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Measurement of Available Soil Moisture

by

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1. Introduction

The following pages are the author's notes taken during 1969 and are devoted, in the main, to a discussion of methods for measuring the resistivity of soil. While this is not a satisfactory method for measuring soil moisture, the material was assembled before its shortcomings were appreciated and is documented here as a background to the study of soil moisture measurement as a function of depth. This material is derived from source material and has been printed to have this information available. It does not reflect the research and literature since that time, nor the present thinking of the author.

A technique based on the measurement of the dielectric constant of the soil as a function of depth was later chosen as the most likely approach to a workable system. Dry soils have dielectric constants of about three to six, whereas water has a dielectric constant of about eighty. Consequently, as water is added to dry soil, the dielectric constant would be expected to rise from about five to higher and higher values rather rapidly as a function of the amount of water added.

Since it was anticipated that it would be necessary to first develop a ground-based instrument and later an airborne instrument for the measurements, the development program was begun by first conducting an experimental program measuring the dielectric constant of soil as a function of water content, and frequency of the impressed electric voltage for the different types of soil most prevalent in Indiana. A constant temperature of 72°F was to be used for the first part of the work, as well as ion-free water. It was intended to use electrical frequencies from 20 kilohertz to as high as necessary, perhaps 1 gigahertz (1×10^9 hertz).

Many anomalous effects have been encountered in the measurements. The measured capacitance between the electrodes of the test capacitor was found to be a very strong function of the soil type (as was expected), of the frequency of the electrical voltage used, of the salinity of the water used to wet the soil, and of the length of time during which the sample is under test, as well as of the amount of water present. These effects have been explained, for the most part, but the real problem is to control them, in order to develop satisfactory field instruments.

The following pages are divided into the readings on five different methods proposed:

2. Objectives

The objectives of this project are two fold:

a. To investigate the possibility of developing hand-carried instruments capable of measuring the available soil moisture content in the field at depths of about three inches, twelve inches, eighteen inches, and forty-eight inches. These measurements should be made with an accuracy of no worse than a few percent. The equipment should be easily hand-portable (weighing not more than 20 pounds); it should be capable of being set up and read in not more than five minutes by a single semi-skilled operator; and it should be reasonably rugged. If it is not possible to make the measurements at depths as precise as those listed above, a reasonable alternative would be to obtain readings which would give average soil moisture contents over the ranges of 2 - 4 inches, 12 - 16 inches, and 36 - 48 inches. Any instruments recommended for development must, of course, do a job superior to that being done by presently available instruments.

b. To investigate the possibility of developing aircraft-carried instruments of reasonable weight, size, complexity, and ruggedness, capable of achieving the measuring capabilities discussed under "a" above.

Since the requirements listed under "a" above are much more likely to be achieved than those listed under "b", the hand-carried type of measuring instrument will be investigated first. This approach has the added advantages that some of the possible methods investigated under "a" may turn out to be useful under "b"; may, by modification, be useful under "b"; or may generate ideas which will be useful under "b".

3. Basic Considerations

The approach to the problem which had been hoped to be followed, by the present writer, had been to try to relate the electrical conductivity (σ) and the permittivity (ε) of the soil to the available moisture in the soil, taking into account the character of the soil, that is, whether it is clay, silt, or sand. These first steps leaned somewhat heavily upon the work of Davis, Lundien, and Williamson (1966), as well as upon the somewhat later work of Nikodem (1966). Based particularly upon the work of Davis, et al., it had been hoped that the measurement of soil conductivity, together with a knowledge of the type of soil, would suffice for the determination of the available moisture content of the soil. Measurement of the conductivity appears to be much more easily accomplished than measurement of the electrical permittivity of the soil, particularly as a function of the depth. Unfortunately, however, Baver (1956, p. 291) makes this comment:

> "Whitney, Gardner and Briggs (1898) proposed an electrical conductivity method for measuring soil moisture in the field. Electrodes were placed in the soil, and the conductivity was measured and interpreted on the basis that any changes in electrical conductivity were brought about by varying amounts of water between the electrodes. It was soon observed, however, that small changes in the salt content of the soil solution affected the conductivity more than the amount of water that was present. In light of this fact, measuring soil moisture by means of electrical conductivity has never proved successful. Multiple electrodes have been used but have not proved entirely satisfactory." (McCorkle, 1931)

This is unfortunate because much work has been done, by geophysicists in particular, on the problem of measuring soil electrical conductivity as a function of depth (Keller and Frischknecht, 1966, Chapter 3), and it appears to be a rather simple problem to develop a practical method for making electrical soil conductivity measurements at varying depths below the surface of the earth.

The work of Davis, et al., (1966) indicates that the electrical permittivity of soil depends upon the percentage of water in the soil, as is shown in Figures 3-1, 3-2, and 3-3, which are abstracted from their report. Consequently, it is believed that the determination of the electrical permittivity of soil, in situ, as a function of depth of the soil sample, will be interpretable as a measure of the available moisture in the soil.

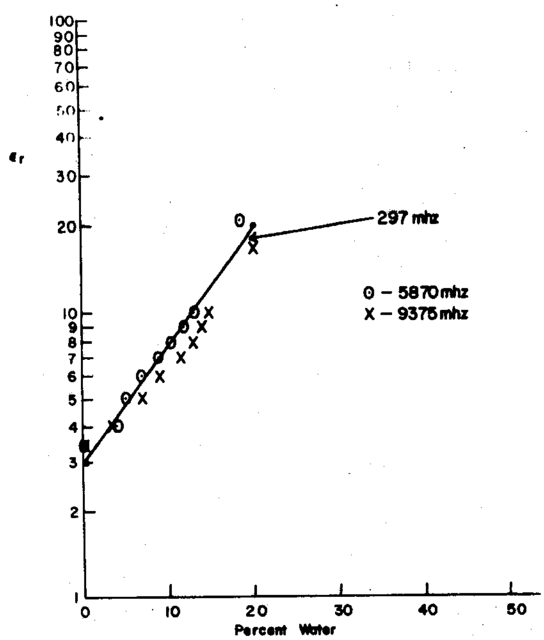

Figure 3-1. ϵ_r vs. the percentage of water content at three different frequencies for sand.

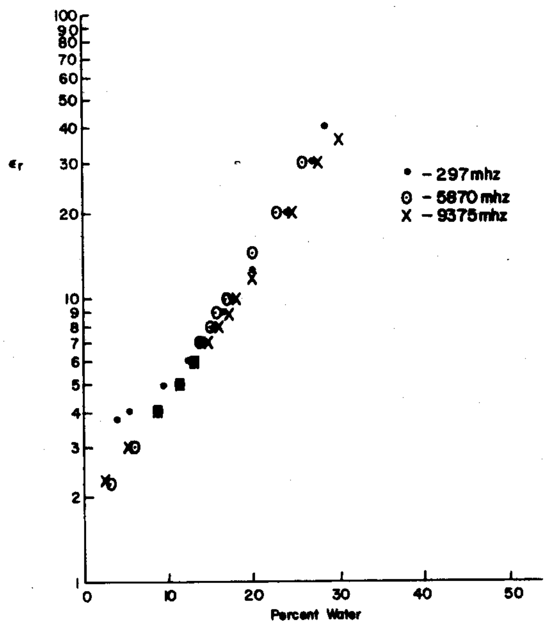

Figure 3-2. ϵ_r vs. percentages of water content at three different frequencies for silt.

The curves of Figure 3-4 (again abstracted from the report of Davis, et al.) show, however, that the relationship between the ε_r and the percentage of moisture content of the soil depends very strongly upon the type of soil, be it clay, silt, or sand. Now, it is a well-known fact that the composition of any given sample of soil is very rarely pure; that is, a given sample taken from an agricultural region is very rarely pure clay, pure silt, or pure sand, but some mixture of the three basic types. Further, the mixture of the three basic soil types occuring at a given point of the earth's surface will be a function of depth below the surface of the earth, the coarser particles forming a higher percentage of the material near the surface than at somewhat greater depths. These complicating factors must be kept in mind in the development of any satisfactory method for measuring the available soil moisture as a function of depth below the surface of the earth.

The curves of Figures 3-1. 3-2, and 3-3 indicate quite a scattering of points, especially at the higher frequencies of operation (5.87 ghz and 9.375 ghz) with considerably less scatter at the lower frequency of 297 mhz. A study of the Davis report indicates that the scatter is due to the method of measurement rather than being inherent in the relation between permittivity and percentage of water.

It seems to be considerably more difficult to measure the electrical permittivity (ε) of the soil, instead of the electrical conductivity (σ), <u>in situ</u>, but several methods seem to have been used successfully to measure ε, and these will be discussed later in this report. At least one, and possibly two, of these methods seems to be adaptable to our use as a ground instrument, and one of these may lead to the development of suitable airborne equipment.

Bayer (1956, p. 292) makes the statement, "It has been shown by Shaw and Baver (1939b) that heat conductivity in soils can be used as an index of soil moisture. Use is made of the principle of the increase in resistance of a wire conductor with increase in temperature to measure the changes in heat conductivity of the soil-water system ... The ability of the soil to conduct heat away from the element determines the temperature rise. Since the heat conductivity of the soil varies with the moisture content, the reading on the microammeter is a measure of soil moisture.

