LARS Information Note 102670
Purdue University

Random Noise in Multispectral Classification

by

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ABSTRACT

Performance of the maximum likelihood classifier used by LARS was tested under various levels of simulated, additive, white gaussian noise. Pictorial and graphical results are presented. Some general problems and considerations in classifying noisy data are discussed.

INTRODUCTION

A problem common to many operational classifiers is that of noise in the data. For instance, the multispectral data used in this report was obtained by an airborne scanner linked optically to twelve sensors, corresponding to twelve wavelength bands. The "signal" received in this manner is subject to the addition of noise in a number of different locations. Thermal noise is introduced at the sensor itself, the amplifiers which are used in recording the data on analog tape, and the A/D system used in digitizing the data.

Consider the general pattern recognition problem where one is using N features to classify data into M categories (classes). Because of the large number of parameters involved, it is not apparent that one should use any specific test as a measure of classification accuracy under the conditions of added noise.

For instance, the noise level in each of the channels (features) in the multispectral sensor system may differ in power and correlation. The noise spectrum undoubtedly will be altered in a different way in each of the channels as it is processed.

PROBLEM INVESTIGATION

It seems reasonable to examine performance initially under very general noise conditions. For the experiment discussed in this paper, a white, additive, zero-mean, gaussianly distributed, discrete random process was approximated and used to corrupt multispectral agricultural data. This "noisy" data was then classified using a pattern recognition scheme discussed by Marill and Green (2).

The data and classifier are part of an operational pattern recognition system developed by the Purdue Laboratory for Applications of Remote Sensing (1). This system has been used successfully for several years in crop and soil classification using an airborne multispectral scanner system. More recently, classification studies have been conducted using data gathered from space platforms, again with much success.

In classifying agricultural data, samples are chosen from carefully selected fields to "train" the classifier. In training, mean values and covariance matrices are estimated for each class represented by the various fields. For instance, samples from three corn fields might be used to estimate the statistics of the class "corn". The recognition scheme is based on the gaussian assumption. Histograms of the various classes are checked to test the validity of that assumption (see ref. 1). From the estimated statistics, features are chosen to be used in classification. Because of errors in estimating the true statistics of the various classes, and because classes may not have a gaussian distribution, a subset of all of the features usually can be found which yields better classification accuracy than the entire set (6), (7). An interclass divergence technique (4) is used to provide a basis for feature selection. Finally, all of the data is classified and compared with known crop cover to determine the accuracy of the classification.

In order to simulate the presence of noise, a number of considerations had to be taken into account. One would like to model the physical process of noise being added to a large data system as accurately as possible. But due to the inaccessability of the various stages of the system, and the large number of noise and data parameters involved, it was decided that samples from the random process would be added directly to the digitized data.

To attempt to obtain independent samples with a gaussian distribution two methods were studied: 1) actual noise generators and 2) computer generation of random numbers. The first method involved sampling the output of a laboratory noise generator, quantizing these samples, and examining their histograms and finite record autocorrelations using the algorithm of Blackman and Tukey (9). It was determined that a good gaussian distribution could be obtained by method (1) only by low-pass filtering the output of the generators available, which of course increases correlation. Slow sampling to reduce correlation was too time consuming for the large number of samples needed. Also, controlling statistical parameters was quite difficult. The second method is a software approach which has great appeal in a problem where most of the analysis is done on a computer. The technique used employed two subroutines from the IBM Scientific Subroutine Package, RANDU and GAUSS. RANDU essentially picks independent samples with a uniform distribution between zero and one using a power residue method. GAUSS simply sums a fixed number of these uniformly distributed samples to approximate one sample from a gaussian distribution. Owing to the central limit theorem (see 10), the approximation is very good for even small numbers of samples, as successive convolutions of identical uniform distributions rapidly approaches a gaussian one. The resulting sample is normalized to have zero mean and unit variance. If L uniformly distributed (on [0,1]) samples are used to produce one approximate to a gaussian sample, then the range of the normalized deviate is $\pm(L/2)/\sqrt{L/12}$ standard deviations (see Appendix C). For L = 12 (as in GAUSS), the range of generated numbers is ± 6 standard deviations, quite reasonable for any physical process. A finite record autocorrelation (fig. 1) with L = 12 shows that the samples generated have little correlation and therefore resemble samples of white noise. A histogram of the same 15,000 samples generated in this manner appears in Figure 1. The second method was used to provide the source of gaussianly distributed random values.

LARS data is stored digitally on magnetic tape. Software was devised to add the generated deviates to the existing data and create a new set of data. In the multispectral data, provision was made to adjust the variance of the "noise" separately for each channel. The word "noise" will represent the set of random samples added to the data. Data is uniformly quantized to the range of 0 to 255 before being stored on magnetic tape. The noise added is measured in the number of units in one standard deviation, where one unit corresponds to one bin of the 256 used. The sample values of signal plus noise were re-quantized uniformly to the range of 0 to 255 with no gain changes. Figure 2 shows an agricultural scene and digitized sensor output for various levels of added noise. Figure 3 shows line 369 of the scene in figure 2 with the same values of standard deviation (sigma).

The agricultural area which was used is shown in figure 4. The fields which were used for training are outlined. A set of histograms for one of the fields appears in figure 5 under different levels of added "noise". The histograms are somewhat ragged due to the relatively small number of samples in that particular field.

This particular area (from flight line C-1, Run 66000600) has been used by LARS ((1), (5), (6)) for a number of studies as it has a representative number of varied crop types. The training fields were carefully selected by LARS staff members. Actual crop types and density of cover has been carefully studied and tabulated.

The classifier was trained for each of seven added noise levels: no noise, 2, 5, 7, 10, 15, and 20 units of standard deviation for all channels. Pattern recognition was carried out on data of each of the above noise levels by using a classifier trained with samples of the same noise level. The optimum feature set corresponding to 4 channels was chosen for the no noise case and used for all of the rest. These four channels corresponded to wavelength bands of .40-.44, .52-.55, .66-.72, and .72-.80 micrometers. Test and training fields are listed in Figure 6.

RESULTS

Accuracy was determined on the basis of per cent correct classification of samples from known classes. Figure 7 is a graph of classification accuracy versus sigma. (About 3200 samples were used in training and 13,000 in classification). Note that one of the lines represents the accuracy based on classification of the training samples only, and another represents the test samples. Results for two particular test classes also are given. Test samples consist of samples from fields of known classes which are not necessarily used in training. Results are displayed with test fields outlined in Figure 8. Blank spaces correspond to points which were below a probability threshold of one percent. That is, the classifier decides which class to announce for a given point. If the probability of the point being from that class is less than the threshold value (which can be changed), no class decision is printed (see (1)). To make the results more explicit, the classification of all data as wheat is printed out in Figure 9. Below is a table of results including the two specific test classes.

PERCENT CORRECT

Added Noise Units of Standard Deviation	Train	Test	Wheat (Test)	Soybeans (Test)
0	97.4	91.8	98.1	95.7
2	92.8	86.3	96.7	89.5
5	79.9	69.1	87.9	60.9
7	70.3	58.9	80.8	44.3
10	57.4	49.1	71.5	33.9
15	42.7	35.0	56.7	17.7
20	38.3	31.7	52.6	15.3

(see figure 7)

As one might expect, the shape of the curve in Figure 7 resembles the complement of the error function. One important feature is that adding a small amount of noise (up to sigma=2) makes only a small difference in performance when the level of added noise is already either very small or very large. This would seem to indicate that schemes designed to reduce perturbations in data caused by added noise would have to make a considerable improvement when the noise level is in these regions.

A problem of interpretation arises. The data without any simulated noise still has intrinsic noise in it. Thus one cannot speak of some signal to noise ratio without knowledge of the noise already in the data. To further complicate matters, the complete absence of any corruption obviously will not imply perfect classification. In the data used here, for instance, the class called corn may have a multivariate density which will overlap the class called wheat in the 4 dimensional observation space corresponding to the four features used to measure the sample values. This overlap will correspond to error regions as the maximum likelihood classifier chooses the class with the highest conditional p.d.f. Consider the classical binary case with one feature in Figure 10. If the estimated statistics for the two classes under the gaussian assumption correctly model the situation, then the shaded area will be the total probability of misclassification (assuming that all observations will be from either of the two classes). The addition of independent gaussian noise will serve to increase the variance of each class, and thus increase the total probability of misclassification. Hence variance due to both noise and the data itself contribute to misclassification. A signal to noise ratio based on the noise itself loses meaning when the various classes alone have different values of variance. There would be no obvious relationship between signal to noise ratio and performance. The reader might refer to Van Trees (10, section 2.6) where the above is discussed as part of what he calls the general gaussian problem. The two class, N features case is developed. In most references, any external noise would become part of the class statistics and would not be considered separately. Hence the performance of data systems of this type cannot be evaluated strictly on the basis of how well external noise is minimized, but must take into account how well the system reproduces the properties of the target.

