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## ATMOSPHERIC EFFECTS ON TIMS ESTIMATED EMITTANCE

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### ABSTRACT

Radiance that reaches the Thermal Imaging Multispectral Scanner (TIMS) consists of attenuated ground radiance, as well as radiance emitted by the intervening atmosphere. The use of LOWTRAN modelled atmospheric spectral parameters in an attempt to compensate for these effects may introduce more errors than it removes. In the warm, humid atmosphere near Hilo, Hawaii, atmospheric effects are extensive. However, where transmission is low, path emittance is high. Thus applying no atmospheric correction theoretically only slightly suppresses and distorts emittance spectra. For TIMS line 94 from Hawaii, LOWTRAN atmospheric corrections resulted in anomalously low emittances in the 9.0 to 10.0 micrometer region in both rocks and vegetation. A black body adjustment discussed in this paper, results in a near uniform calculated emittance for vegetation, and shows an emittance trough at long wavelengths for basaltic lava flows.

### INTRODUCTION

The Thermal Imaging Multispectral Scanner (TIMS) measures radiance in the 8 to 12 micrometer region, which is part of the so-called thermal window (Swain and Davis, 1978). Despite this name, the overall atmospheric transmission in this region may however be as low as 60 to 70%, under warm, humid conditions and long pathlengths. Similarly, the atmosphere in the path between the object viewed, and the sensor, may contribute a path radiance of as much as 30 to 40% of the radiance impinging on the sensor. Because atmospheric corrections applied to remotely sensed data may be large, and differ greatly between different bands, the spectral emittance estimations of ground objects determined from atmospherically corrected data may be altered significantly. If the correction is slightly inappropriate, the data may be corrupted. In this paper we suggest that when radiance is inverted to the more geologically useful emittance, atmospheric corrections be applied with caution, if applied at all. We shall discuss below a simple, alternative atmospheric normalization, which we call a black body adjustment. This approach may be followed where significant absorption is suspected, such as in moist, humid areas.

The radiation from the ground surface, as well as that emitted by the atmosphere, is absorbed as a function of the secant of the view angle, for as the view angle increases, the effective distance also increases (Chandrasekhar, 1960). Furthermore, as the path length increases, the amount of particles in the path radiating energy also increases. Thus to completely remove the atmospheric effects on the data, we need to identify three atmospheric properties for each spectral band: the transmission by the atmosphere of the radiance from the ground, the radiance emitted by the intervening atmosphere, and finally the transmission of that radiance emitted by the atmosphere. For the six TIMS spectral bands there are thus 18 variables that describe the atmospheric effect.

These variables are not, however, unrelated. The emittance by the atmosphere is equal to one minus the transmission, if reflectance is insignificant. This means that as transmission is decreased, there is a somewhat compensating increase in radiation by the atmospheric

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constituents in the path. Bartolucci *et al.* (1988) have pointed out that in certain warm humid atmospheric conditions, the emittance and transmission effects very nearly cancel each other out, and that ignoring atmospheric effects may result in less errors than from other noise sources in the data.

A great deal of research effort has been concentrated on approximations, simplifications and linearizations of the radiance transfer equations in order to make accurate estimates of sea temperatures from aircraft or satellite altitudes (see, for example, Saunders 1967, McMillin 1975, Maul 1983, Holyer 1984, and Callison 1985). These temperature based approaches do not normally attempt to separate the effects of atmospheric transmission from path radiance.

However, in geological analyses, it is emittance of the rocks, rather than their temperatures, that is generally of interest. Thus for emittance determinations, the LOWTRAN model (Kneizys, 1983) has generally been employed to identify all the atmospheric parameters for the data (Kahle *et al.*, 1988). LOWTRAN is a single parameter band model for molecular absorption, and gives integrated emittance and transmission of the atmosphere for selected bandwidths, and view angles.