"Readings with this apparatus are not affected by the presence of electrolytes or changes in soil temperature. Readings can be made over the entire moisture range from saturation to the air-dried state. Results obtained by this technique for several widely different soils are shown in Figure 3-5," (Shaw and Baver, 1939a).

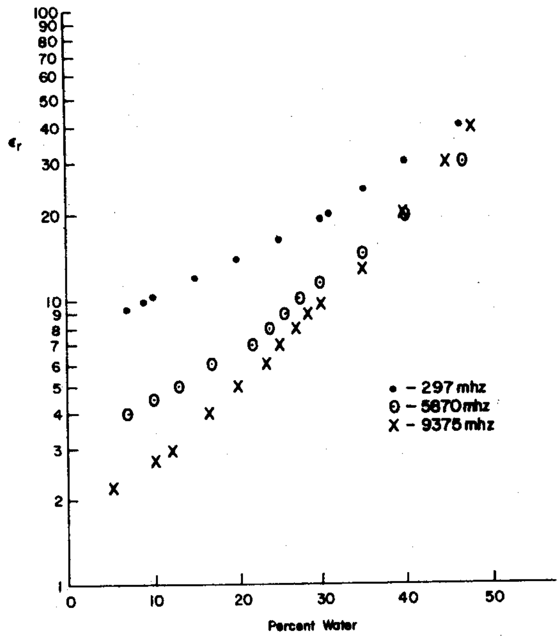

Figure 3-3. ε_r vs. percentage of water content at three different frequencies for clay.

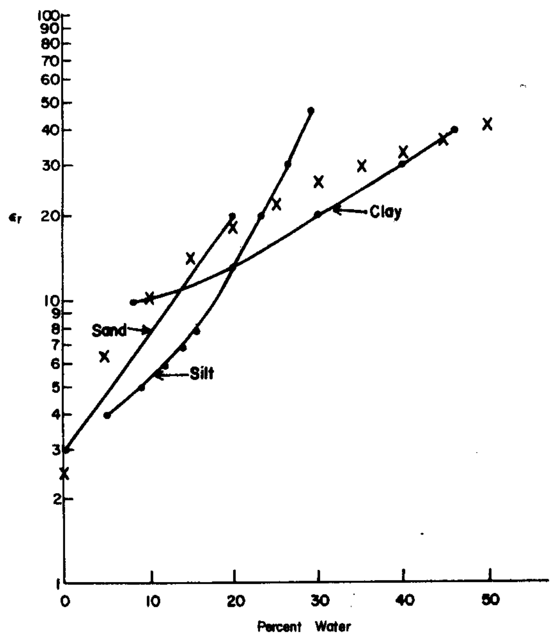

Figure 3-4. ε_r vs. the percentage of water content (f=297 mhz) (from Davis, et al., 1966). Values of x given by ε=0.78w + 2.5, where w is the percentage of water present. (Josephson and Blomquist, 1958.)

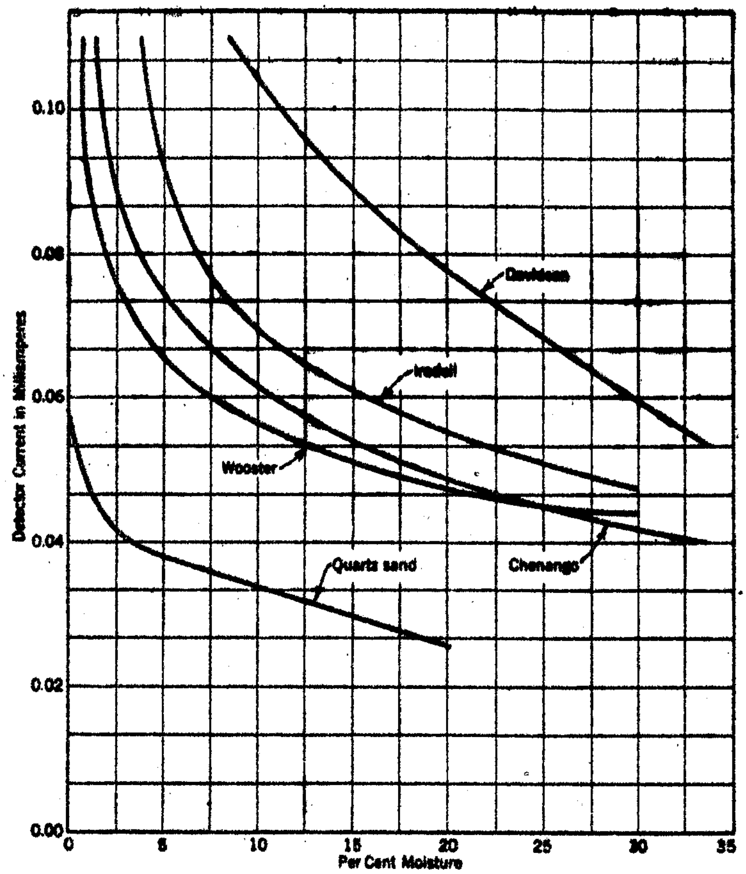

Figure 3-5. The relation of heat conductivity to the moisture content of soils. (A high detector current signifies a low heat conductivity.) (Courtesy Baver, 1956).

Gardner (1965), however, makes the comment, "Thermal conductivity and electrical capacitance measurements in porous blocks, although favorably reported on from time to time in the literature, have not come into general use (see Fletcher, 1939; Shaw and Baver, 1940; Anderson and Edlefsen, 1942; de Plater, 1955 and Bloodworth and Page, 1957). This probably results from the fact that electrical conductivity is so easily measured and porous blocks for such use are so easily constructed..."

Because of this statement, and the fact that the thermal conductivity method seems to lend itself best to situations involving repeated measurements on buried equipment, no further attention is planned, for the present, to the method.

Gardner (1965, pp. 99-104) gives rather detailed consideration to the measurement of the resistivity of buried porous blocks of various materials, as a means for measuring soil moisture. Again, this method seems to possess the same undersirable characteristics, with regard to the present application, as does the thermal conductivity method so it, too, will be given no further consideration at present.

4. Galvanic Resistivity Measurement Method

This is a method of measuring the resistivity of a somewhat extended mass of material, and it has been much used in measuring the resistivity of sections of earth. An excellent discussion of the basic theory of the method and its applications is given in Chapter 3 of Keller and Frischknecht (1966).

The method consists basically of injecting a current I into the earth (or other material) at two points (A and B of Figure 4-1) and measuring the voltage (ΔV) between two other points, M and N. The basic theory of the method is set forth in Appendix 4A of this section. It is shown there that if the conductivity of the earth is isotropic and homogeneous, and if the interface between the earth and the atmosphere is planar, the resistivity of the earth is given by:

$$\rho = K \frac{\Delta V}{I} = \left(\frac{\Delta V}{I}\right) \left(\frac{2\Pi}{\frac{1}{\overline{AM}} - \frac{1}{\overline{BM}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{BN}}} \right) . \qquad (4\text{-}1)$$

Here K is the geometrical factor shown in parentheses, $\overline{AM}$ is the distance from A to M, and so forth. If the earth is not uniform, (4-1) may be used as a definition for the "apparent resistivity" of the earth.

Several different values are used for the distances, $\overline{AM}$, etc. and the resulting configurations are given different names, and they have somewhat different characteristics.

The Wenner array consists of four electrodes arranged in a straight line with $\overline{AM} = \overline{MN} = \overline{NB} = a$, with the result that

$$K \equiv K_W = 2\Pi a \quad . \qquad (4\text{-}2)$$

The Lee modification of the Wenner array has a third potential electrode (0), midway between M and N. Potential differences then are measured between M and 0 and between N and 0. The geometrical factor for one half of the Lee array is

$$K_L = 4\Pi a \quad . \qquad (4\text{-}3)$$

The two values of ρ, resulting from using the two different voltage readings, M to 0 and 0 to N, are obtained without moving the current probes. These two different readings for ρ give an indication as to whether or not there are horizontal variations in ρ.

In the Schlumberger array, M and N have a spacing of b; A and B have a spacing of 2a, with $b << a$. The electrodes are placed symmetrically about the center point of the array. For this array

$$K_S = \Pi(a^2/b - b/4) \quad . \qquad (4\text{-}4)$$

In the polar dipole system the electrodes are arranged as shown in Figure 4-2. If $b = c$

$$K_P = \Pi(a^3/b^2 - a) \quad . \qquad (4\text{-}5)$$

Keller and Frischknecht (on p. 97) give the value of K_P if $b \neq c$.