Another type of classifier known as a per field classifier (11) was used on the above data. This classifier performs pattern recognition on a group of samples under the assumption that all are from the same class. This type of classifier seems particularly useful on many types of data where field boundaries can be distinguished. Results (not shown here) for this classifier showed near perfect accuracy under all levels of noise. An obvious explanation is that this classifier uses a number of samples to estimate the statistics of the group, then uses the statistics to make a decision. The effects of noise and data variance are essentially averaged out. A simple weighted distance minimization to the statistics of the known classes provides the decision rule. This type of classifier would then require careful training. In conjunction with a cluster technique to determine boundaries (such as (12)), this type of classifier might provide a highly automated pattern recognition system.

ACKNOWLEDGEMENT

This study was done under the supervision of Professor David A. Landgrebe and with the help of the LARS Data Processing and Data Analysis staff.

FUTURE RESEARCH

It is obvious that a large number of assumptions and approximations were made in this study. The large number of parameters and the size of the system made this necessary. Because of these compromises and the open ended nature of the work, the following topics are suggested for further investigation.

1. Determination of classification accuracy of noisy data using statistics generated with no added noise.

2. Determination of the effect of noise on the optimum number of features.

3. Determination of the differences in adding noise with a continuous distribution to samples which are already quantized, then requantizing, versus adding noise to the unquantized data.

4. Determination of the effects of varying noise levels in channels separately on feature selection and classification.

5. Examination of the effects of noise and/or data time correlation on classification.

BIBLIOGRAPHY

(1) K. Fu, D. Landgrebe, and T. Phillips, "Information Processing of Remotely Sensed Data", Proceedings of the IEEE, April, 1969, p. 639.

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(3) S. Kullback, Information Theory and Statistics, Dover Inc., New York, 1968.

(4) T. Marill and D. Green, "On the Effectiveness of Receptors in Recognition Systems", IEEE Transactions on Information Theory, IT-9, January, 1963, p.11.

(5) LARS Research Bulletin, No. 844, September, 1968.

(6) K. Fu and P. Min, "On Feature Selection in Multiclass Pattern Recognition", LARS Technical Report No. TR-EE68-17, July, 1968.

(7) G. Hughes, "On the Mean Accuracy of Statistical Pattern Recognizers", IT-14, January, 1968, p. 55.

(9) R. Blackman and J. Tukey, The Measurement of Power Spectra, Dover, New York, 1958.

(10) A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York, 1965.

(11) T. Huang, "Per Field Classifier For Agricultural Applications", LARS Information Note 060569.

(12) A. Wacker, "A Cluster Approach to Finding Spatial Boundaries in Multispectral Imagery", LARS Information Note 122969.

APPENDIX A

The Noise Adding Program

The program essentially does the following:

1. Reads user generated values for the number of units standard deviation desired for each channel.
2. Creates a new ID record for the noise run and stores the values of standard deviation used by each channel.
3. Reads lines of data from the run to which noise is to be added and generates the needed number of noise samples.
4. Adds these together and writes these "new" lines of data on to a new tape.
5. Retains the standard data storage tape format so as to be compatible with the LARS processors.

Note that RANDU generates about 540 million random numbers in sequence and then repeats. In order to keep the sequence unique, each run used a noise starting number which would have been the next in sequence for the previous run. Twelve numbers are used from RANDU for each gaussian sample. There were 222 data points per channel per line, 12 channels per data point, and 950 lines in 66000600. This gives about 2.5 million data samples. Then about 30 million numbers (12 uniform for one gaussian) or about 5.7% of the possible number of noise samples available were used for each run.

APPENDIX B

File Information

The run used as clean data was 66000600. The generated noisy data runs are stored on tape 88. Run numbers and noise levels are given below

Run	Sigma (all channels)
66000610	10
66000611	5
66000612	2
66000613	15
66000614	7
66000615	20

Classification results are stored on tape 100. Files 1 thru 8 are strictly for experimental purposes. Channels 1, 6, 10 and 11 were used to classify the data.

Files 9 thru 15 contain the results used in this work. The serial number for each trial with the corresponding run classified is given. Only odd lines and columns were classified.

Serial Number	Classification of Run
715010009	66000600
715010010	66000610
715010011	66000611
715010012	66000612
715010013	66000613
715010014	66000614
715010015	66000615

APPENDIX C

Moments of the Approximately Gaussian Samples

RANDU and GAUSS in the IBM Scientific Subroutine Package use the following technique for generating numbers with a distribution which is approximately gaussian.

Let $X_1, X_2, \ldots, X_N$ be independent, identically distributed, random variables with uniform probability densities on the interval [0, 1]. Then, by the central limit theorem,

$$X_s = X_1 + X_2 + \ldots + X_N = \sum_{i=1}^{N} X_i$$

is approximately gaussian, the approximation improving with increased N. X_s is normalized by defining a new random variable $Z = \frac{X_s - E(X_s)}{\sigma_{X_s}}$ where E stands for the expected value, and sigma, the standard deviation. Z will then be normal with zero mean and unit variance.

All that remains to be found are $E(X_s)$ and σ_{X_s}.

The mean $E(X_s) = E(X_1 + X_2 + \ldots + X_N)$

$$= E(X_1) + E(X_2) + \ldots + E(X_N)$$

$$= 1/2 + 1/2 + \ldots + 1/2$$

$$= N/2 \qquad (1).$$

$$\sigma_{X_s}^2 = E\{[X_s - E(X_s)]^2\} = E(X_s^2) - E^2(X_s) \qquad (2).$$

$$E^2(X_s) = N^2/4 \text{ from } (1). \qquad (3).$$

$$E\left(X_s^{\,2}\right) = E\left[\left(\sum_{i=1}^{N} X_i\right)^2\right] = E\left[\left(X_1 + X_2 + \ldots + X_N\right)\left(X_1 + X_2 + \ldots + X_N\right)\right]$$

$$= E\left(\sum_{i=1}^{N} X_i^{\,2} + 2\sum_{j<k}^{N}\sum^{N} X_j X_k\right) \qquad (4).$$

$$\text{But } E\left(\sum_{i=1}^{N} X_i^{\,2}\right) = \sum_{i=1}^{N} E\left(X_i^{\,2}\right) = NE\left(X_i^{\,2}\right) = N\int_{-\infty}^{\infty} x_i^{\,2} p_{X_i}(x_i)dx_i$$

$$= N\cdot\int_0^1 x^2 dx = N\frac{x^3}{3}\Big|_0^1 = N/3 \qquad (5).$$

$$\text{Also } E\left(2\sum_{j<k}^{N}\sum^{N} X_j X_k\right) = 2\sum_{j<k}^{N}\sum E\left(X_j X_k\right) = 2\sum_{j<k}^{N}\sum E\left(X_j\right)E\left(X_k\right)$$

$$= 2\sum_{j<k}^{N}\sum 1/4$$

Now there are $(N-1) + (N-2) + \ldots + 2 + 1 = \frac{N(N-1)}{2}$ of these terms.

So the above $= 2\cdot\frac{N(N-1)}{2}\cdot 1/4$ or $E\left(2\sum_{j<k}^{N}\sum X_j X_k\right) = \frac{N(N-1)}{4} = \frac{N^2-N}{4}$ (6).

Putting (5) and (6) into (4) one has $E\left(X_s^{\,2}\right) = N/3 + \frac{N^2-N}{4}$

$$= \frac{4N}{12} + \frac{3N^2+3N}{12} = \frac{3N^2+N}{12} \qquad (7).$$

Putting (3) and (7) into (2) one gets $\sigma_{X_s}^{\,2} = \frac{3N^2+N}{12} - \frac{3N^2}{12}$

$$= N/12 \text{ or } \sigma_{X_s} = \sqrt{N/12} \qquad (8).$$

This of course is a special result of the theorem which states that the variance of the sum of independent R. V. is equal to the sum of the individual variances.

$$\sum_{i=1}^{N} \sigma x^2 = N\left[E\left(X^2\right) - E^2(X)\right] = N(1/3 - 1/4) = N/12$$

So the normalization formula is

$$Z = \frac{\left(\sum_{i=1}^{N} X_i\right) - N/2}{\sqrt{N/12}}$$

This is equal to $\left(\sum_{i=1}^{12} X_i\right) - 6$ for GAUSS, where N=12.