LOWTRAN has been found to be useful in single band radiant temperature studies, for example those of Wilson and Anderson (1986) and Luvall *et al.* (1990). When applied to the TIMS spectral data, however, problems have been found. For example, in Hawaii, Kahle *et al.* (1988) found it necessary to empirically normalize the LOWTRAN corrected radiances, by assuming vegetation to be a black body radiator. They ascribe the problem to the non-representative nature of the radiosonde data: it was collected some distance away, and possibly not coincident with the aircraft overpass.

In our use of LOWTRAN we found other problems, which we shall describe below. We emphasize, however, that we do not in any way assume that the cause is due to errors in LOWTRAN. Other than non-representative radiosonde data, other possible causes include drifts in the TIMS spectral calibration, and errors in the TIMS reference black body temperature calibrations (Palluconi and Meeks, 1985).

## TIMS DATA PROCESSING

### Raw video to radiance conversion and noise suppression

TIMS data is recorded as 8 bit raw video data. Before and after each line the instrument stares at a cold and hot reference black body, respectively, and response of the TIMS detectors in each channel is recorded. Thus each line of spectral data may be converted from the raw video signal to radiance data by assuming there is a linear relation between the radiance of the hot and cold black bodies (Palluconi and Meeks 1985).

In Figure 1D and 1E the 9.9 micrometer (band 4) hot and cold reference black body DN response is plotted for 500 lines of TIMS data over the Pacific Ocean, near Hilo, Hawaii, from Flight Line 94, collected on September 30, 1988 at 19h31 GMT. Figure 1C shows the average video DN response for the 9.9 micrometer band for the average of the near-nadir, central 21 pixels. The equivalent radiance data (Figure 1B), does not have the long wavelength noise, for example between lines 400 and 500, of the raw DN values. This data however is more spiky, due to the additive effect of short wavelength noise in the raw data and the reference black body data. We therefore use a 21 line moving average on the hot and cold reference data, to filter out high frequency noise from this source. Figure 1A shows the success of this approach.

In Figure 2, we show the average radiance across the plus or minus 38 degree scan of each spectral band of the 500 lines of data graphed in Figure 1. In Figure 3, these spectral radiances have been converted to spectral estimates of temperatures. An emittance of 0.986 was assumed, and no atmospheric correction was applied.

The radiances appear to be slightly higher at the end of the scan line, compared to the beginning. When the scan each side of nadir is overlaid, it is found that there is an apparent offset of six pixels in the nadir position in the data. This amounts to only 0.76 degrees, but this has a large effect at high view angles. The offset may be due to either the instrument or the aircraft having a slight tilt to one side.

#### Atmospheric correction using LOWTRAN

Figure 4. shows the effect of a LOWTRAN correction to the data of Figure 2. LOWTRAN default aerosol and ozone parameters supplemented temperature and moisture information from Hilo, Hawaii radiosonde data, collected approximately four and a half hours after the TIMS data. The model was run for zero and 38 degrees, for each TIMS spectral interval. Nadir transmission and radiance data are presented as the lower two curves in Figure 5.

The LOWTRAN model transmission by the atmosphere of the path radiance was estimated by substituting the angular parameters into the path radiance part of the radiative transfer equation. In this way an equivalent atmospheric black body radiation is estimated, as well as the atmospheric transmission. Using these parameters, the three modelled path parameters may be used to estimate atmospheric path transmission of ground radiance, as well as the transmitted radiance of the path itself, for each pixel's view angle, after the empirically determined angular offset of the instrument is removed.

The result of the application of the LOWTRAN angular and spectral corrections is shown for the estimated sea temperatures of each pixel, for the average of the 500 lines, in Figure 4.