A variation of the polar dipole array has been used, as shown in Figure 4-3. This is discussed in some detail by Keller and Frischknecht. They also discuss arrays in which one of the current electrodes is placed at a relatively great

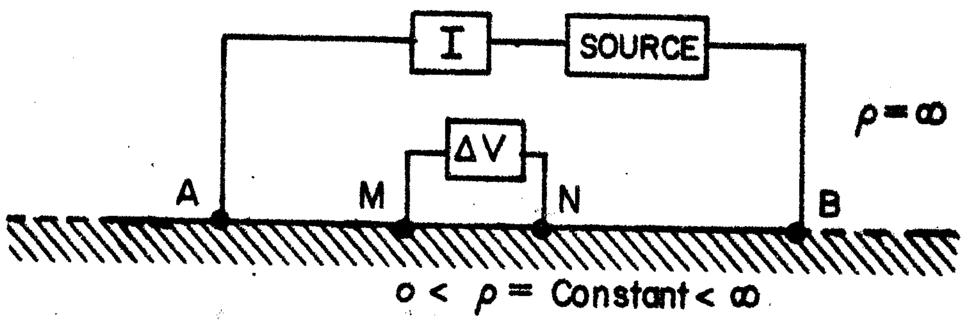

Figure 4.1. Galvanic resistivity method for measuring resistivity (ρ).

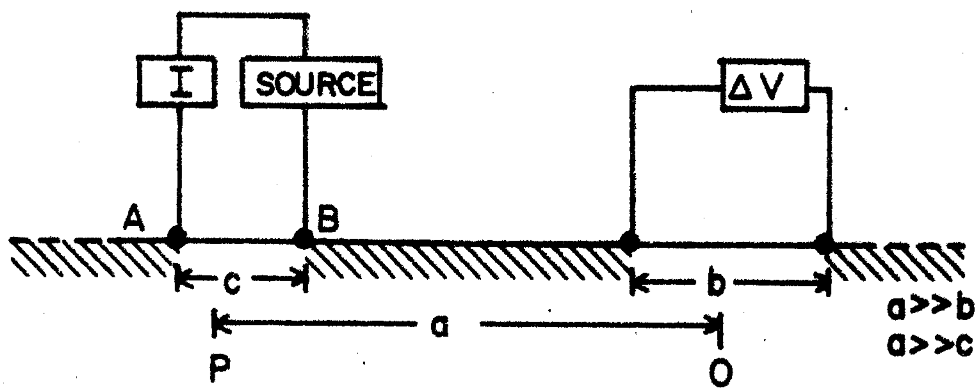

Figure 4.2. Polar dipole system.

distance from the other three electrodes and in which one of the current electrodes and one of the potential electrodes are widely spaced from each other, and both at relatively large distances from the other two electrodes.

In Section 16, Keller and Frischknecht discuss the problems and considerations of instrumenting this method of measuring the resistivity of the earth.

In Section 18, Keller and Frischknecht discuss the application of the method to a medium, the properties of which are anisotropic. That is, ρ has one value, ρ_ℓ, when measured horizontally, and another value, ρ_t, when measured vertically. Neither value of ρ is considered to be a function of depth, however, and the work seems to have no applicability to the present problem.

Keller and Frischknecht limit their concern to impressed currents which are either DC or AC with a maximum frequency of a few dozen hertz. Wait and Conda (1958), on the other hand, are interested in measurements made at frequencies of the order of 15 khz. Consequently, they are concerned with displacement currents and coupling between the current and potential circuits. Their work, therefore, is confined to consideration of the Wenner array and the so-called <u>right angle</u> array, shown in Figure 4-4. The latter is especially effective in reducing undesired coupling between the current and potential circuits. In the present work there appears to be no reason for using frequencies greater than a few dozen herts, so the modifications of Wait and Conda seem to be of no importance to us at present.

Wait and Conda do consider the problem of a two-layered earth, as illustrated in Figure 4-5, but they consider both ρ_1 and ρ_2 to be anisotropic, which is a complication apparently not needed in the problem under investigation. Keller and Frischknecht also consider the two-layered earth but with isotropic resistivities, and it is their analysis which is discussed in the following section.

4.1 <u>Resistivity Measurements of a Two-Layered Earth</u>

Keller and Frischknecht treat this problem in their Section 19, and they refer to it as "The Single Overburden Problem." They set up the problem as shown in Figure 4-6, where $\rho_0 = \infty$, $\rho_1 \neq \rho_2$, A is the location of a point current source of strength I, and M is the point at which it is desired to measure the potential, Ψ_M. By using the methods of geometrical optics, which form an accurate analog for the present electrical problem, they show that

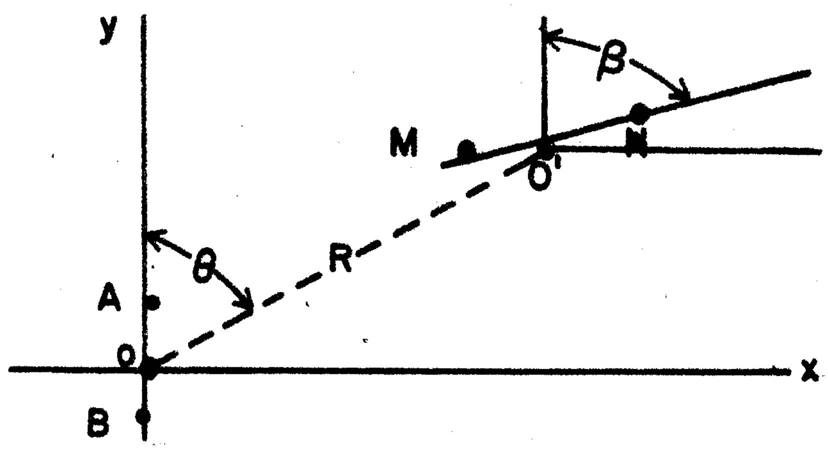

Figure 4-3. A variation of the polar dipole array.

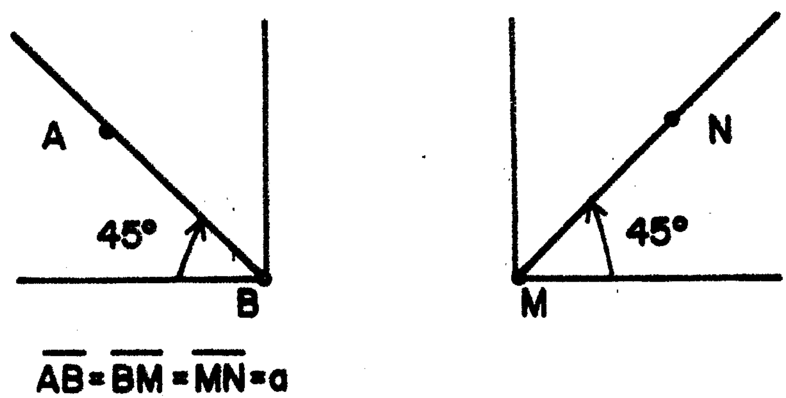

Figure 4-4. The right angle array.

$$\Psi_M = \frac{\rho_1 I}{2\Pi a} \left[1 + 2 \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (2nt/a)^2]^{1/2}} \right] , \tag{4-6}$$

if A and M are on the surface between the ρ_0 and ρ_1 regions, and with $K_{12} = (\rho_2 - \rho_1) / (\rho_2 + \rho_1)$. Keller and Frischknecht state, without proof, that the series in (4-6) converges if K_{12} is "not large."

It then is straightforward to determine the apparent resistivity for the Wenner array:

$$\rho_{a,w} = 2\Pi a \frac{\Delta V}{I} = \rho_1 \left[1 + \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (2nt/a)^2]^{1/2}} - 2 \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (nt/a)^2]^{1/2}} \right] . \tag{4-7}$$

Similarly, for the Schlumberger and polar dipole arrays, it can be shown that the apparent resistivities are, respectively,

$$\rho_{a,s} = \rho_1 \left[1 + 2 \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (2nt/a)^2]^{3/2}} \right] \tag{4-8}$$

and

$$\rho_{a,d} = \rho_1 \left[1 - \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (2nt/a)^2]^{3/2}} + 3 \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (2nt/a)^2]^{5/2}} \right] \tag{4-9}$$

For each of the values given by (4-7), (4-8), and (4-9), with the overburden thickness, t, very large compared to the potential electrode spacing, a, it is easily shown that

$$\rho_{a,w} \cong \rho_{a,s} \cong \rho_{a,d} \cong \rho_1 \quad . \tag{4-10}$$

This is very useful information, in case one is interested in measuring only the overburden resistivity, ρ. It also is a very logical result, based on physical reasoning.

A similarly logical result also is obtained from (4-8), (4-9), and (4-10) if a >> t. Then

$$\rho_{a,w} \cong \rho_{a,s} \cong \rho_{a,d} \cong \rho_2 \quad . \tag{4-11}$$

In case a/t or t/a cannot be considered to be vanishingly small, Keller and Frischknecht have prepared the curves of Figure (4-7), for the Wenner array, which are self-explanatory. Obviously, similar curves could be plotted for the Schlumberger and polar dipole arrays. Keller and Frischknecht point out that curves of the type given in Figure (4-7) differ most between the three arrays if the substratum (ρ_2) is either highly resistive or highly conductive (Figures (4-8) and (4-9)). These results do not appear to be important in the present application and are not discussed further here.

