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Various Measures of Moisture Content of Air

by

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The most variable constituent of air is water vapor, and each reference seems to have its own method of describing it--precipitable centimeters per 1000 yards, grams per cubic meter, molecules per cubic meter, millimeters of mercury of water vapor partial pressure, mixing ratio, relative humidity or specific humidity. It seemed desirable to make a chart relating these units to each other.

Reference was found which said that ". . . both air and water are practically perfect gases . . ."[1] so the equation of state of an ideal gas was used to compute the number of moles and the mass density of the water vapor.

$$pV = NRT$$

Where R is 8.31×10^7 ergs/mole $^\circ K$

T is in degrees Kelvin

p is in dynes/cm^2

V is in cm^3

N is number of gram moles.

Since

$$\text{Number of moles} = \frac{\text{mass}}{\text{molecular weight}} \quad \text{or } N = \frac{m}{M}$$

and the mass density is

$$\rho = \frac{m}{V}$$

the equation of state of a gas is also written

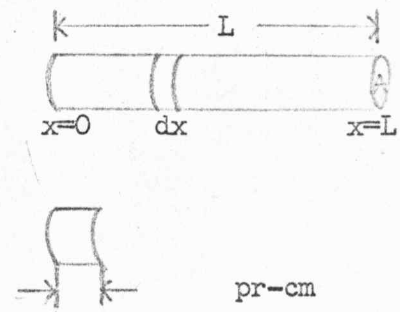
$$p = \frac{m}{V}\frac{R}{M}T \quad \text{or,} \qquad p = \rho\frac{RT}{M}$$

[1] Lichty, Lexter, Thermodynamics, (McGraw-Hill, New York 1948), p. 102.

The partial pressures (in mm Hg) of water vapor corresponding to temperatures in Centigrade degrees were taken from the Institute of Physics Handbook,[2] and number of moles and mass density in grams per cubic centimeter were calculated. It seems to be more common, and the units are less unwieldy in the case of water vapor in air, to use the units of grams per cubic meter; therefore, the results were so recorded. Number of molecules was determined by multiplying the number of moles by Avogadro's number, 6.02×10^{23} molecules per mole.

Quoting from the Handbook of Geophysics,[3] "It is customary to express the amount of water vapor concentration in the atmosphere in precipitable centimeters. This measurement of concentration is the thickness of the layer of liquid water that would be formed if all the water vapor traversed by a light beam were condensed in a container of cross sectional area equal to that of the beam."

This means that for the path length, L, of the light beam, and a cross sectional area, A, the precipitable centimeters (pr-cm) of H_2O can be found by summing up the amount of water per unit, dx, thus,



$$\int_0^L n(x)\ A\,dx = A.(\text{pr-cm})\cdot\left(N_o\ \frac{\text{molecules}}{\text{gram-mole}} \cdot \frac{1}{18\ \text{grams/gram-mole}} \cdot 1\ \text{g/cm}^3\right)$$

$n(x)$ = number of H_2O molecules/cm^3
pr-cm = precipitable centimeters
N_o = Avogadro's number

[2] American Institute of Physics Handbook, 2nd edition (McGraw-Hill, New York, Toronto, London, 1963), p. 4-275.

[3] United States Air Force, Handbook of Geophysics (Macmillan, New York, 1961), p. 16-2.

Solving for the pr-cm

$$\text{pr-cm} = \frac{18}{N_o} \int_o^L n(x)\ dx$$

or

$$\text{pr-cm} = \frac{18\ \overline{n(x)}^{L}\ L}{N_o}$$

where $\overline{n(x)}^{L}$ is the average H_2O molecule number density in the beam path. Path length was taken to be 1000 yards to correspond with the tables and charts which report this quantity as precipitable centimeters per thousand yards.

The methods of describing moisture content which are proportional to the mass density of the water present are shown on one graph(Graph 1). The different scales of the Y axis are the grams of water per cubic meter, which is called absolute humidity, number of moles per cubic meter, number of molecules per cubic meter, and the precipitable centimeters per 1000 yards.

Other methods of describing water content are related to the partial pressure of water vapor. This is given on graph 2 in several commonly used units, millimeters of mercury, dynes/cm^3, millibars and atmospheres. Specific humidity is also shown on this graph, since it is directly proportional to the other scales.

Specific humidity is the number of grams of water per kilogram of normal air. Another common method of expressing moisture content is called the mixing ratio, which is the number of grams of water per kilogram of dry air. Air was considered to be at standard conditions which then has a pressure of 1.013 x 10^3 dynes/cm^2.