#### Black body atmospheric adjustment

An alternative atmospheric correction technique, that does not rely on the use of models, is to carry out a black body temperature adjustment to the data. For this approach, no attempt is made to separate transmission and radiance by the atmosphere. We know that the calculated temperature for each pixel, and for each band in Figure 3, should be exactly the same, and should represent the sea temperature. An adjustment is thus calculated for each pixel and band that will normalize that pixel to the one temperature. In order to estimate the temperature of the sea, we use a generalization of Saunders' (1967) two look angle method for nadir and 60 degrees. The procedure is based on a linearization of the black body functions.

$$L_{sea} = (\sec\theta L_0 - L_\theta) / (\sec\theta - 1) \quad (1)$$

Where:

- $L_{sea}$  = Radiance at sea surface
- $\theta$  = View angle
- $L_0$  = Radiance measured by sensor at nadir
- $L_\theta$  = Radiance measured by sensor at  $\theta$  degrees

A constant spectral emittance of the sea of 0.986 for each band was assumed, and an estimate made of the sea temperature in each band. The results are shown in Table 1. We used the 10.7 micrometer (band 5) calculated temperature of 29.4 degrees Celsius, which was the highest, to estimate sea temperature. This temperature is used to calculate an expected radiance in each band. The averaged observed radiance for each pixel is then divided by the expected radiance for that band. This normalization factor includes angular, transmission and path emission effects.

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Table 1. Estimated temperatures using the generalization of Saunders (1967)

| Band                 | 8.4  | 8.8  | 9.1  | 9.9  | 10.7 | 11.4 (micrometers)     |
|----------------------|------|------|------|------|------|------------------------|
| Temperature Estimate | 27.9 | 29.0 | 29.2 | 29.3 | 29.4 | 29.2 (degrees Celsius) |

The advantage of this approach is that it is entirely based on the instrument data itself. Any spectral drift, or angular offset, is automatically removed. The disadvantages are that path radiance is not removed, and thus emittance features in spectral bands that have low atmospheric transmission will tend to be suppressed. Furthermore, the normalization becomes less appropriate as the differential between the temperature of the sea and the object of interest becomes greater.

#### A black body adjustment of hypothetical data

To evaluate these effects on emittance, we used the Hilo LOWTRAN atmospheric parameters as a model atmosphere. We assumed a hypothetical emittance for a rock that alternated between 0.96 and 0.86. Figure 6 shows the hypothetical emittance curves for the six TIMS spectral bands, and emittance calculated for the rock at 30 degrees Celsius if no atmospheric correction is applied, as well as emittance if a 30 degrees Celsius black body adjustment is made. The black body adjustment, like the raw data, preserves the overall spectral shape of the emittance curves. The results are similar in all bands, except the 8.4 micrometer band (band 1), where the black body adjustment represents an improvement over no correction.

Even where the rock is ten degrees warmer than the sea from which the black body adjustment factors were calculated, the application of these factors is an improvement over no correction in the 8.4 micrometer band, as is shown in Figure 7. Thus the technique is highly robust, but in warm, humid climates it is not worth applying unless atmospheric transmission is low, for example, less than 65%.

#### Emittance estimation

The optimum band normalization approach of Warner and Levandowski (1990) was used to estimate emittance, which is of greater interest geologically than radiance or temperature data. Briefly, for the optimum band normalization technique, an arbitrary emittance of 0.96 is first assumed for each spectral band, and an equivalent temperature calculated for each band. The band with the highest temperature is assumed to have the highest emittance, and that temperature is taken to represent the actual temperature of the object. This temperature is then used to calculate emittances for each band. The major advantages of this approach include the ability to deal with unknown spectral rock emittances, as well as preservation of six spectral bands.

### EVALUATION OF ATMOSPHERIC CORRECTIONS

Figure 2 shows that the difference in radiance between nadir, at the center of the scan line, and the 38 degree view angles at either side, is 50 to 100  $\text{mW sr}^{-1} \mu\text{m}^{-1} \text{m}^{-2}$ . This is less than the noise associated with any one pixel, and is not necessarily identifiable in a single line of data. It is thus not necessary to remove this angular atmospheric effect for cosmetic improvement of imagery.

When the radiance data is converted to equivalent black body temperatures without any atmospheric corrections, an angular effect is again seen (Figure 3). There is a reduction of about one half of a degree Celsius in each band at 38 degrees compared to nadir. Also, each band gives a slightly different temperature. The 8.4 micrometer band, which gives a nadir temperature of 25.5 degrees Celsius, is about 1 and a half degrees lower than the other 5 bands. The highest calculated temperature is for the 9.9 micrometer band at 27.4 degrees Celsius.