4.1.1 Logarithmic Curve Matching

All of the computed apparent resistivity curves which have been shown in the illustrations have been plotted on a logarithmic coordinate system. An advantage of logarithmic curve plotting is that it permits a wide range of values for the variables to be presented on a single graph, though this is not the primary reason here. Consider (4-8), which was developed for the apparent resistivity which would be measured over a single overburden:

$$\rho_{a,s} = \rho_1 \left[1 + 2 \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + (2nt/a)^2]^{3/2}} \right] \quad . \tag{4 12}$$

This equation may be rewritten so that all the variables occur as dimensionless ratios: a/t and ρ_a/ρ_1 :

$$\frac{\rho_{a,s}}{\rho_1} = 1 + 2 \sum_{n=1}^{\infty} \frac{K_{1,2}^n}{[1 + 4n^2(\frac{t}{a})^2]^{3/2}} \quad . \tag{4-13}$$

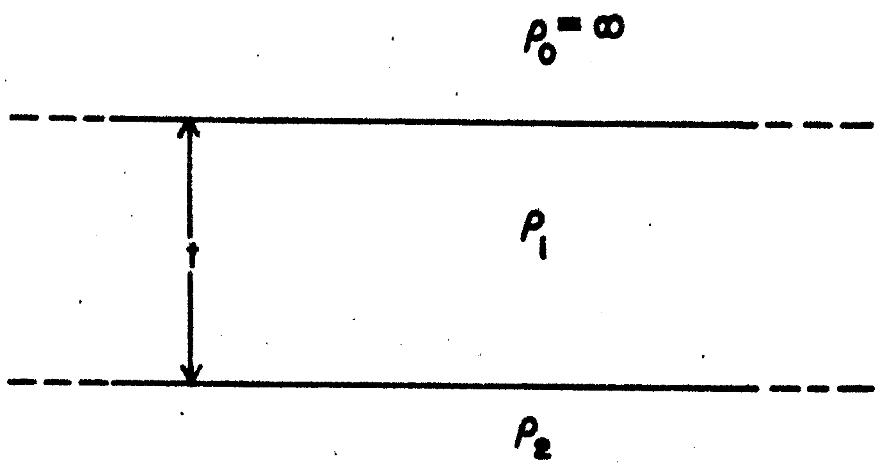

Figure 4-5. Two-layered earth.

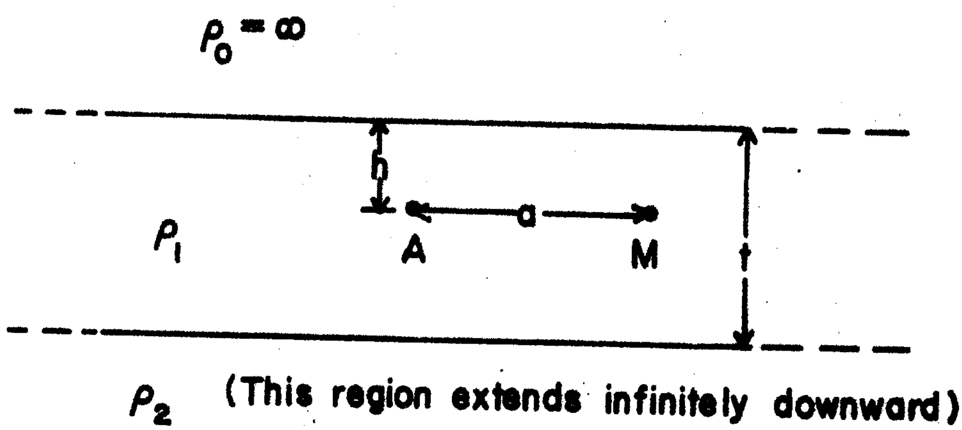

Figure 4-6. The single overburden problem.

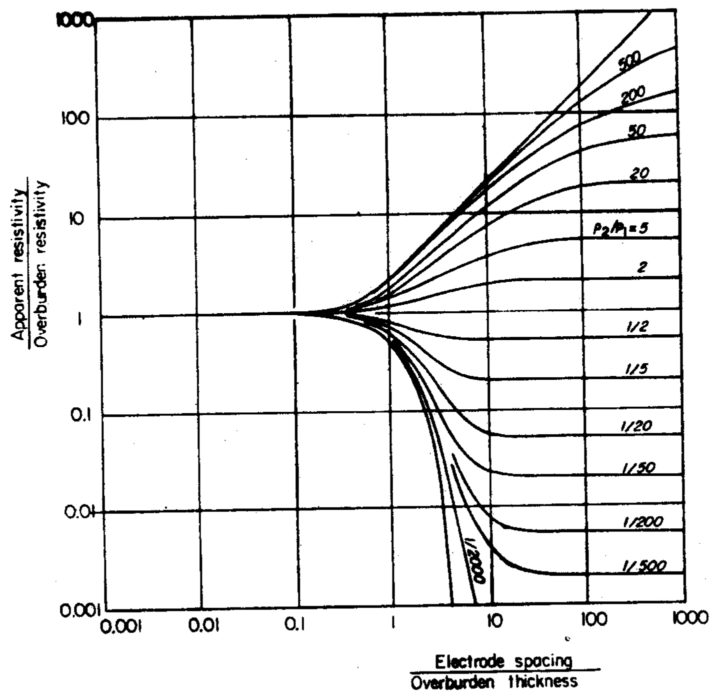

Figure 4-7. Relation between the apparent resistivity which would be measured with a Wenner Array over a single overburden. (Courtesy Keller and Frischknecht, 1966)

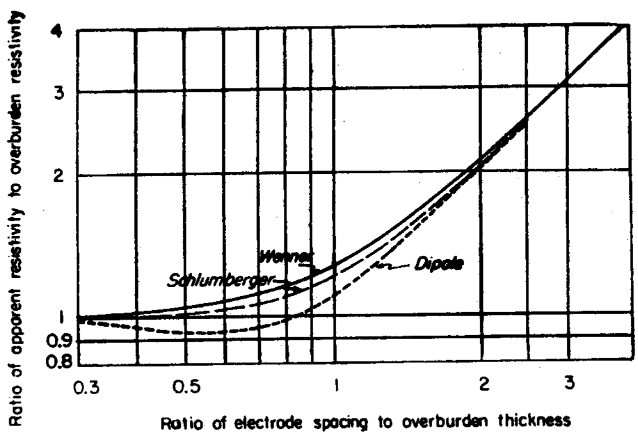

Figure 4-8. Comparison of resistivity sounding curves which would be at the surface of an overburden which rests upon an insulating substratum. The spacing factor for the Wenner Array has been multiplied by 1·38 so the 45° asymptote will pass through the point 1·1. The spacing factor for the polar dipole array has been multiplied by 1/2 so that its asymptote will pass through the same point. (Courtesy Keller and Frischknecht, 1966)

The equations for the apparent resistivity measured with any of the electrode arrays can be expressed in terms of these same dimensionless ratios. This means that the theoretical curves computed with these equations can be plotted without regard to the system of units used, so long as they are consistent, and without regard to the absolute magnitudes of any of the resistivities, electrode spacings, or bed dimensions. In practice, it is most convenient to measure the apparent resistivity in terms of the resistivity of the overburden ρ_a/ρ_1 and the spacing in terms of the thickness of the overburden, a/t.

In normal field surveys, the proper values for ρ_1 and t are not known, usually being the object of the survey. Thus, dimensionless ratios expressed in terms of the overburden resistivity and thickness cannot be used in plotting field data. The coordinates for a point plotted on an apparent resistivity curve should be (a/t, ρ_a/ρ_1). In a logarithmic coordinate system, the coordinates of the same point would be (log a - log t, log ρ_a - log ρ_1). If a series of such points are to be plotted in logarithmic coordinates, the quantities log t and log ρ_1 are the same for all points. If values of 1 are arbitrarily assigned to the parameters t and ρ_1, each of the points along the apparent resistivity curve will be shifted a constant distance (log t) horizontally and a constant distance (log ρ_1) vertically. The shape of a curve plotted in logarithmic coordinates is preserved, even when the ordinate and abscissa of each point along the curve are multiplied by arbitrary constants. The preservation of curve shape in logarithmic coordinates is the basis for the curve-matching method of interpretation.

A field curve, which is a plot of apparent resistivity as a function of electrode spacing obtained in a field survey, will have the same shape as a curve computed from theoretically derived expressions provided both are plotted to the same logarithmic scales. These field curves may be compared directly with a set of theoretical curves by superposition.

In superposition, field data are plotted on a sheet of logarithmic graph paper which has exactly the same scales as the graph paper on which a set of theoretical curves have been plotted. The sheet with the field data (the field curve) is laid over the sheet with the theoretical curves and is moved around until the points on the field curve correspond, or match with one of the theoretical curves. The only restriction in moving the field curve around is that the coordinate axes of both sets of curves must be kept parallel

The technique for curve matching is illustrated in Figure 4-10, using the data listed in Table 4-1.

These data have been plotted on logarithmic coordinates on Figure 4-10. A set of dashed two-layer theoretical curves has been superimposed on top of the field data, and moved around until a match was found between the field data and the theoretical curves. The origin of coordinates for the dashed theoretical curves ($a/t = 1$, $\rho_a/\rho_1 = 1$) is known as the theoretical cross. This point corresponds to a point on the field plot with corresponding values for ρ_a and a. In the example, these two points establish the conditions:

$a/t = 1$ on the dashed theoretical curves when a is 70 m on the field curve, and

$\rho_a/\rho_1 = 1$ on the theoretical curves when ρ_a is 0.72 ohm-m on the field curve.