Petterssen[4] gives equations for computing specific humidity and the mixing ratio, without explaining the constants.

$$q = \frac{623\ e}{p - .377\ e} \qquad \text{and} \qquad m = \frac{623\ e}{p - e}$$

where e is the vapor pressure of water in millibars.

p is the atmospheric pressure in millibars

q is the number of grams of water per kilogram of normal air

m is the number of grams of water per kilogram of dry air.

According to Petterssen, "The specific humidity differs so slightly from the mixing ratio, that the difference is insignificant in view of the inaccuracy in humidity measurements." Figures for q and m agree with each other to two significant figures up to a vapor pressure of 30 mm Hg, which corresponds to approximately 30 grams of water per cubic meter. For greater water vapor content these equations are not valid and differ too much from the theoretical calculations. For this reason, the calculations for specific humidity are given to correspond to a water content of 30 g/m^3 or less. A justification for this was found in the Handbook of Geophysics,[5] "At the highest surface air temperature of 55° C the air could hold about 100 grams (of water) per cubic meter. However, the amount actually present in the atmosphere is a complex function of various weather parameters. Thus the highest water vapor content recorded is about 30 grams per cubic meter . . ."

The calculations for specific humidity were made in two ways—first, by Petterssen's formula and, second, by multiplying the number of grams per cubic meter by the volume of one kilogram of air:

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[4]Petterssen, Sverre, Introduction to Meteorology (McGraw-Hill, New York & London, 1941), p. 19.

[5]Handbook of Geophysics, p. 8-5.

$$q = g(H_2O)/m^3 \times 10^{-6}\ m^3/cm^3 \times 10^3 g\ (air)/\rho(air)$$

The density of air at 0° C is $1.2929 \times 10^{-3} g/cm^3$ and is proportional to temperature changes in this way:[6]

$$\rho\ (\text{gas at } T) = \rho_0 \left(\frac{273.16}{T}\right)$$

Although the g/m^3 of water differs considerably for temperatures from $285^\circ K$ to $313^\circ K$, the density of air differs also, and the specific humidity for a given vapor pressure is remarkably the same throughout the temperature range.

The two methods of calculation gave figures which differed by less than 2%. The latter method of calculation was considered to be more exact so the specific humidities given on Graph 2 were made by this method.

A mnemonic device for relating the methods of describing moisture content by mass and by pressure is given here. It was noted that the values of mm Hg and g/m^3 were the same when $T = 289^\circ K$. In order to show this, consider the equation of state of an ideal gas, rearranged

$$\frac{m}{V} = \frac{p\,M}{R\,T}$$

$$\frac{m}{V} \text{ in } (g/m^3) = p\ (\text{mm Hg}) \cdot \left(\frac{13{,}333 \frac{\text{dynes/cm}^2}{\text{mm Hg}} \times 10^6\ cm^3/m^3 \times 18 \text{ (molecular weight of) } H_2O}{8.31 \times 10^7 \text{ ergs/mole } ^\circ K \times 289^\circ K} \right)$$

Numerically, the portion within the parentheses is equal to 1. For the temperature range used here ($285^\circ K$ to $305^\circ K$), the grams per cubic meter of water vapor are very nearly equal to the water vapor partial pressure in mm Hg. The difference goes from 0% to 6%. Graph 3 shows the relationship between vapor pressure and g/m^3, which, by definition, is the absolute humidity.

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[6] American Institute of Physics Handbook, p. 3-59.

[1]Lichty, Lexter, Thermodynamics, (McGraw-Hill, New York, 1948), p. 102.

[2]American Institute of Physics Handbook, 2nd edition, (McGraw-Hill, New York, Toronto, London, 1963), p. 4-275.

[3]United States Air Force, Handbook of Geophysics, (Macmillan, New York, 1961), p. 16-2.