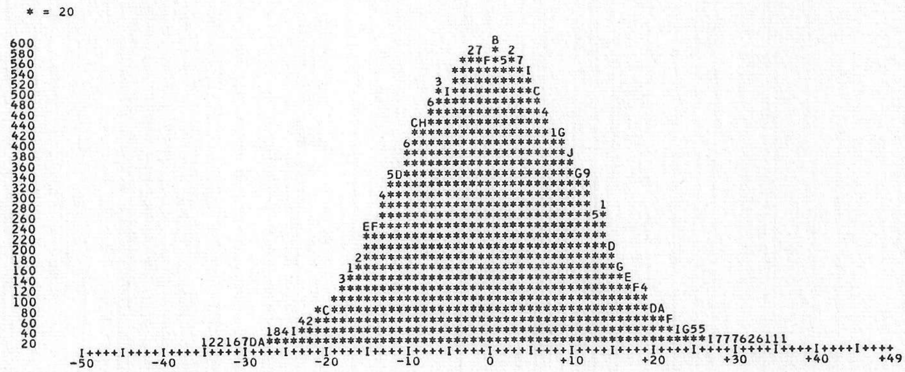
The range of possible values of $\sum_{i=1}^{N} X_i$ is obviously zero to N.

So the range of the normalized deviate is $\frac{-N/2}{\sqrt{N/12}}$ to $\frac{N-N/2}{\sqrt{N/12}}$ or

$\pm \frac{N/2}{\sqrt{N/12}}$ standard deviations.

FIGURES - ILLUSTRATIONS

HISTOGRAM OF QUANTIZED NOISE
IN THE RANGE OF -50 TO +49



0 SAMPLES OUTSIDE -50 TO +49

NO. SAMPLES = 15000
SAMPLE VAR = 102.01180

SAMPLE SIGMA = 10.10009
REQUESTED SIGMA = 10.000

MAX VALUE = 36.000
MIN VALUE = -35.000

SAMPLE MEAN = -0.11827
REQUESTED MEAN = 0.0

PCT. WITHIN ONE SIGMA = 69.5
PCT. IN NORM. DIST. = 68.3

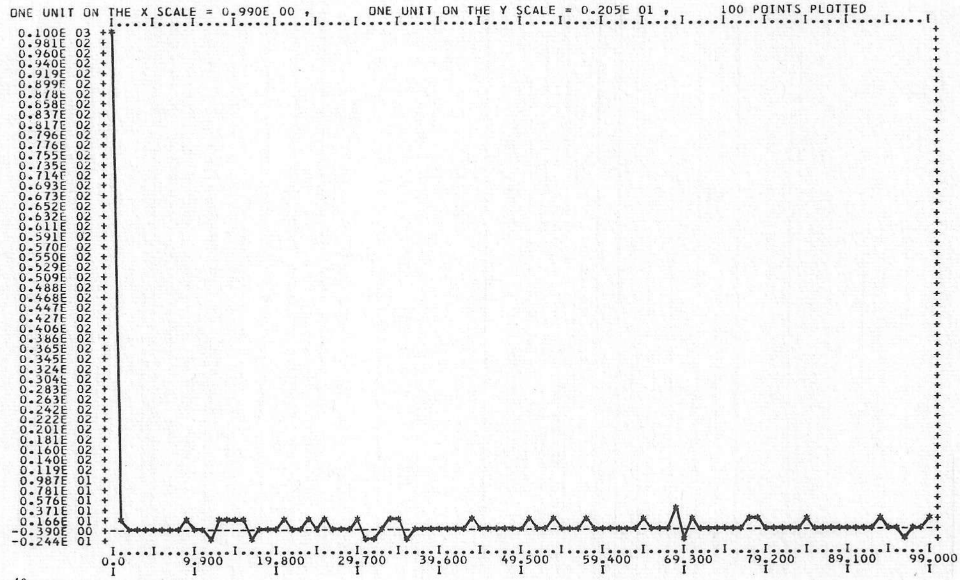
PCT. WITHIN TWO SIGMA = 95.8
PCT. IN NORM. DIST. = 95.5

/&

Histogram of 15,000 Points

FINITE RECORD AUTOCORRELATION

ONE UNIT ON THE X SCALE = 0.990E 00 , ONE UNIT ON THE Y SCALE = 0.205E 01 , 100 POINTS PLOTTED

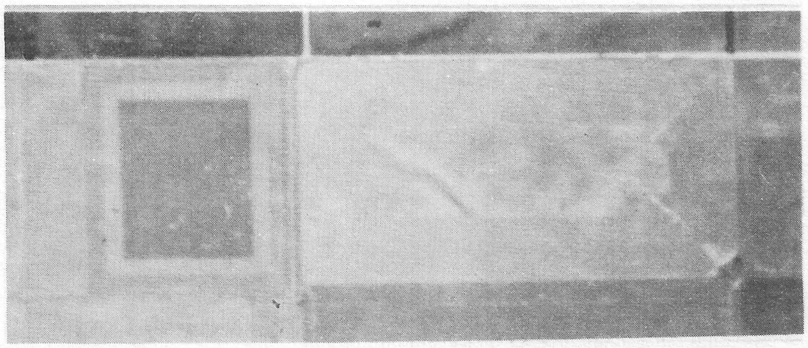


/&

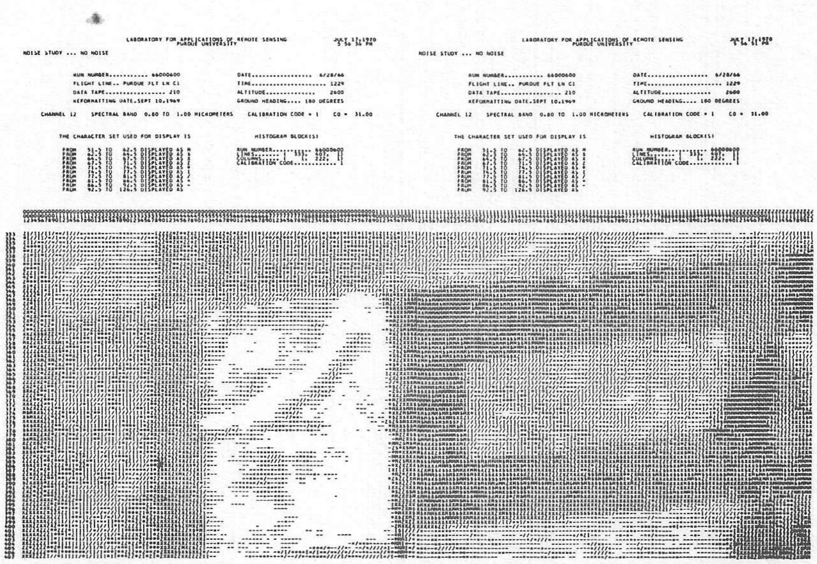
Autocorrelation - 100 Lags

Random Sample Statistics

Figure 1



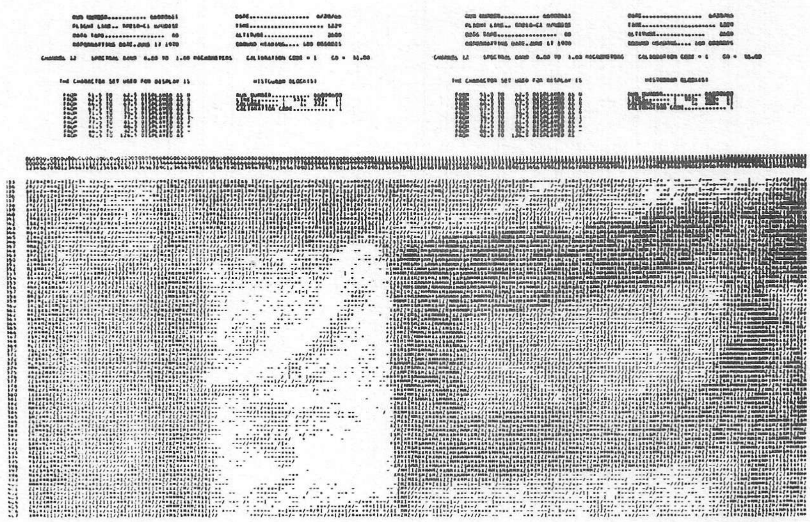
Agricultural Scene from C-1
(B&W Photograph)