In Figure 4 the modelled spectral and angular LOWTRAN parameters are used to calculate atmospherically corrected temperatures. The calculated temperature for each band has increased as expected, and there is a smaller differential between the 9.9 and 8.4 micrometer bands, which has been reduced from 1.9 to 1.1 degrees Celsius. Unfortunately the angular effect across the scan line has not been reduced, but increased from 0.5 to as much as 0.9 degrees Celsius for the 8.4 micrometer band.

Figure 8 shows the TIMS estimated emittance of vegetation, where flight line 94 covers an area near Hilo, on the main island of Hawaii. Salisbury (1986) has shown that vegetation emittance features tend to be very minor, and to a limited extent can be regarded as a near black body for TIMS data. The LOWTRAN corrected data has an anomalously low emittance at all wavelengths, except at 11.4 micrometers. If no correction at all is applied, a weak absorption feature is found at 8.4 micrometers, and a small peak at 9.9 micrometers. The black body adjustment results in a near flat emittance spectrum.

Comparing the estimated emittance for vegetation to that of nearby basalt lava flows in Figure 9, the LOWTRAN corrected data has a spectral shape very similar to the vegetation. By contrast, both the use of no correction, and the black body adjustment, result in a distinct long wave fall-off in emittance. This is consistent with the emittance spectra of mafic minerals.

#### CONCLUSIONS AND RECOMMENDATIONS

The LOWTRAN model should be applied with caution to thermal multispectral data. We emphasize that there are many possible areas for problems with the application of LOWTRAN: for example, we relied on the default tropical ozone and aerosol parameters in LOWTRAN, and there was a minute delay between the collection of the radiosonde data, and the aircraft overpass. Furthermore, the instrument may have experienced drift since the spectral calibration, and the black body temperature estimations may be incorrect.

When LOWTRAN was used to correct TIMS data from Hawaii, we found that the range of estimated temperature between the different bands was reduced by almost half, but the view angle effects were almost doubled. When this data was converted to emittance, the spectra were dominated by high emittance at long wavelengths, and low emittance between 9.1 and 9.9 micrometers. This was found both for rocks and vegetation, and clearly reduced the value of emittance data for interpreting geological information.

Atmospheric transmission and path radiance tend to compensate for each other, and even where atmospheric transmission is low, net radiance impinging on the sensor may not be reduced significantly. Ignoring atmospheric effects in such bands will not make a large difference in estimated temperature, but will tend to reduce spectral emittance effects. For example, failing to correct for a transmittance of approximately 70% will increase the estimated emittance of 0.86 to approximately 0.88. If emittance is higher, the TIMS estimated emittance will more closely reflect the real emittance.

If strong atmospheric absorption occurs, a black body adjustment, calculated by flying over a large water body, may be applied. This is simple to calculate, and tends to be robust, even when applied to bodies with temperatures that differ greatly from the normalizing body. The black body adjustment takes into account spectral, angular and instrument off-horizontal effects.

Water is the ideal black body to use for data normalization. A generalization of the approach of Saunders (1967) can be used to estimate the equivalent black body temperature of the sea. If no nearby water body is available, vegetation could be used. However vegetation is less likely to be uniform over the large area needed to average out noise in the data and to investigate the view angle effects.

The data used for this study was from a tropical area which is both humid and warm. Under these circumstances absorbance and reradiation from atmospheric water is high. In drier, and especially cooler, areas, atmospheric effects should be greatly reduced. Thus the need for atmospheric corrections is likely to be less in such areas.

If the LOWTRAN parameters are used to correct for atmospheric effects, the central pixels should not be assumed automatically to represent nadir. A small offset of nadir position has a significant effect at large scan angles.

The 21 line moving average of hot and cold reference bodies produced a marked reduction in high frequency noise in the data. This clearly has the advantage over a low pass filter of the radiance data, since the black body filter does not reduce the spatial resolution at all. Calibration of data from future multi-spectral thermal scanners would be improved if the instrument stared at the black body for a longer period: for example, for the equivalent of ten pixels.