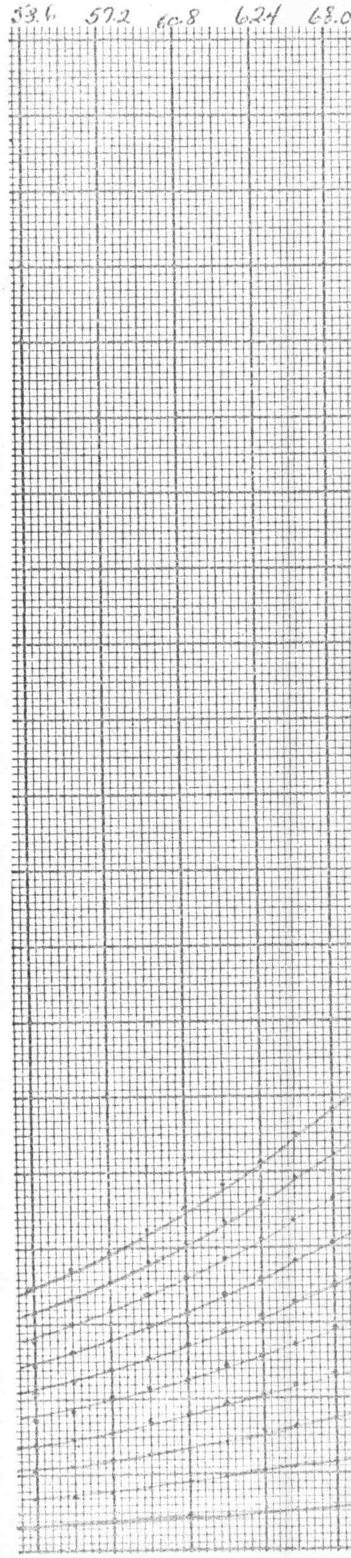
[4]Petterssen, Sverre, Introduction to Meteorology, (McGraw-Hill, New York & London, 1941), p. 19.

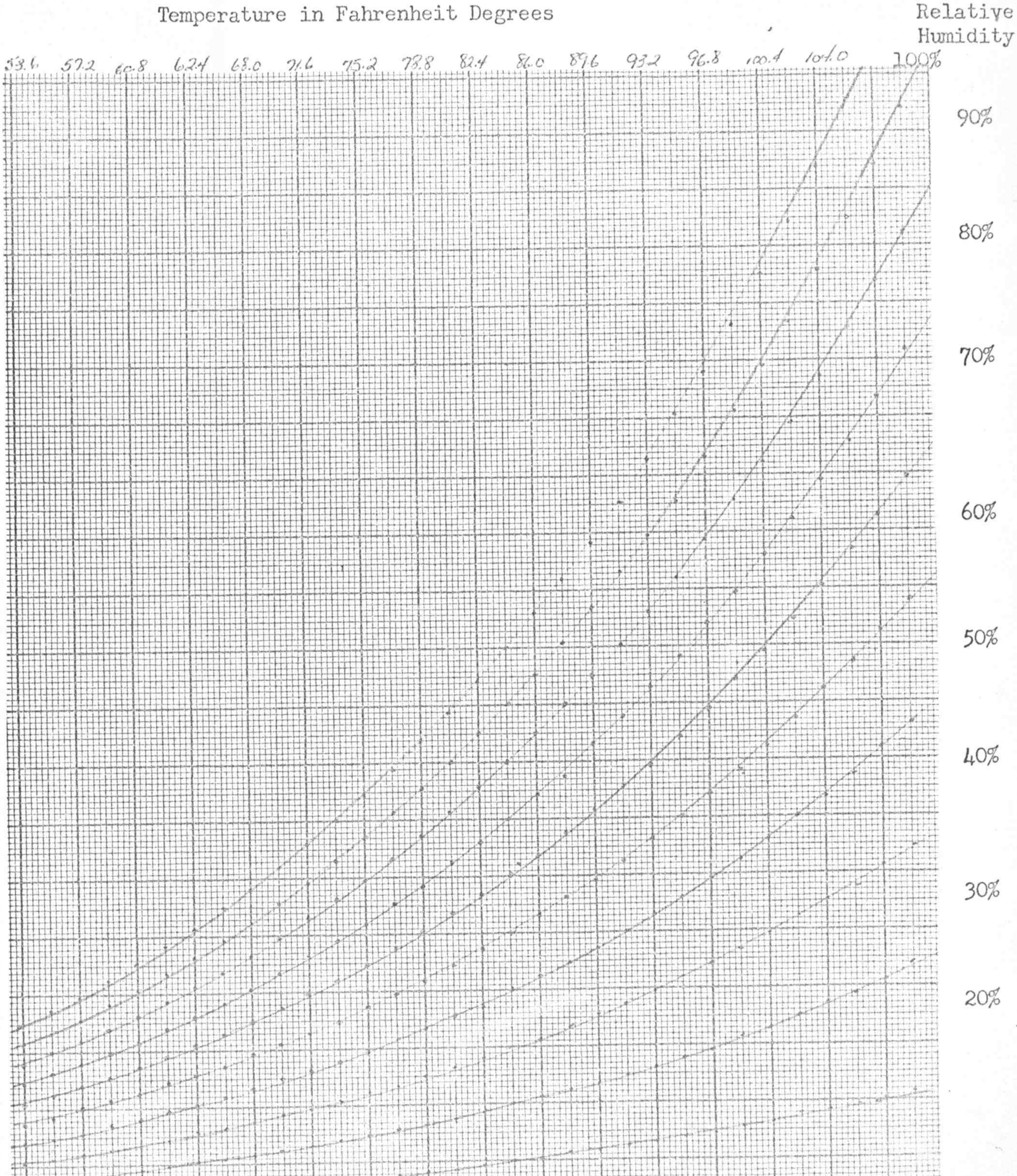
[5]Handbook of Geophysics, p. 8-5.