These two conditions constitute a set of two equations with two unknowns t and ρ_1. Solving these equations:

$$t = a = 70 \text{ m}$$

$$\rho_1 = \rho_a = 0.72 \text{ ohm-m.}$$

4.1.2 Critique of Section 4.1.1

This procedure appears to be much too complicated for a rapid collection of data in the field. It does seem, however, that it could be used rather simply to obtain the resistivity, ρ_1 of the overburden. If it is assumed that the overburden has a thickness of four inches, and if a set of values of electrode spacing, a, of perhaps one inch were used, the curves of Figure 4-7 reveal that the ratio of ρ_a,w to ρ_1 would be essentially unity and the value of ρ_a,w read would be essentially equal to ρ_1, especially if the ratio of ρ_2 to ρ_1 differed by not too large an amount from unity as is known to be the case, in general. If there were some doubt as to the validity of this last assumption, two different values of (a) might be used (not too difficult to achieve, mechanically, it would seem) of perhaps a = 1 inch and a = 0.5 inch, and the resulting values of ρ_a,w compared. If the two measured values of ρ_a,w differed by no more than a few percent, the

Table 4-1. Field data obtained from a Schlumberger sounding at Searles Lake, California (Keller and Frischknecht, 1966).

Spacing (ft.)	Apparent Resistivity
15	1.23
25	0.85
40	0.73
50	0.75
60	0.74
80	0.79
100	0.86
150	1.02
200	1.12
250	1.18
300	1.33
400	1.53
500	1.70
600	1.88
800	2.50
1000	2.85
1200	3.46
1400	3.95
1600	4.80
2000	5.80

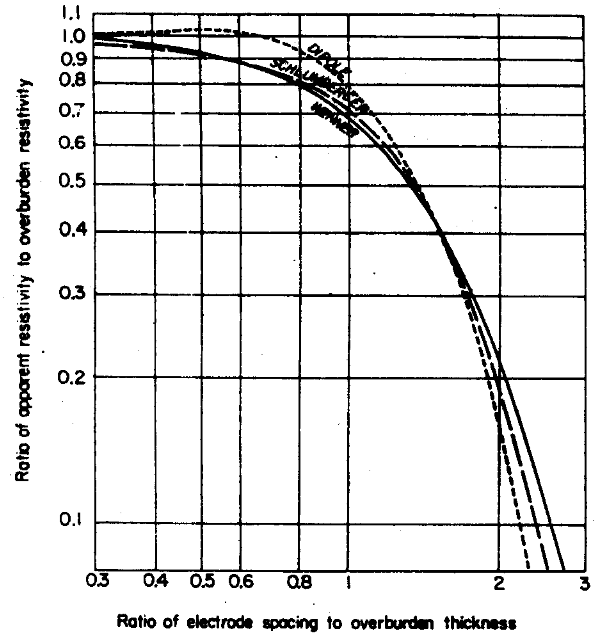

Figure 4-9. Comparison of resistivity sounding curves which would be obtained at the surface of an overburden which rests upon a perfectly conducting substratum. The spacing factor for the polar dipole curve has been multiplied by the factor 1/2. (Courtesy Keller and Frischknecht, 1966)

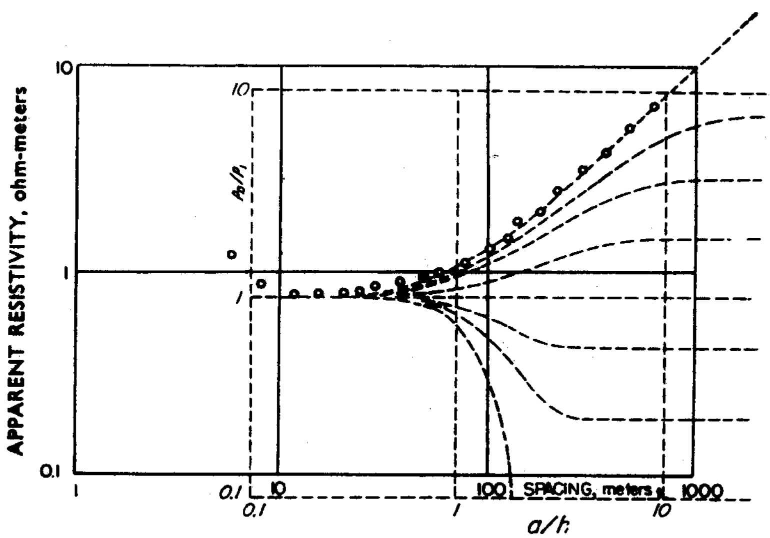

Figure 4-10. Example of the interpretation of a field curve by superposition with a set of two-layer (single overburden) resistivity curves. (Courtesy Keller and Frischknecht,1966)

value of $\rho_{a,W}$ measured by using the smaller of the two values of a, could be assumed to be very close to the true value of ρ_1. All of this reasoning would, of course, need to be verified by carefully conducted experimental work.

When one considers the problem of measuring the resistivity at depths of 12, 18, and 48 inches, one does not seem to be able to use the theory developed by Keller and Frischknecht for the two-layered earth. An examination of Figure 4-7 reveals that for $a>t$, as is necessary for measuring ρ_1, the procedure discussed above under "Logarithmic Curve Matching" would need to be followed, and this appears to be a too complicated and time-consuming process for use in making rapid field measurements by semi-skilled operators. Also, this particular method is valid for only a two-layered earth. Keller and Frischknecht (pp. 135-178) do discuss "Interpretation of Resistivity Soundings When There Are More Than Two Horizontal Boundaries," but this procedure seems to be completely unreasonable for rapid field work.

It would appear that a more practical method for measuring the resistivity at different soil depths would be to use an apparatus of a type suggested by the sketch of Figure 4-11. In practice, the framework F would be placed horizontally upon the earth -- probably it would be desirable to use a leveling device to ensure that F would be truly horizontal -- and then each of the rods would be driven into the earth to the desired depth. These depths would be 14 inches to measure the available soil moisture content in the band from 12 to 16 inches, and 42 inches to measure the moisture content in the band from 36 to 48 inches. The two outer rods (A and B) would be used as the current probes and the two inner rods (M and N) as the potential probes. Then, if the total length, 3a, of the array is but a small fraction of the depth of the uninsulated tips of the rods (A, B, M, N) below the surface of the earth, the apparatus should function as a Wenner array immersed in a homogeneous, isotropic conducting medium and the theory developed in the first part of Appendix 4A should apply, with the result that

$$\rho = 4\pi a \left(\frac{\Delta V}{I}\right) \quad . \tag{4-14}$$

There are many practical considerations to be taken into account in the development of the device described above, and whether or not such a device can be developed into a useful tool seems to depend upon whether or not the problems

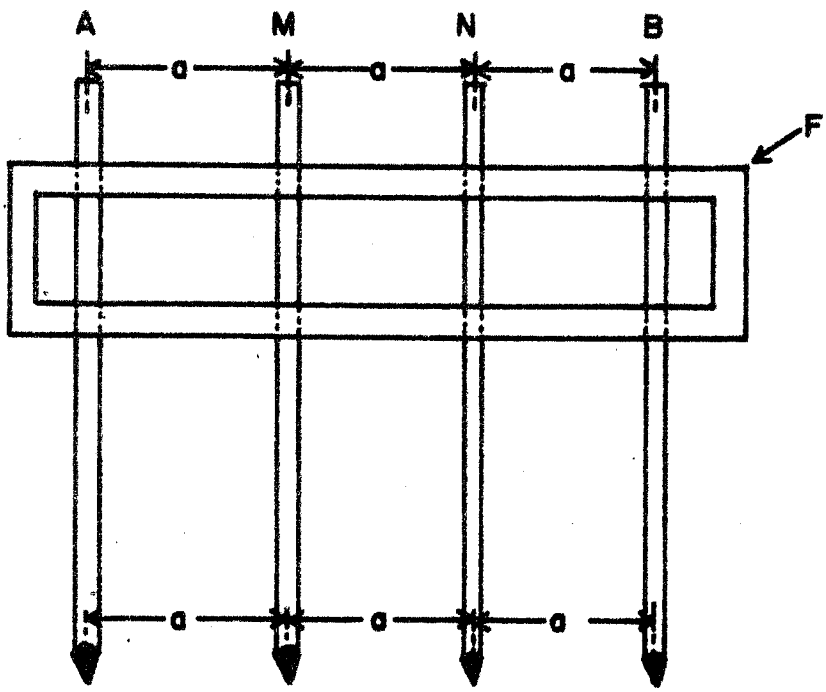

Figure 4-11. Extended Wenner Array. A,B,M,N are slender stainless steel rods of perhaps 3/16" or 1/4" diameter, each covered with a thin coating of a tough, abrasive-resistant insulating material. The spacing between these four rods is held at "a" inches by the framework F and the rigidity of the rods. The rods are free to move (under axial force) through the openings in the framework F. The bottom tips of the four rods are sharply pointed and free of insulation.

associated with these practical considerations can be solved. In the first place, the spacings between the probe tips must be held to the, as yet unknown, spacing, a, within an unknown specified tolerance. The spacing, a, must be small enough so that the currents from the probes, A and B, are confined to a volume small enough so that the calculated value of ρ would be the average value within a region of not more than a few inches across.