#### ACKNOWLEDGEMENTS

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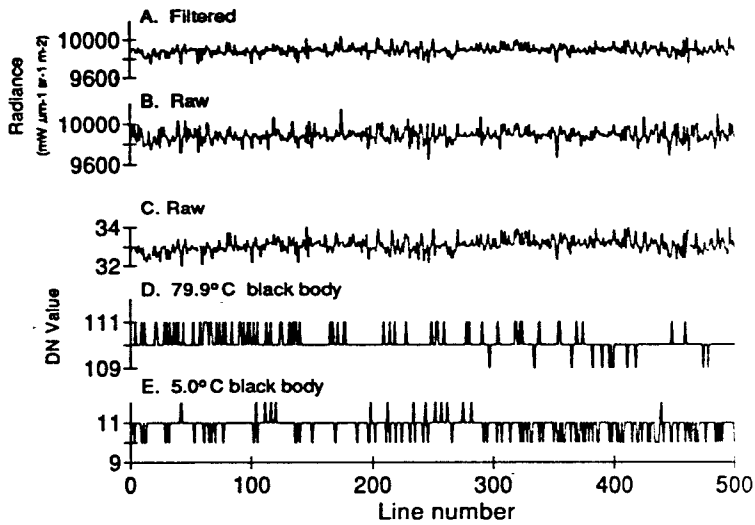


Figure 1. A comparison of 9.9 micrometer (band 4) data along 500 lines of TIMS data of line 94, of Hawaii. A. Radiance data of the average of the central (near nadir) 21 pixels, with a moving average of 21 lines applied to the black body reference video DN values. B. Radiance data of central 21 pixels converted with raw, unfiltered reference black body data. Note the increased high frequency noise compared to A. C. DN values of average of central 21 pixels of raw video data. Note some high frequency noise level similar to A, as well as long wavelength features, not found in A or B. D. and E. Hot and cold reference black body DN values used in converting raw video data to radiance.

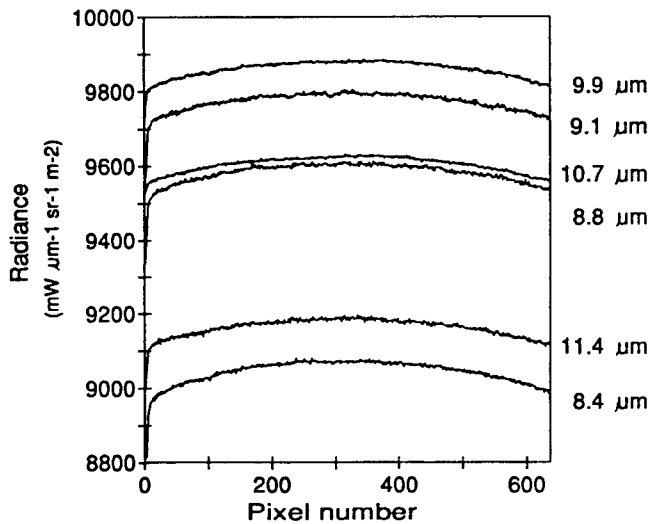


Figure 2. Average of 500 lines of 6 band TIMS spectral radiances plotted against pixel number. The nadir view angle is at the center of each scan, and extremities are at an angle of approximately 38 degrees.