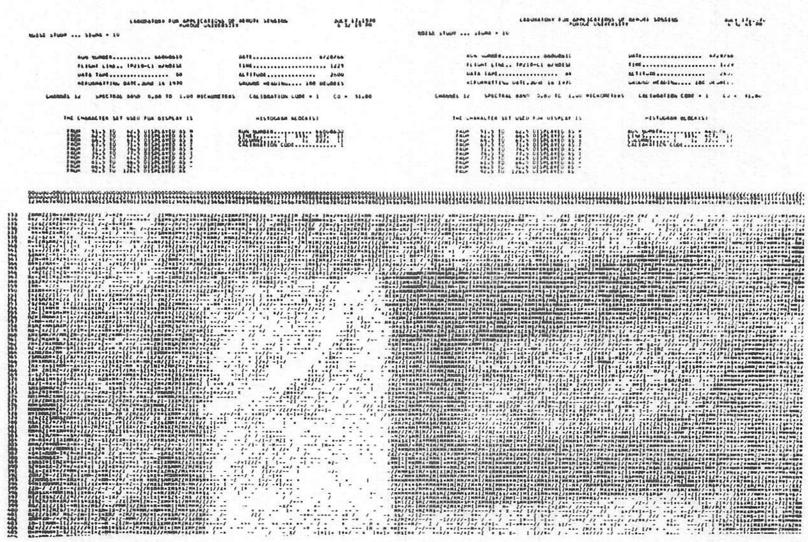
Scanner Output of the Same Scene
in the 0.80 to 1.00 Micrometer Band
Run 66000600

Data With No Added "Noise"