[6]American Institute of Physics Handbook, p. 3-59.

Specific Humidity $g(H_2O)/kg(air)$	Partial Pressures of H_2O: Atmos-pheres	Dynes/cm^2 $(x\ 10^3)$ or Millibars	mmHg
	.079	80	60
	.075	76	57
	.071	72	54
	.067	68	51
	.063	64	48
	.059	60	45
	.055	56	42
31.8	.051	52	39
29.4	.047	48	36
26.9	.043	44	33
[illegible].5	.040	40	30
22.1	.036	36	27
19.6	.032	32	24
17.3	.028	28	21
14.7	.024	24	18
12.2	.020	20	15
9.8	.016	16	12
7.3	.012	12	9
4.9	.008	8	6
2.5	.004	4	3

Temperatu




Temperature in Fahrenheit Degrees
Relative Humidity
53.6
57.2
60.8
62.4
68.0
71.6
75.2
78.8
82.4
86.0
89.6
93.2
96.8
100.4
104.0
100%
90%
80%
70%
60%
50%
40%
30%
20%

Precipitable Centimeters per 1000 yds.	Molecules per Cubic Meter (x 10^{23})	Moles per Cubic Meter	Absolute Humidity Grams per Cubic Meter
49	20.1	3.33	60
5.21	19.0	3.16	57
4.94	18.0	3.00	54
4.66	17.0	2.83	51
4.39	16.0	2.66	48
4.12	15.0	2.50	45
3.84	14.0	2.33	42
3.57	13.0	2.17	39
3.29	12.0	2.00	36
3.02	11.0	1.83	33
2.74	10.0	1.67	30
2.47	9.0	1.50	27
2.20	8.0	1.33	24
1.92	7.0	1.17	21
1.65	6.0	1.00	18
1.37	5.0	0.83	15
1.10	4.0	0.67	12
.82	3.0	0.50	9
.55	2.0	0.33	6
.27	1.0	0.17	3