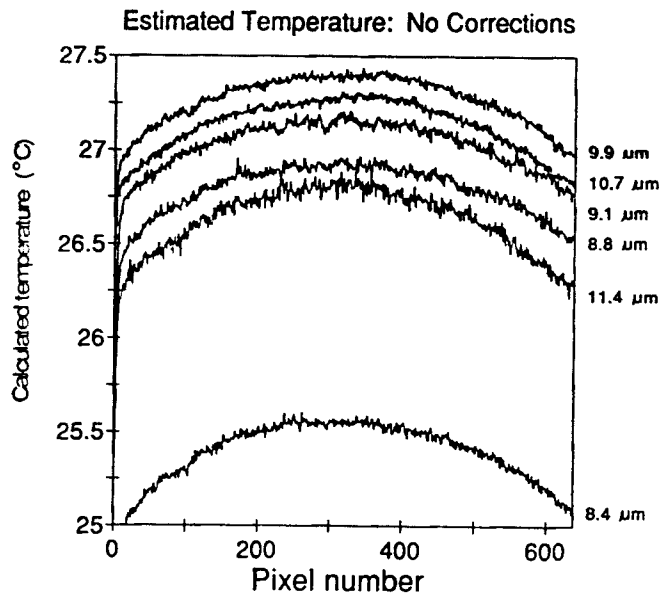


Figure 3. Radiance data from the average of 500 lines shown in Figure 2 converted to estimated black body temperatures, assuming no atmospheric effects.

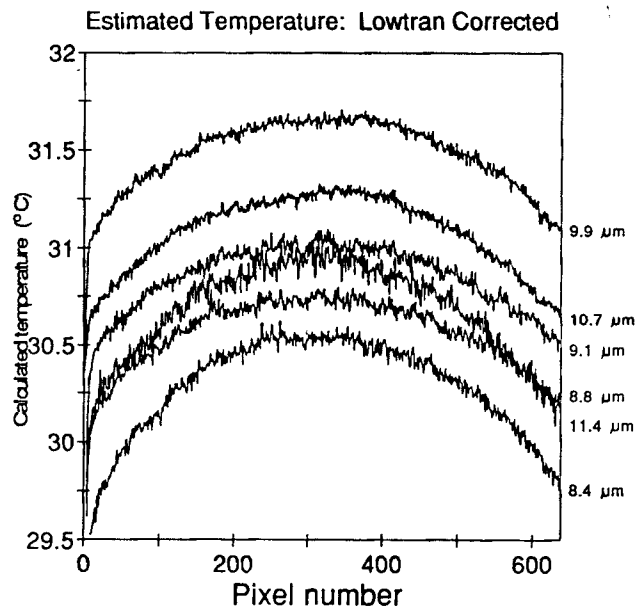


Figure 4. Radiance data from the average of 500 lines shown in Figure 2 converted to estimated black body temperatures, using LOWTRAN modelled atmospheric parameters. Note the reduction in differences of temperature estimates between bands, and the increased view angle effects on the data, when compared to the uncorrected data.

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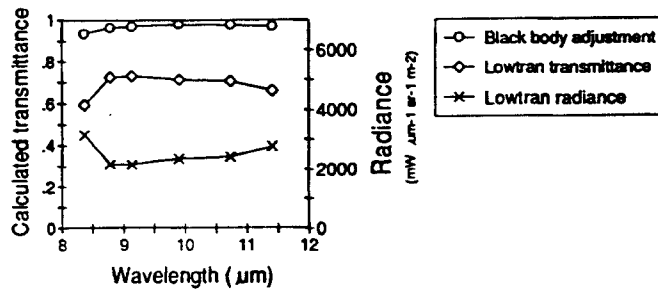


Figure 5. LOWTRAN modelled atmospheric parameters compared to black body adjustment parameters for Hilo, Hawaii. Radiosonde data from 23h00 GMT, September 30, 1988 of Hilo, Hawaii, was used to provide water vapor information. LOWTRAN aerosol and ozone default values for the tropical model were applied. Modelled radiance tends to increase as calculated transmittance decreases. The black body adjustment was calculated based on an assumed ocean temperature of 29.4 degrees C.

