	0.06	0.03	0.05	0.15	0.03	0.01	0.01	0.15	0.05	0.13
3	t_1t_2	0.12	0.12	0.09	0.30	0.08	0.09	0.00	0.11	0.12	0.06
	t_1t_3	0.18	0.01	0.01	0.28			0.05	0.09	0.44*	0.28
	t_2t_3	0.03	0.04	0.10	0.17	0.24	0.02	0.17	0.13	0.15	0.23
4	t_1t_2	0.09	0.02	0.44*	0.13	0.22*	0.08	0.13	0.14	0.11	0.05
	t_1t_3	0.37	0.09	0.86*	0.04	0.01	0.13	0.33	0.02	0.71*	0.14
	t_2t_3	0.05	0.04	0.14	0.02	0.13	0.08	0.20	0.12	0.32*	0.15
5	t_1t_2	0.10	0.04	0.07	0.03	0.08	0.03	0.21	0.06	0.20*	0.03
	t_1t_3	0.14	0.09	0.36	0.08	0.15	0.06	0.20	0.07	0.16	0.08
	t_2t_3	0.02	0.11	0.16	0.07	0.01	0.04	0.00	0.10	0.13	0.05

Area No.	Times Compared	|Gradient| |Δx|	|Gradient| |Δy|	Local Gradient No Threshold $\sigma n^2 = 1.2$ |Δx|	Local Gradient No Threshold $\sigma n^2 = 1.2$ |Δy|	Threshold at Median |Δx|	Threshold at Median |Δy|	|Gradient| Threshold=6.0 |Δx|	|Gradient| Threshold=6.0 |Δy|	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$ |Δx|	Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$ |Δy|
6	t_1t_2	0.13	0.05					0.19	0.34	0.71*	0.34*
	t_1t_3	0.01	0.02	0.15*	0.59	0.07	0.02	0.03	0.01	0.01	0.00
	t_2t_3	0.10	0.03	0.04	0.02	0.31	0.05	0.01	0.07	0.01	0.02
7	t_1t_2	0.03	0.04	0.45*	0.18	0.13	0.13	0.05	0.02	0.10	0.05
	t_1t_3	0.03	0.01	0.03	0.04	0.03	0.03	0.06	0.11	0.05	0.11
	t_2t_3	0.02	0.04	0.05	0.24	0.04	0.15	0.08	0.05	0.18	0.31
8	t_1t_2	0.01	0.07	0.07	0.02	0.11	0.02	0.09	0.09	0.05	0.09
	t_1t_3	0.01	0.06	0.42	0.11	0.59*	0.30	0.09	0.03	0.01	0.05
	t_2t_3	0.42*	0.13*	0.24	0.26	0.15	0.08*	0.16*	0.05	0.37*	0.05
9	t_1t_2	0.16	0.10	0.09	0.08	0.06	0.06	0.16	0.08	0.13	0.01
	t_1t_3	0.18	0.13*	0.09	0.03	0.49	0.16	0.23	0.22*	0.03	0.12*
	t_2t_3	0.05	0.02	0.03	0.01	0.24	0.05	0.06	0.01	0.00	0.02
10	t_1t_2	0.11	0.03	0.02	0.12	0.14	0.10	0.12	0.12	0.17	0.06
	t_1t_3	0.06	0.23*	0.18	0.53*	0.43	0.05	0.08	0.27*	0.20	0.19*
	t_2t_3	0.10	0.05	0.18	0.16	0.22*	0.13	0.02	0.08	0.11	0.09
11	t_1t_2	0.03	0.02	0.16	0.05	0.09	0.05	0.01	0.09	0.03	0.04
	t_1t_3	0.17	0.08	0.01	0.31	0.02	0.06	0.25	0.10	0.02	0.30
	t_2t_3	0.09	0.07	0.21	0.05	0.03	0.09	0.14	0.10	0.11	0.27

Area No.	Times Compared	|Gradient|		Local Gradient No Threshold $\sigma n^2 = 1.2$		Threshold at Median		|Gradient| Threshold=6.0		Local Gradient Threshold=11.0 $\sigma n^2 = 1.2$	
		$\|\Delta x\|$	$\|\Delta y\|$	$\|\Delta x\|$	$\|\Delta y\|$	$\|\Delta x\|$	$\|\Delta y\|$	$\|\Delta x\|$	$\|\Delta y\|$	$\|\Delta x\|$	$\|\Delta y\|$
12	t_1t_2	0.04	0.02	0.03	0.05	0.03	0.08	0.02	0.08	0.00	0.06
	t_1t_3	0.25	0.22	0.19	0.11	0.33*	0.49*	0.01	0.23	0.33	0.09
	t_2t_3	0.07	0.26	0.04	0.29	0.15	0.04	0.01	0.27	0.05	0.23

Blanks indicate false shift positions.

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