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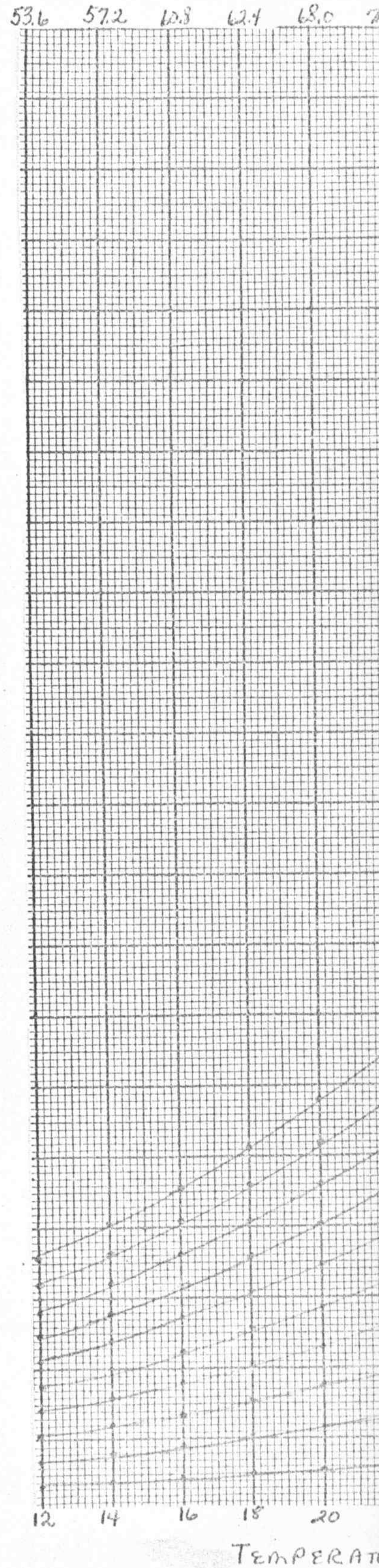
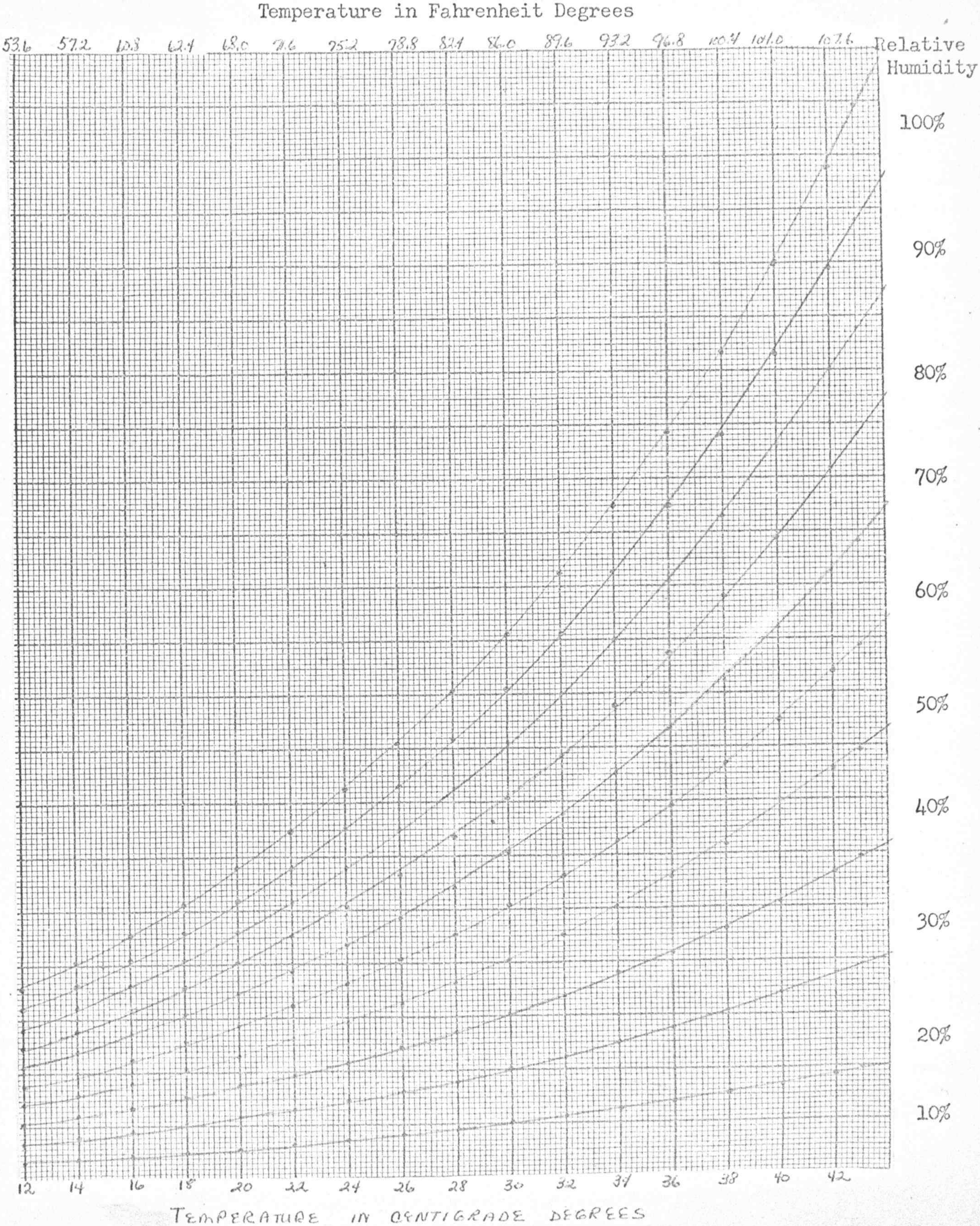


Water Content as a Function of Temperatu



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