Figure 2a



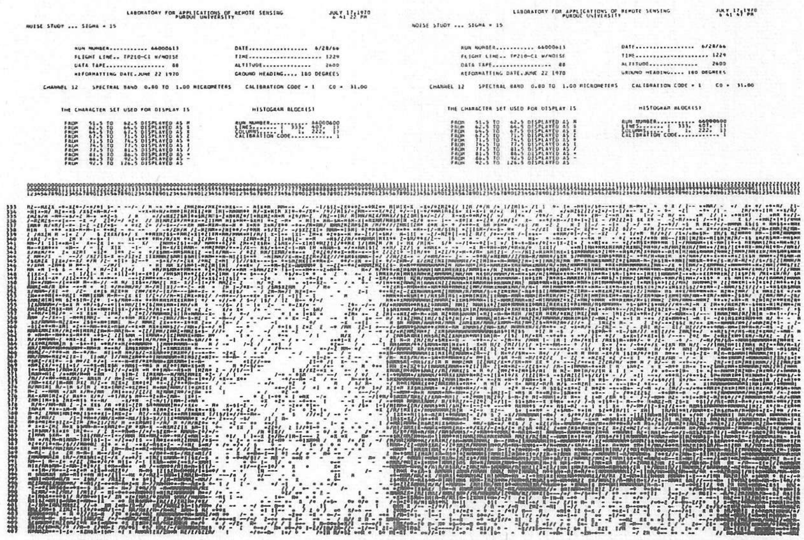
Sigma = 5



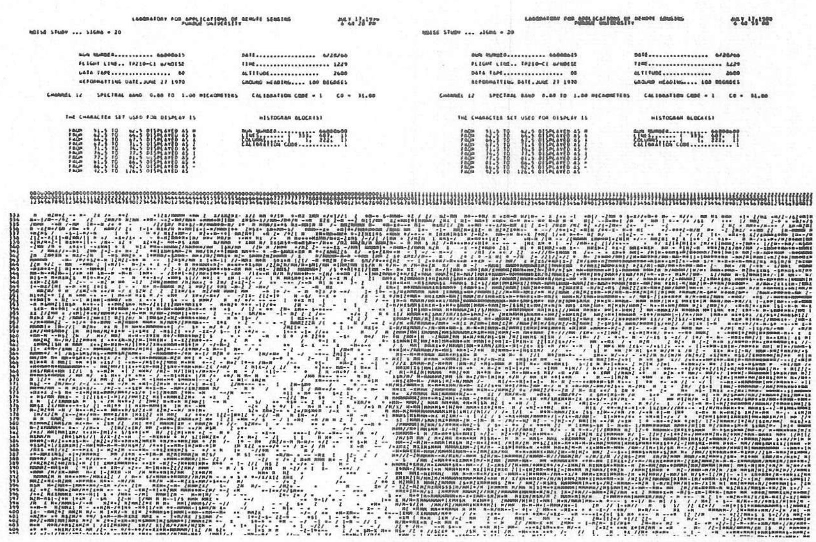
Sigma = 10

Scanner Data from Flight Line
C-1, with Noise Added

Figure 2b



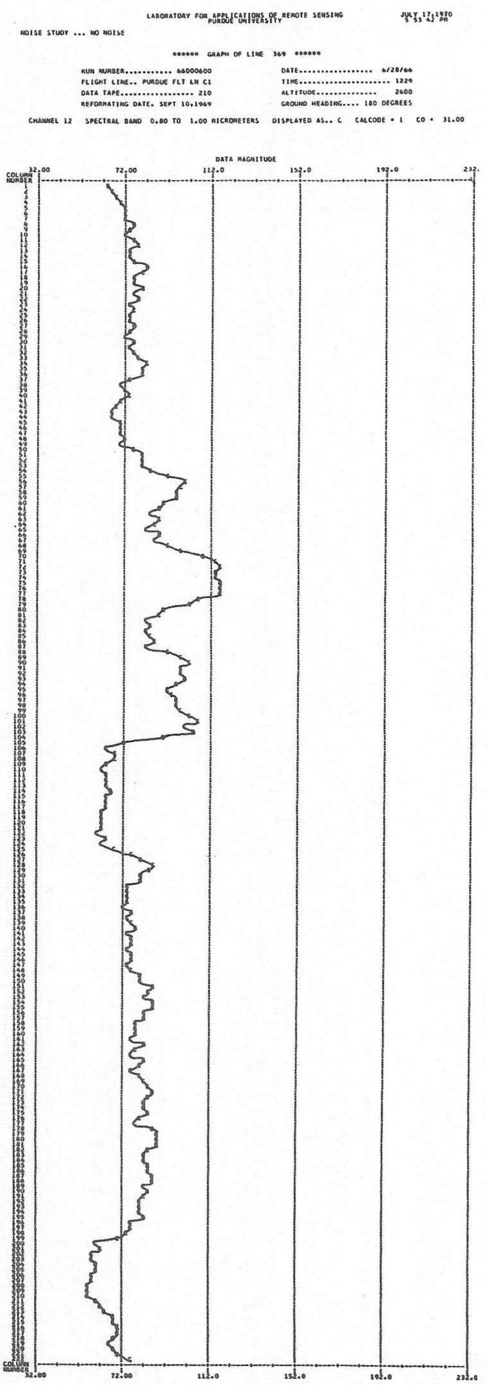
Sigma = 15



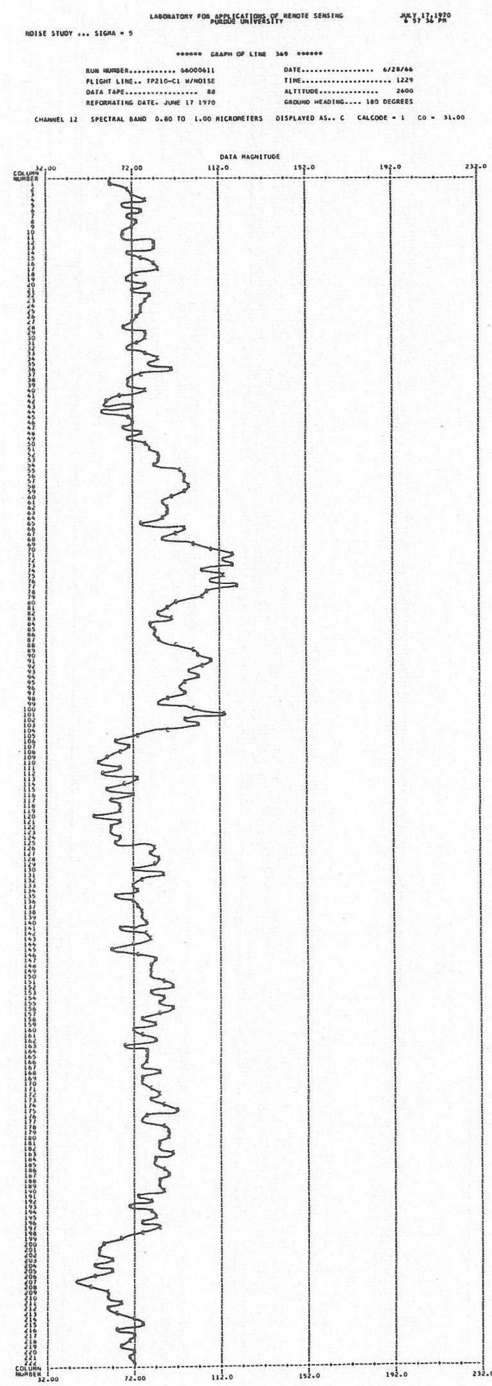
Sigma = 20

Scanner Data from Flight Line
C-1, with Noise Added

Figure 2c



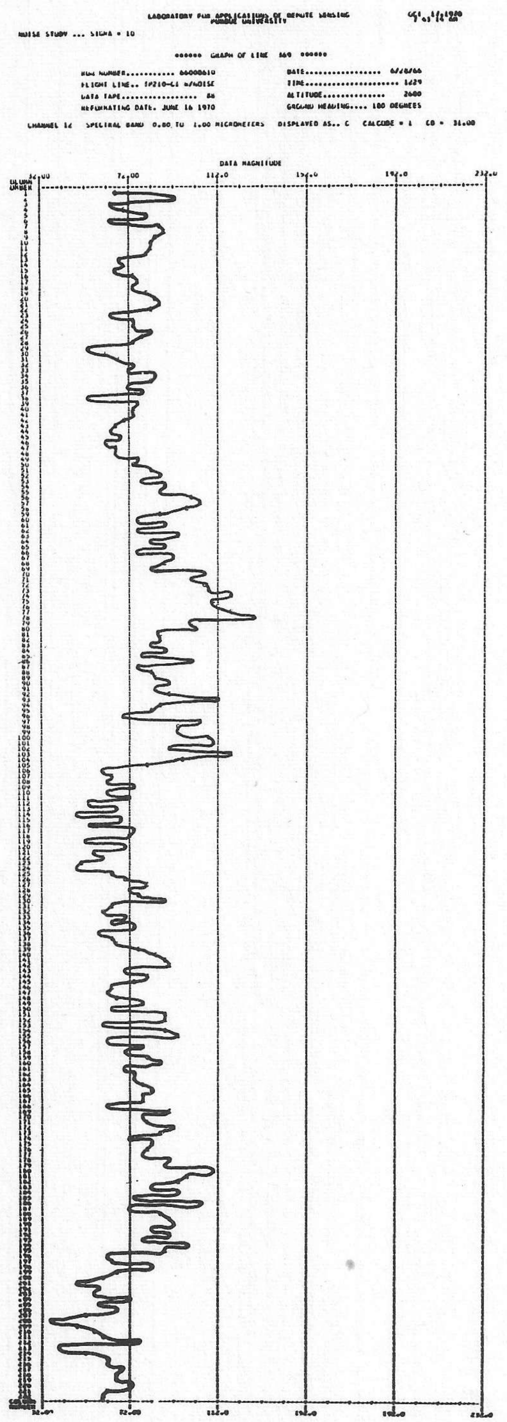
No Added Noise



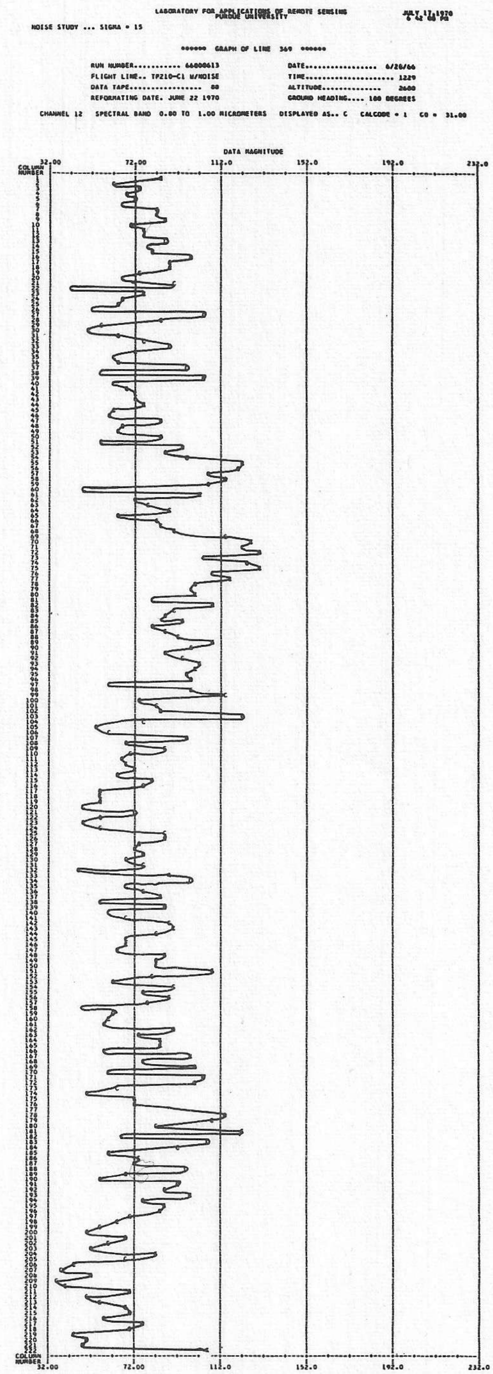
Sigma = 5

Plot of Scan Line 369 from
Figure 2 (C-1, Run 66000600)

Figure 3a



Sigma = 10

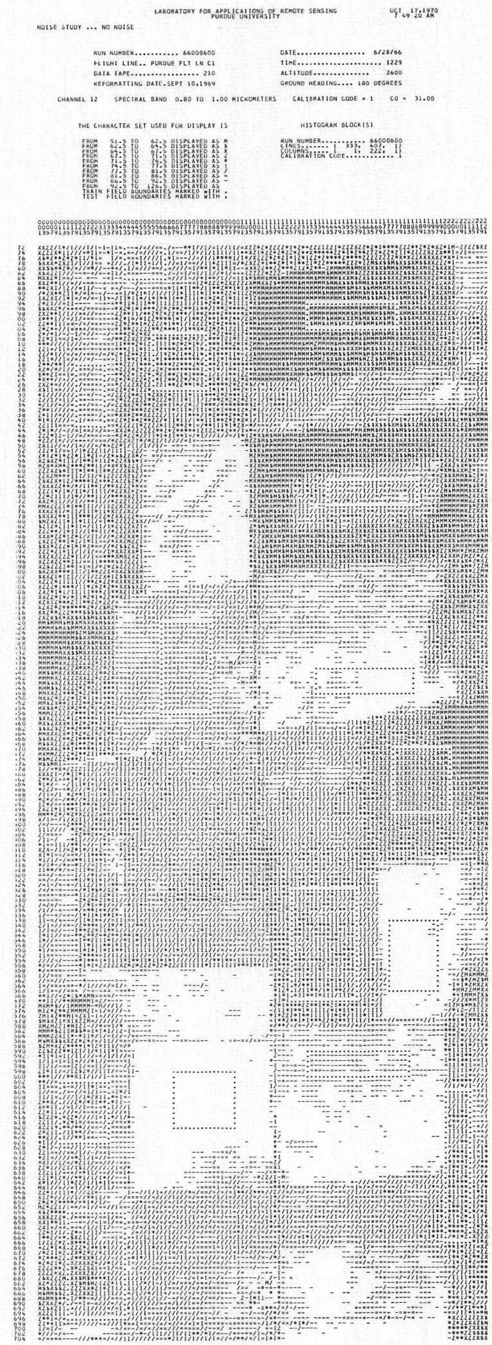


Sigma = 15

Plot of Scan Line 369 from
Figure 2 (C-1, Run 66000600)

Figure 3b

B&W Photograph



Scanner Data in
0.80 to 1.00 Micrometer Band
with Training Fields Outlined

Area from C-1 Classified

Figure 4

LABORATORY FOR APPLICATIONS OF REMOTE SENSING
PURDUE UNIVERSITY
JULY 17,1970
6 07 49 PM

NOISE STUDY ... NO NOISE

CLASS....