In driving the probes into the earth, it will be necessary that the spacing of their tips remain at the distance "a" within some, as yet unknown, tolerance. This will require that the probe rods must be of great enough diameter to prevent bending during insertion, and yet small enough to leave the natural condition of the soil undisturbed.

In order to avoid polarization effects, it will be necessary to use AC current rather than DC, as the current inserted into the current probes A and B. The frequency of this current must be low enough so that the capacitive current flowing through the insulated sides of the probes A and B will be inappreciable when compared with the conductive current flowing through the uninsulated lower tips of the probes A and B. Also, the magnitude of the current inserted into the earth must be great enough to guarantee that the voltage readings between the potential terminals, M and N, will be large enough to be reliable. This current must not be so great, however, as to cause any appreciable changes in the properties of the soil, the resistivity of which is being measured, nor so great as to require the hand-carried power supply to be unduly heavy and large.

It is believed that the answers to the above-discussed problems can be found only through an experimental program.

Appendix 4A Basic Theory of the Galvanic Resistivity Measurement Method.

Consider a DC point current source embedded in an unbounded, homogeneous, isotropically conducting medium. A radially directed current of density

$$\vec{J} = \hat{r} J_r \ , \tag{4A-1}$$

will flow, and a radially directed electric field intensity, of

$$\vec{E} = \hat{r} E_r \tag{4A-2}$$

will exist. Both E_r and J_r will be functions of the spherical coordinate r (centered at the current source), but independent of the spherical coordinates Θ and φ.

$\vec{E}$ and $\vec{J}$ must satisfy the equations

$$\vec{E} = \rho\vec{J} \tag{4A-3}$$

and

$$\nabla \cdot \vec{J} = 0 , \tag{4A-4}$$

where ρ is the resistivity of the medium. If a potential function, Ψ, is defined as usual, then

$$\vec{E} = -\nabla\Psi \tag{4A-5}$$

and

$$\nabla^2\Psi = 0 \tag{4A-6}$$

In polar coordinates, Ψ does not depend upon Θ and φ, and (4A-6) becomes

$$\frac{d}{dr} \left(r^2 \frac{d\Psi}{dr}\right) = 0 . \tag{4A-7}$$

This may be integrated twice to give

$$\Psi (r) = \frac{-C}{r} + D , \tag{4A-8}$$

where C and D are undetermined constants. Ψ must go to zero as $r \to \infty$, so D = 0. In order to evaluate C, the following procedure is used. The total current, I, flowing from the source, is

$$I = \oint_s \vec{J} \cdot \vec{ds} = \frac{1}{\rho} \oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\rho} \oint_s \left(- \frac{C}{r^2}\right) ds$$

$$I = - \frac{C}{\rho r^2} \oint_s ds = \left(- \frac{C}{\rho r^2}\right) (4\Pi r^2) = - \frac{4\Pi C}{\rho} . \tag{4A-9}$$

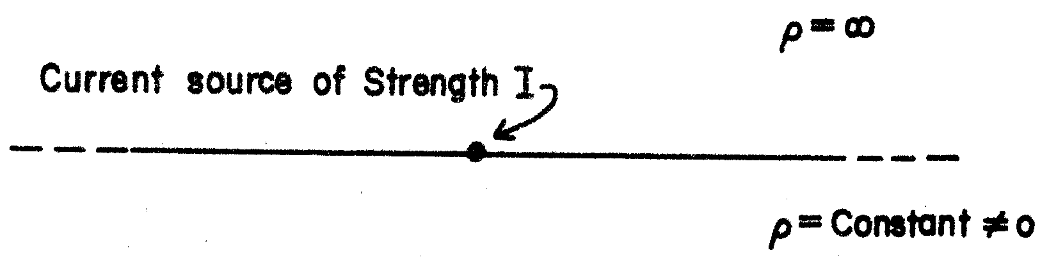

Figure 4A-1. Current source at an interface.

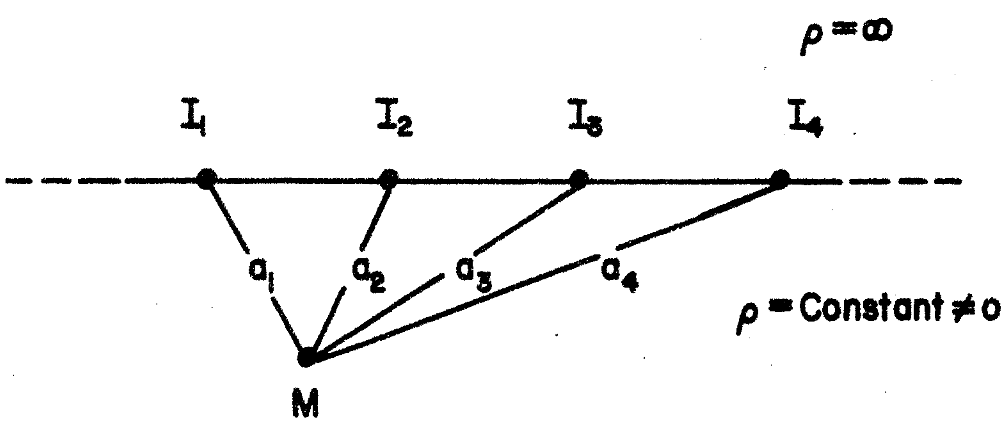

Figure 4A-2. Several current sources at an interface.

Consequently,

$$C = \frac{-\rho I}{4\Pi} , \qquad (4A-10)$$

and from (4a-8),

$$\Psi = \frac{\rho I}{4\Pi r} . \qquad (4A-11)$$

As noted at the beginning of this appendix, (4A-11) has been derived for an unbounded medium. Now, if the situation is as shown in Figure 4A-1, the surface through which current flows in (4A-9) is $2\Pi r^2$ rather than $4\Pi r^2$, and (4A-11) becomes

$$\Psi = \frac{\rho I}{2\Pi r} . \qquad (4A-12)$$

Now, if we have a situation as shown in Figure 4A-2, with point current sources on the surface, I, I_2, I_3, and I_4, the potential at M is

$$\Psi_M = \frac{\rho}{2\Pi} \left(\frac{I_1}{a_1} + \cdots + \frac{I_4}{a_4} \right) . \qquad (4A-13)$$

Consequently, for the array of Figure 4-1, with a current source (I) at A and a current sink (-I) at B,

$$\Psi_M = \frac{\rho}{2\Pi} \left(\frac{I}{\overline{AM}} - \frac{I}{\overline{BM}} \right) . \qquad (4A-14)$$

$$\Psi_N = \frac{\rho}{2\Pi} \left(\frac{I}{\overline{AN}} - \frac{I}{\overline{BN}} \right) . \qquad (4A-15)$$

Then,

$$\Delta V = \Psi_M - \Psi_N = \frac{\rho I}{2\Pi} \left(\frac{1}{\overline{AM}} + \frac{1}{\overline{BN}} - \frac{1}{\overline{BM}} - \frac{1}{\overline{AN}} \right) , \qquad (4A- 6)$$

and (4-2), (4-3), (4-4), and (4-5) follow directly.

Keller and Frischknecht (pp. 122 196) discuss many refinements and extensions of the basic method, which has been described herein. Their additional material pertains to situations involving several horizontally stratified homogeneous layers, horizontal inhomogeneities (vertical faults), non-horizontal surfaces of inhomogeneity (dipping beds), etc., which are of importance in making measurements to relatively great depths but do not appear worthy of discussion here, where the depths of interest vary from a very few inches to a very few feet.

5. Use of Open-Wire Transmission Lines and Capacitors to Measure the Electrical Properties of Soil.

Kirkscether (1960) seems to have been the first to use a transmission line terminated in soil to measure the electrical properties of the earth. Since then, staff members of the Stanford Research Institute (SRI) have used the method quite extensively to measure the electrical properties of a forest environment (from above soil level to tree-top level) as well as those of the earth. Only the three reports by SRI listed as references seem to cover the subject in sufficient detail (Parker and Hagn, 1966; Goldstein, Parker, and Hagn, 1967; and Parker and Makarabhiromya, 1967).

The basic theory of the method is very simple, but, for completeness, it will be outlined here, together with a brief discussion of the measurement procedure.

An open two-wire transmission line is inserted vertically into the earth, which is assumed to be a homogeneous isotropic medium, with electrical permittivity, ε, electrical permeability, μ, and electrical conductivity, σ. The transmission line, of course, is made of two conductors with equal diameters, and a definite (and, supposedly successful) effort is made to keep them equally spaced as they are inserted into the earth. The end of the transmission line which is inserted into the earth is, of course, open circuited. The input impedance of the open-circuited transmission line (of length ℓ_1), as measured at the surface of the earth is

$$Z_1 = Z_{oc}(\ell_1) = \frac{Z_o}{\tanh \gamma\ell_1} \tag{5-1}$$

where Z_o is the characteristic impedance of the transmission line, and

$$\gamma = \sqrt{j\omega\mu\,(\sigma + j\omega\varepsilon)} \quad , \tag{5-2}$$

the propagation constant of the TEM wave, on the transmission line, moving through the earth.