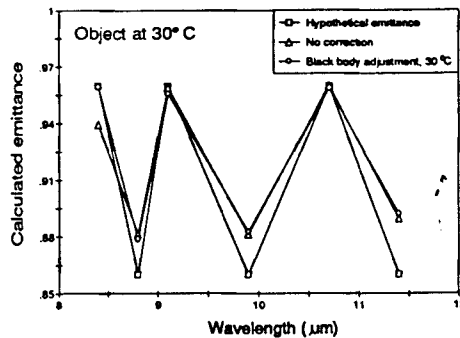


Figure 6. Hypothetical emittance curves for a body that has 0.96 emittance in 8.4, 9.1 and 10.7 micrometer bands, and 0.86 emittance in 8.8, 9.9 and 11.4 micrometer bands. An atmospheric model equivalent to the LOWTRAN parameters estimated for Hilo (see Figure 5) have been assumed. If no correction is applied the optimum band normalization technique of calculating emittance results in overestimating emittance in the 8.4 micrometer band, where atmospheric transmission is comparatively low. The black body adjustment correctly calculates a 0.96 emittance for this band. For low emittance features in the hypothetical object spectrum, both ignoring atmospheric corrections and the black body adjustment result in slightly high estimates of emittance.

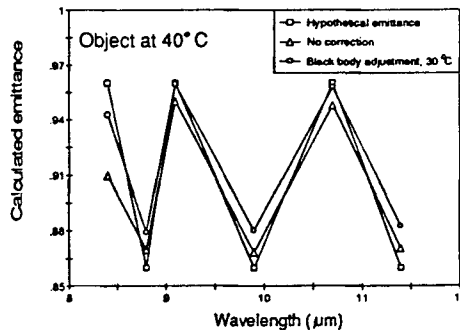


Figure 7. Same as Figure 6, except for a body at 40 degrees Celsius, 10 degrees above the temperature used to calculate the black body adjustment.

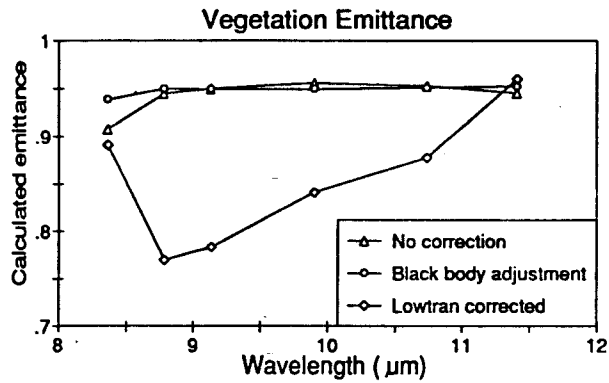


Figure 8. Calculated TIMS emittance spectra for vegetation from Line 94, Hawaii, for no atmospheric correction, black body adjustment and LOWTRAN corrected data.

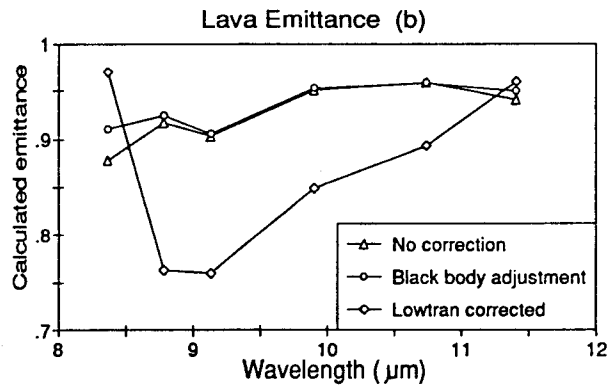
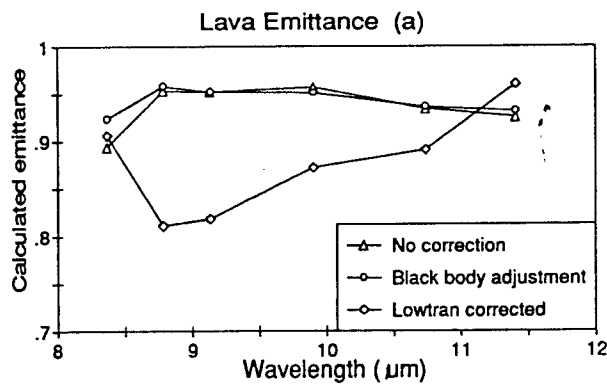


Figure 9. Calculated TIMS emittance spectra of two basaltic lava flows from Line 94, Hawaii, for no atmospheric correction, black body adjustment and LOWTRAN corrected data.

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