FIELD 31-12
RUN NO. 66000600
OTHER INFORMATION WHEAT

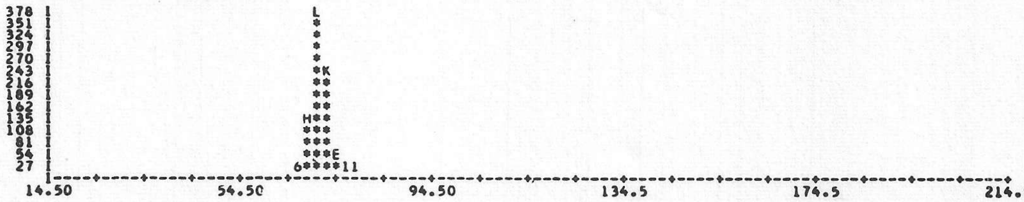
TYPE
NO. OF SAMPLES 782

LINES 285- 317 (BY 2)
COLUMNS 109- 199 (BY 2)

HISTOGRAM(S) FOR...FIELD 31-12

CHANNEL 1 0.40 - 0.44 MICROMETERS

EACH * REPRESENTS 27 POINT(S).



No Added Noise

LABORATORY FOR APPLICATIONS OF REMOTE SENSING
PURDUE UNIVERSITY
JULY 17,1970
6 31 17 PM

NOISE STUDY ... SIGMA = 7

CLASS....

FIELD 31-12
RUN NO. 66000614
OTHER INFORMATION WHEAT

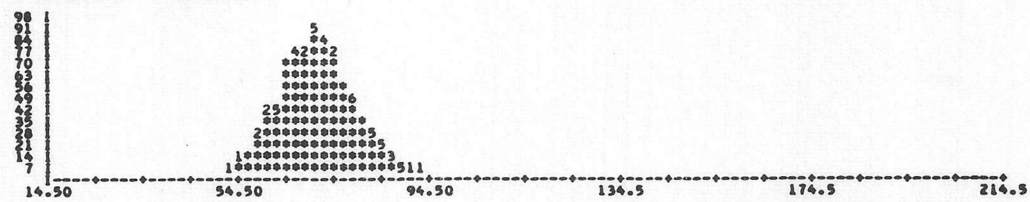
TYPE
NO. OF SAMPLES 782

LINES 285- 317 (BY 2)
COLUMNS 109- 199 (BY 2)

HISTOGRAM(S) FOR...FIELD 31-12

CHANNEL 1 0.40 - 0.44 MICROMETERS

EACH * REPRESENTS 7 POINT(S).



Sigma = 7

LABORATORY FOR APPLICATIONS OF REMOTE SENSING
PURDUE UNIVERSITY
JULY 17,1970
6 27 44 PM

NOISE STUDY ... SIGMA = 15

CLASS....

FIELD 31-12
RUN NO. 66000613
OTHER INFORMATION WHEAT

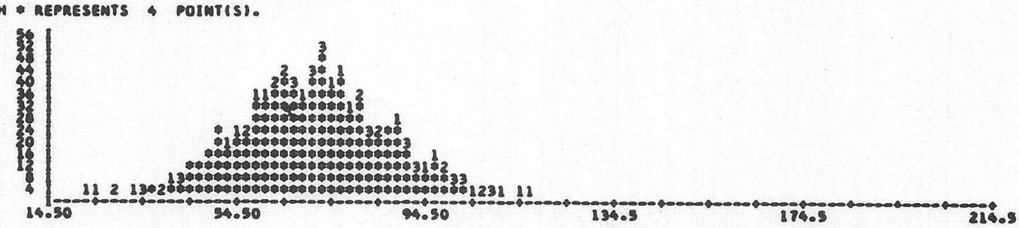
TYPE
NO. OF SAMPLES 782

LINES 285- 317 (BY 2)
COLUMNS 109- 199 (BY 2)

HISTOGRAM(S) FOR...FIELD 31-12

CHANNEL 1 0.40 - 0.44 MICROMETERS

EACH * REPRESENTS 4 POINT(S).



Sigma = 15

Histogram of One Wheat Field
in the 0.40 to 0.44 Micrometer Band

Figure 5

LABORATORY FOR APPLICATIONS OF REMOTE SENSING
PURDUE UNIVERSITY

OCT 17,1970
7 49 05 AM

NOISE STUDY ... NO NOISE

FIELD CARDS ADDED TO FIELD BOUNDARY STORAGE

FIELD NUMBER	RUN NUMBER	FIELD DESIGNATION	FIRST LINE	LAST LINE	LINE INTERVAL	FIRST SAMPLE	LAST SAMPLE	SAMPLE INTERVAL	FIELD TYPE	OTHER INFORMATION
1 *	66000600	25-6	65	81	2	69	89	2	3	SOYBEANS1
2 *	66000600	31-13	237	253	2	141	167	2	3	SOYBEANS2
3 *	66000600	36-7	307	327	2	59	81	2	3	SOYBEANS3
4 *	66000600	7-23	773	777	2	135	179	2	3	SOYBEANS4
5 *	66000600	36-4	167	177	2	33	77	2	1	CORN1
6 *	66000600	36-9	267	283	2	45	61	2	1	CORN2
7 *	66000600	36-8	319	341	2	21	31	2	1	CORN3
8 *	66000600	12-9	603	625	2	13	33	2	1	CORN4
9 *	66000600	6-2	365	373	2	145	185	2	3	OATS1
10 *	66000600	1-11	421	455	2	63	83	2	3	OATS2
11 *	66000600	7-1	591	599	2	135	181	2	3	OATS3
12 *	66000600	31-12	295	303	2	134	175	2	4	WHEAT1
13 *	66000600	6-14	471	495	2	177	201	2	4	WHEAT2
14 *	66000600	7-2	607	665	2	203	211	2	4	WHEAT3
15 *	66000600	6-10	439	447	2	139	183	2	6	RED CL1
16 *	66000600	1-1	539	565	2	175	195	2	6	RED CL2
17 *	66000600		599	619	2	69	95	2	6	RED CL3
18 *	66000600	7-24	731	737	2	129	177	2	6	ALFALFA1
19 *	66000600	7-24	749	755	2	131	171	1	6	ALFALFA2
20 *	66000600	7-22	809	817	1	155	183	1	6	ALFALFA3
21 *	66000600	6-8	527	569	1	127	155	2	7	RYE1
22 *	66000600	36-1	97	115	1	49	85	1	5	&R 00 -
23 *	66000600	12-10	655	695	2	17	41	2	9	W&E&T4

* - INDICATES TRAINING FIELDS

Training Fields

LABORATORY FOR APPLICATIONS OF REMOTE SENSING
PURDUE UNIVERSITY

OCT 17,1970
7 50 11 AM

NOISE STUDY ... NO NOISE

FIELD CARDS ADDED TO FIELD BOUNDARY STORAGE

FIELD NUMBER	RUN NUMBER	FIELD DESIGNATION	FIRST LINE	LAST LINE	LINE INTERVAL	FIRST SAMPLE	LAST SAMPLE	SAMPLE INTERVAL	FIELD TYPE	OTHER INFORMATION
1	66000600	36-7	291	341	2	43	97	2		SOYBN VOLUNTR CORN
2	66000600	6-9	489	519	2	115	161	2		SOYBEANS
3	66000600	7-27	643	663	2	125	197	2		SOYBEANS
4	66000600	12-7	647	699	2	51	87	2		SOYBEANS
5	66000600	12-2	647	675	2	93	111	2		SOYBEANS
6	66000600	12-3	705	797	2	33	63	2		SOYBN W. PRT PLT ERL
7	66000600	7-23	759	785	2	121	197	2		SOYBN PLT CIRC PATTR
8	66000600	36-8	307	349	2	19	35	2		CORN
9	66000600	6-11	401	421	2	111	199	2		CORN
10	66000600	12-9	589	643	2	3	43	2		CORN DIFF VARIETIES
11	66000600	31-11	327	335	2	109	197	2		OATS
12	66000600	6-2	365	377	2	131	183	2		OATS DITCH W END
13	66000600	1-11	413	467	2	45	93	2		OATS
14	66000600	7-1	583	605	2	121	193	2		OATS
15	66000600	31-12	285	317	2	109	199	2		WHEAT
16	66000600	6-1	347	353	2	107	205	2		WHEAT
17	66000600	6-1	385	393	2	109	203	2		WHEAT
18	66000600	6-14	459	509	2	167	211	2		WHT 2 VARIETIES
19	66000600	7-2	581	689	2	203	211	2		WHEAT
20	66000600	12-10	649	699	2	3	43	2		WHEAT 2 VAR LODGING
21	66000600	1-1	357	399	2	61	95	2		RED CL HAY
22	66000600	6-10	433	453	2	113	197	2		RED CL HAY
23	66000600	1-6	559	581	2	49	109	2		RED CL PASTURE
24	66000600	6-7	521	561	2	173	215	2		RED CL PASTURE
25	66000600	12-8	589	633	2	49	109	2		RED CL PASTURE
26	66000600	7-29	613	619	2	121	183	2		RD CL DIVERIED ACRES
27	66000600	7-28	629	637	2	123	191	2		RED CL HAY
28	66000600		675	695	2	127	195	2		RED CL
29	66000600	7-24	729	737	2	121	195	2		ALFALFA HAY
30	66000600	7-24	745	757	2	121	195	2		ALFALFA HAY
31	66000600	6-8	525	577	2	119	163	2		RYE