Now, if a second transmission line, identical to the first but twice as long ($\ell_2 = 2\ell_1$), replaces the first, we have for its input impedance at the surface of the earth:

$$Z_2 = Z_{oc}(\ell_2) = \frac{Z_o}{\tanh 2\gamma\ell_1} \tag{5-3}$$

Now, from (5-1) and (5-3), one finds that

$$\frac{Z_2}{Z_1} = \frac{\tanh \gamma\ell_1}{\tanh 2\gamma\ell_1} = \frac{1}{2}\,(1 + \tanh^2 \gamma\ell_1)\ . \tag{5-4}$$

This leads to

$$\tanh \gamma\ell_1 = \sqrt{\frac{2Z_2 - Z_1}{Z_1}} \tag{5-5}$$

and

$$Z_o = Z_1 \tanh \gamma\ell_1 = \sqrt{Z_1(2Z_2 - Z_1)}\ . \tag{5-6}$$

Thus, by measuring Z_1 and Z_2, one can calculate Z_o and γ. In general, both Z_o and γ turn out to be complex. Let us write

$$\gamma = \alpha + j\beta\ . \tag{5-7}$$

Then, as shown by Parker and Hagn (1966, p. 11),

$$\varepsilon_r = \left(\frac{c}{\omega}\right)^2 (\beta^2 - \alpha^2)\ , \tag{5-8}$$

$$\sigma = 2\alpha\beta\,\frac{c^2\,\varepsilon o}{\omega}\ , \tag{5-9}$$

where c is the velocity of electromagnetic waves in free space (3×10^8 meters $\cdot$ sec^{-1}); ε_r is the relative permittivity, or dielectric constant, K, of the earth; and $\omega = 2\Pi f$, where f is the frequency of the electromagnetic wave used.

As mentioned at the beginning of this section, the theory of the measurement is based upon the assumption that the earth surrounding the transmission line is homogeneous. If this is not true, one measures some sort of average of the earth's properties. Consequently, if the properties of the earth are known to vary with depth, one must use discretion in his choice of values of ℓ_1 and ℓ_2 in order to insure that the values of ϵ_r and σ which he measures are satisfactorily close to the actual values which exist close to the surface. Obviously, the, this method is limited to making measurements which are an average of the values of ϵ_r and σ within a very few inches of the surface of the earth. To make measurements at greater depths it might be possible to develop a modification of the method, which, to the writer's knowledge, has not yet been done. One could, of course, dig a pit in the earth and make measurements in the sides of the pit at various depths below the earth's surface but, obviously, this is out of the question for rapid measurements in a tilled area.

A second question which arises has to do with the extent to which the electromagnetic field extends in a horizontal direction beyond the conductors comprising the transmission line. This question has been investigated by Parker and Hagn (1966, pp. 1-4) and the results are exhibited in Figure 5-1. This figure shows that about 90% of the power flowing along the line is contained within a cylinder having a radius (about the midpoint between the two conductors) equal to the spacing of the two conductors, for a 300-ohm line. Consequently, one can conclude that the horizontal spreading of energy is quite modest.

No work seems to have been done on the extent to which the electromagnetic field extends beyond the open-circuited end of the transmission line. This would seem to depend upon the loss tangent of the soil.

If the two conductors of the transmission line are inserted but a very few inches into the soil, and if the operating frequency is not too high, the two conductors can be considered as the plates of a capacitor, and the theory becomes somewhat simpler. Then (with $|\gamma\ell| << 1$),

$$Y_{oc} = \frac{1}{Z_{oc}} \cong Y\ell = (G + j\omega C)\,\ell\,, \qquad (5\text{-}10)$$

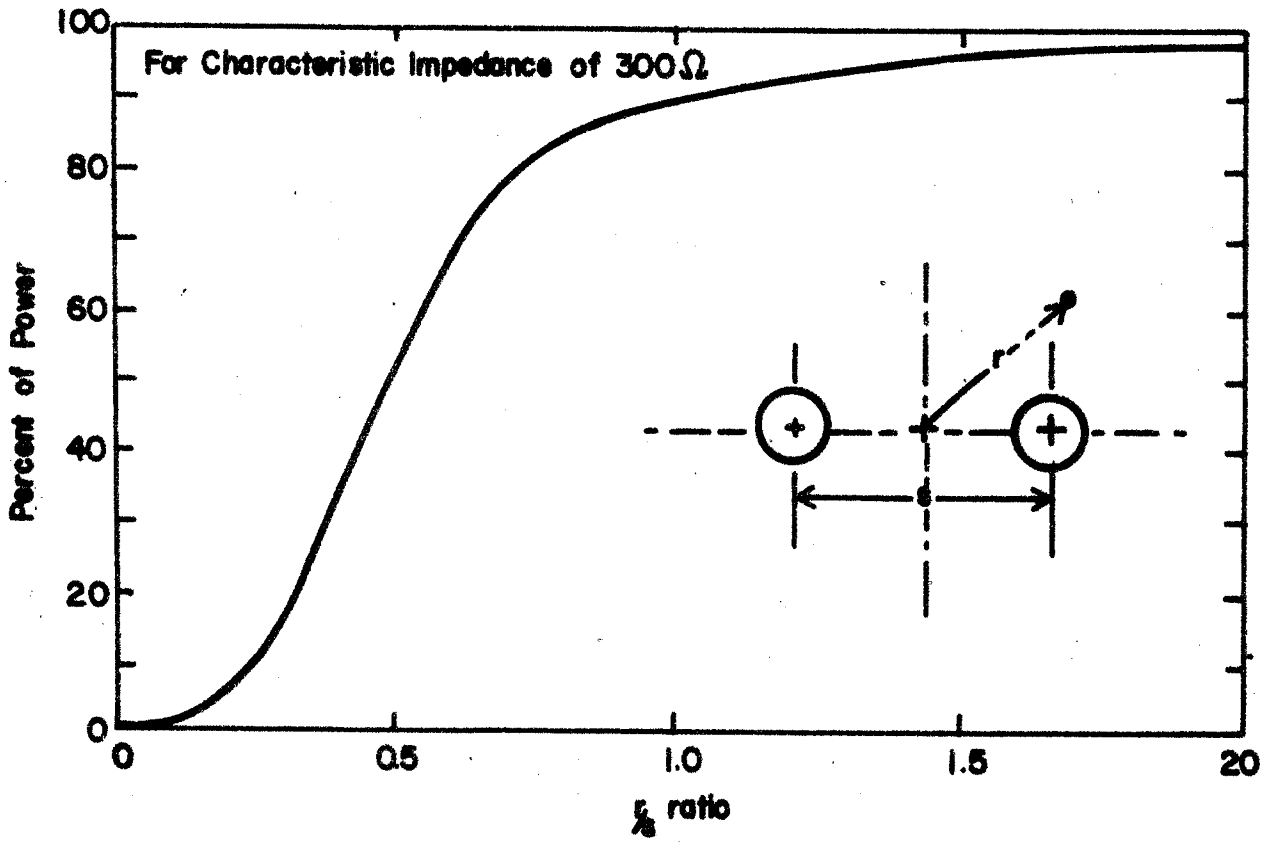

Figure 5-1. Relative power distribution in the vicinity of an open wire line (OWL). (Courtesy Parker and Hagn, 1966.)

where

Y_{oc} = input admittance of the open-circuited transmission line, or capacitor

ℓ = length of transmission line;

Y = admittance, per unit length, of the transmission line;

G = conductance, per unit length, of the transmission line; and

C = capacitance, per unit length, of the transmission line.

With Y_{oc} (a complex quantity), ω, and ℓ of (5-10) measurable, G and C can be calculated.

Next, measure the characteristics of the transmission line in free space, thus obtaining the free-space values (denoted by primes): C' and G'. If the magnetic permeability of the earth can be assumed to be the same as that of free space, as is almost invariably the case, one can calculate the properties of the soil from the equations given by Kirkscether (1960):

$$\sigma = \frac{8.85\ G_{mhos}}{C'_{\mu\mu f}} \qquad (5\text{-}11)$$

and

$$\varepsilon_r = \frac{C}{C'} \; . \qquad (5\text{-}12)$$

Kirkscether shows that, if the experimental arrangement is treated as a transmission line with the two different lengths ℓ_1 and $\ell_2 = 2\ell_1$, then the equations are redundant and one may calculated the μ of the soil from two different equations, which gives one a check on the accuracy of his work.

All four of the references cited early in this section give useful advice on how to best carry out the experiments and avoid difficulties.