Test Fields

Fields Used from C-1, Run 66000600

Figure 6

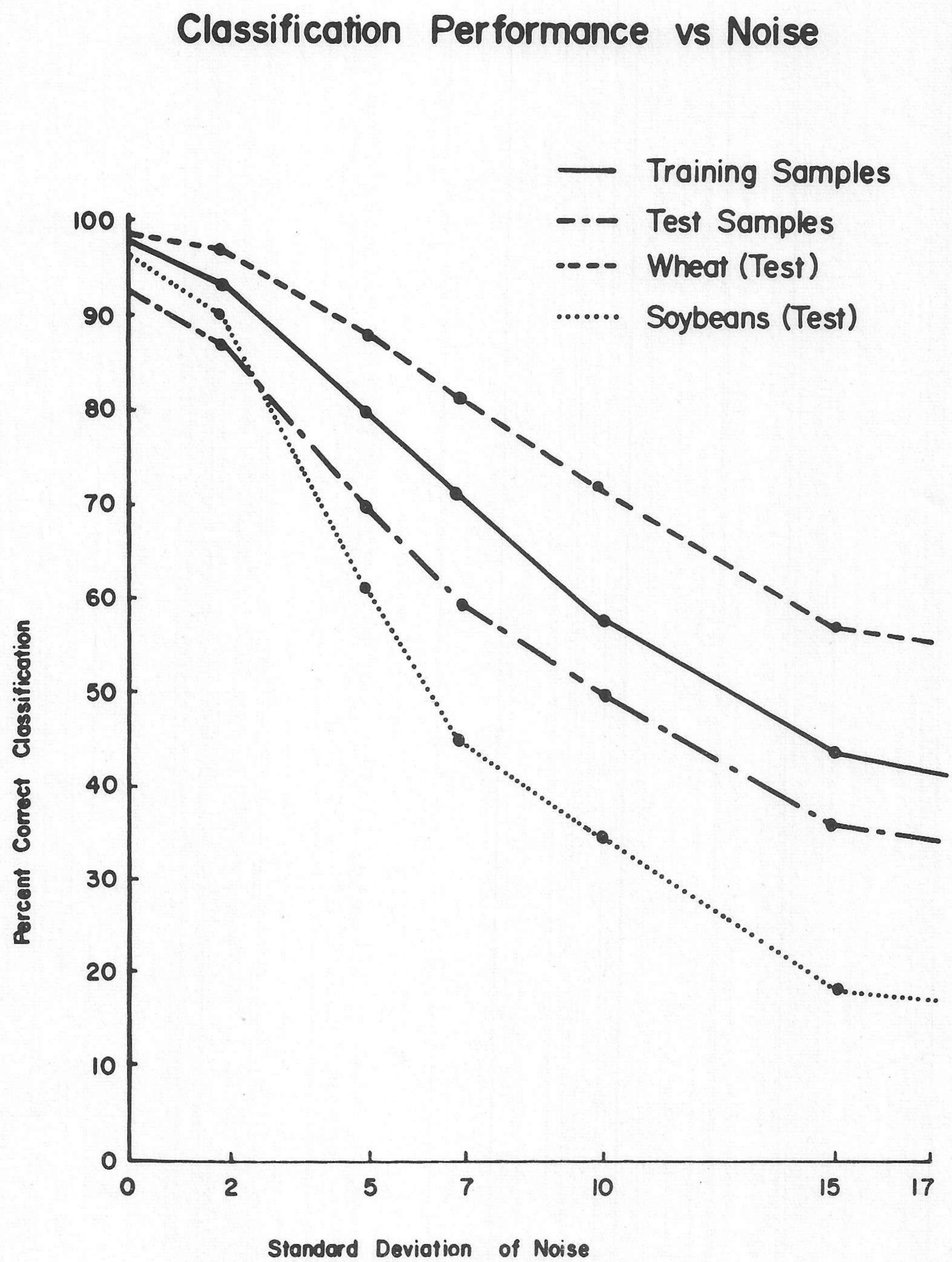
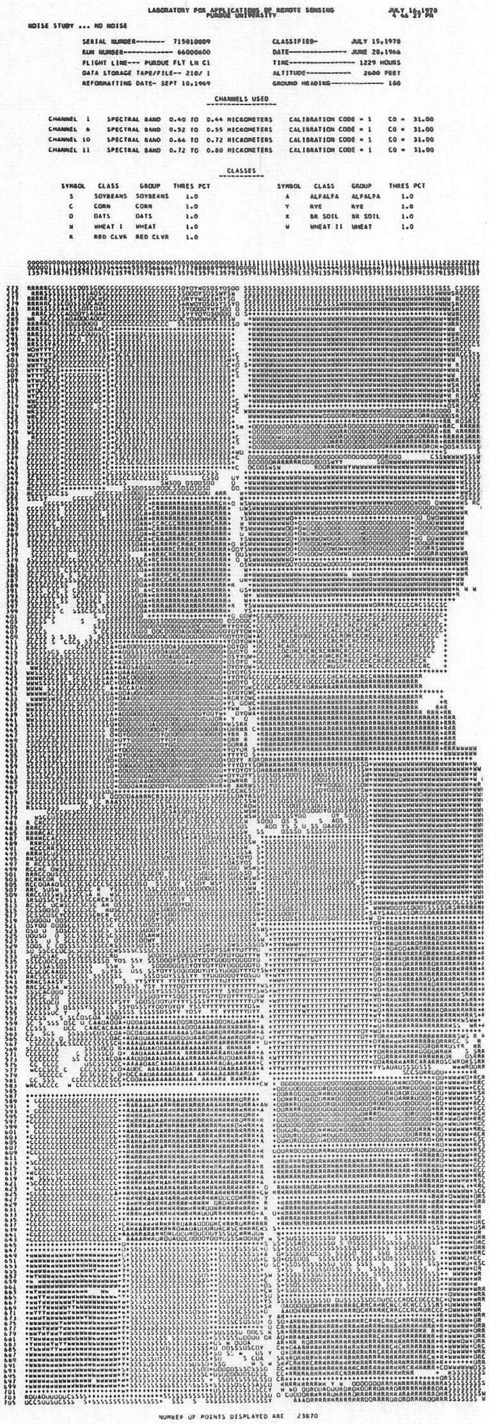
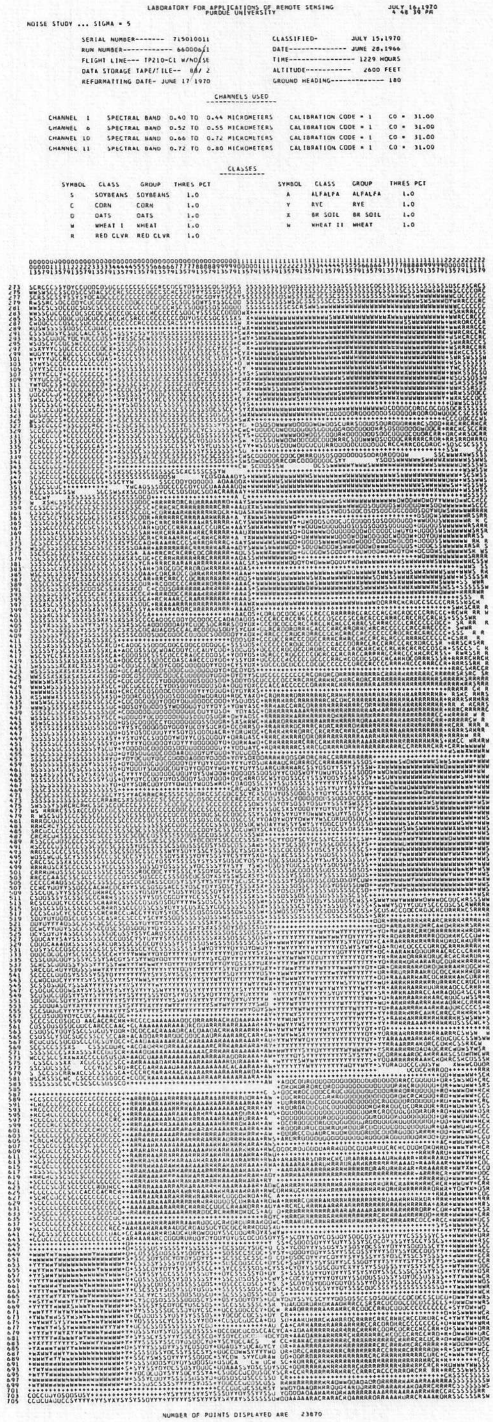