6. Use of Magneto-Telluric Fields

Apparently Tikhonov (1950) in the USSR and Kato and Kikuchi (1950) in Japan were the first to point out that the electrical characteristics of the deep strata of the earth's crust could be determined from a combined analysis of geomagnetic and telluric (earth-current) field variations. These variations have frequencies of the order of 10^{-3} to 1.0 Hz. A rather extensive series of investigations of the method have been made, and the work to date seems to be well summarized by Cagniard (1953), Wait (1962), and Wu (1968). Each of these three papers include a long list of references.

The method seems to have a vertical resolution, in determining characteristics of the earth, of the order of about one kilometer, so it is completely in applicable to the present problem.

7. Radiophase Method

This method of mapping earth conductivity (Barringer, 1968) measures the relative magnitudes and phases of the horizontal and vertical components of the electric and magnetic field vectors, near the surface of the earth, produced by a distant commercial radio transmitter emitting 50-1000 kilowatts of C-W power at about 20 khz. From these measurements, made in an aircraft flying at an altitude of a few hundred feet, it is possible to determine the conductivity of the earth. If the earth has two or more strata of differing conductivities, it is possible to use two or more frequencies to interpret the results.

The measurements reported seem to have been rather preliminary, but it appears that the vertical resolution is entirely inadequate for our purposes. Also, the quantity measured is the conductivity of the earth which, as discussed previously herein, is not a valid indicator of available soil moisture content.

8. Use of the Mutual Electromagnetic Coupling of Loops Over the Earth

James R. Wait (1954, 1955) and other referenced by Wait have made a theoretical investigation of the mutual electromagnetic coupling between two wire loops located over the earth. In his 1955 paper, Wait considers the case when both loops are located at some distance above the earth. He obtains equations and curves for the mutual coupling between two small loops, both on a flat homogeneous ground, and also, for the case where both loops are situated at a finite height above the earth-air interface. He neglects the displacement currents in the ground, which seems to invalidate his work insofar as application to our problem is concerned. Furthermore, he states, "The extension of these results to a stratified ground is very difficult in general; however, certain special cases are amenable to analytical treatment..." and then he gives three references to such work.

. No ready application of Wait's work is evident to the present writer, and no later work has been located.

9. Transmission Line Synthesis Approach to Measuring Earth Permittivity (and, thus, Earth Moisture) as a Function of Depth.

It can be shown that, under certain conditions, for a non-uniform transmission line, the characteristics impedance of the line, as a function of length, can be determined by measuring the input impedance of the line as a function of frequency (Sharpe, 1963; Heim and Sharpe, 1967; Berger, 1966; Youla, 1964; Stadamore, 1965; Wohlers, 1966; and Becher and Sharpe, 1969).

Most of the work done to date involves only lossless lines. In order to insert a two-wire transmission line into the earth and measure the dielectric constant of the earth by determining the characteristic impedance of the line as a function of length, it appears that much needs to be done yet with regard to lossy transmission lines. Also, presently available methods require the solution of an involved integral equation, and these solutions appear to require the use of a digital computer. This, of course, is not feasible for field work, so it appears that there are two problems requiring more work: a more complete analysis of the lossy line problem, and a simplification of computational procedures.

If the ground-truth measurement of soil moisture as a function of depth can be made by the use of transmission lines, as outlined above, it may be possible to extend the method to airborne radars, because of the close analogy between the theory of transmission lines and of uniform plane electromagnetic waves.

References

1. Barringer, A. R. and J. D. McNeill (1968), "Radiosphase - A New System of Conductivity Mapping," Proceedings of the Fifth Symposium on Remote Sensing of Environment, Willow Run Laboratories, The University of Michigan. April 16-18, 1968, 157-167.

2. Baver, L. D. (1956), Soil Physics, 3rd Edition, Wiley.

3. Becher, W. D. and C. B. Sharpe (1969), "A Synthesis Approach to Magneto-telluric Exploration," private communication.

4. Berger, Henry, "Generalized Nonuniform Transmission Lines," IEEE Trans. on Circuit Theory, CT-13, No. 1, March, 1966, 92.

5. Cagniard, Louis (1953), "Basic Theory of the Magneto-Telluric Method of Geophysical Prospecting," Geophysics, 18, July, 1953, 605-635.

6. Davis, B. R., J. R. Lundien, and A. N. Williamson, Jr. (1966), "Feasibility Study of the Use of Radar to Detect Surface and Ground Water," Tech. Report No. 3-727, U. S. Army Engineer Waterways Experiment Station, Corps of Engineers, Vicksburg, MS.

7. Gardner, Walter, H. (1965), "Water Content," Chapter 7 in Methods of Soil Analysis Part 1, C. A. Black, Editor-in-Chief, American Society of Agronomy, Inc., Madison, Wisconsin.

8. Goldstein, N. E., H. W. Parker, and G. H. Hagn (1967), "Three Techniques for Measurement of Ground Constants in the Presence of Vegetation," Stanford Research Institute, Special Tech. Report 30, Contract DA-36-039-AMC-00040 (E), March, 1967.

9. Hein, D. A. and C. B. Sharpe (1967), "The Synthesis of Nonuniform Lines of Finite Length - Part I," IEEE Trans. on Circuit Theory, CT-14, No. 4, Dec., 1967, 394-403.

10. Josephson, B. and A. Blomquist, "The Influence of Moisture in the Ground, Temperature, and Terrain on Ground-Wave Propagation in the VHF-Band," (1958). IRE Transactions on Antennas and Propagation, Vol. AP-6, No. 2, April, 1958, 169-172.

11. Keller, G. V. and F. C. Frischknecht, (1966), Electrical Methods in Geophysical Prospecting, Pergamon Press, New York.

12. Kirkscether, Erik J. (1960), "Ground Constant Measurements Using a Section of Balanced Two-Wire Transmission Line," IRE Trans. on Antennas and Propagation, AP-8, No. 3, 307-312.

13. McCorkle, W. H. (1931), "Determination of Soil Moisture by the Method of Multiple Electrodes," Texas Agricultural Experiment Station, Bulletin 426.

14. Nikodem, H. J. (1966), "Effects of Soil Layering on the Use of VHF Radio Waves for Remote Terrain Analysis," Proceedings of the Fourth Symposium on Remote Sensing of Environment, April 12-14, 1966, Institute of Science and Technology, The University of Michigan.

15. Parker, H. W. and G. H. Hagn, (1966), "Feasibility Study on the Use of Open-Wire Transmission Lines, Capacitors, and Cavities to Measure the Electrical Properties of Vegetation," Stanford Research Institute, Special Tech. Report 13, Contract DA-36-039-AMC-00040 (E), August, 1966.

16. Parker, H. W. and W. Makarabhiromya (1967), "Electrical Constants Measured in Vegetation and in Earth at Five Sites in Thailand," Stanford Research Institute, Special Tech. Report 43, Contract DA-36-039-AMC-00040 (E), December, 1967.

17. Sharpe, C. B. (1963), "The Synthesis of Infinite Lines," Quarterly of Applied Mathematics, XXI, No. 2, July, 1963, 105-120.

18. Shaw, Byron and L. D. Baver (1939a), "Heat Conductivity as an Index of Soil Moisture," Journal of the American Society of Agronomists, Vol. 31, 886-891.

19. Shaw, Byron and L. D. Baver (1939b), "An Electrothermal Method for Following Moisture Changes of the Soil, *In Situ*," Proceedings of American Society of Soil Scientists, Vol. 4, pp. 78-83.

20. Stadamore, H. A. (1965), "Analysis and Synthesis of Nonuniform Transmission Lines Incorporating Loss," IEEE Trans. on Circuit Theory, CT-12, No. 2, June, 1965, 285-288.

21. Wait, James R. (1954), "Mutual Coupling of Loops Lying on the Ground," Geophysics, 19, April, 1954, 290-296.

22. Wait, James R. (1955), "Mutual Electromagnetic Coupling of Loops Over a Homogeneous Ground," Geophysics, 20, No. 3, July, 1955, 630-637.

23. Wait, James R. and A. M. Conda (1958), "On the Measurement of Ground Conductivity at VLF," IRE Trans. on Antennas and Propagation, AP-6, 273-277.

24. Wait, James R. (1962), "Theory of Magneto-Telluric Fields," Journal of Research of the NBS-D., Radio Propagation, 66D, No. 5, Sept.-Oct., 1962, 509-541.

25. Wohlers, M. R. (1966), "A Realizability Theory for Smooth Lossless Transmission Lines," IEEE Trans. on Circuit Theory, CT-13, No. 4, Dec., 1966, 356-363.

26. Wu, Francis T. (1968), "The Inverse Problem of Magnetotelluric Sounding," Geophysics, 33, December, 1968, 972-979.

27. Youla, D. C. (1964), "Analysis and Synthesis of Arbitrarily Terminated Lossless Nonuniform Lines," IEEE Trans. on Circuit Theory, CT-11, No. 3, September, 1964, 363-372.

ACKNOWLEDGEMENTS

Figures 65, 66, 67 and 68 and Section 20, pp. 120-122 of Electrical Methods in Geophysical Prospecting, G. V. Keller and F. C. Frischknecht, 1966, Pergamon Press, Inc., have been reprinted as figures 4-7, 4-8, 4-9 and 4-10 and Table 4-1 of this paper, permission pending.

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