Classification Performance vs Noise
Training Samples
Test Samples
Wheat (Test)
Soybeans (Test)
Percent Correct Classification
100
90
80
70
60
50
40
30
20
10
0
0
2
5
7
10
15
17
Standard Deviation of Noise

Figure 7



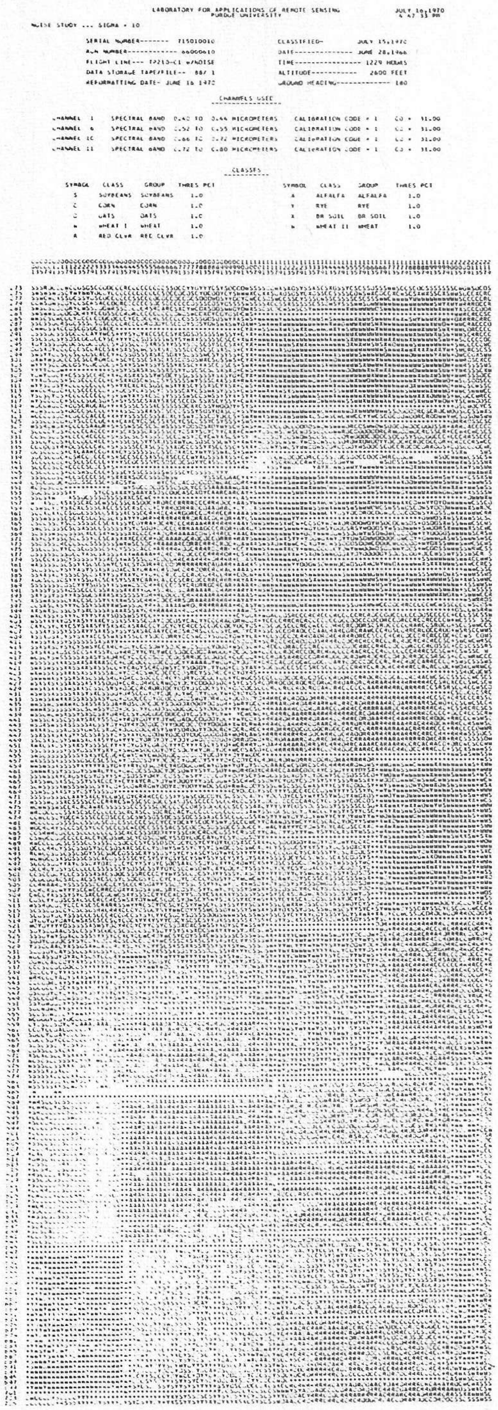
No Added Noise

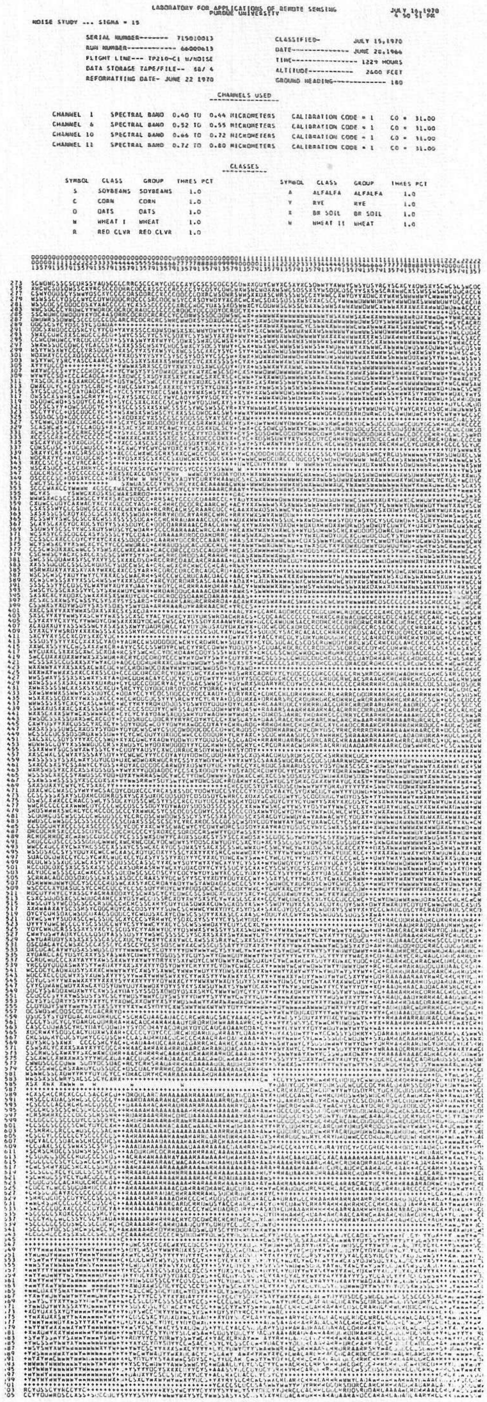


Sigma = 5

Classification Results for C-1
Test Fields are Outlined

Figure 8a



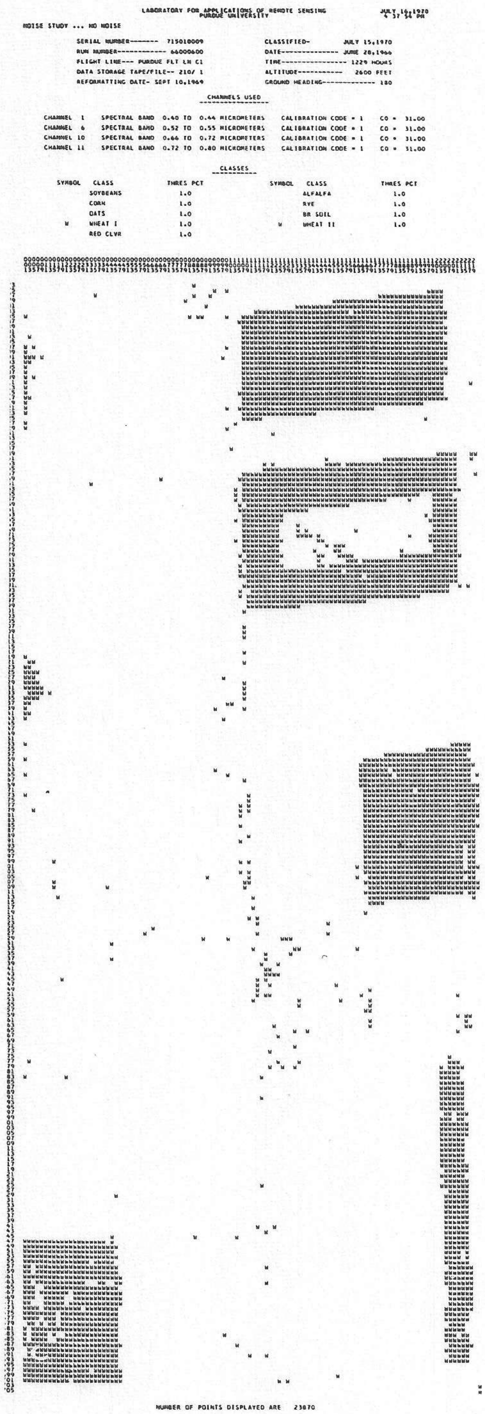


Sigma = 10

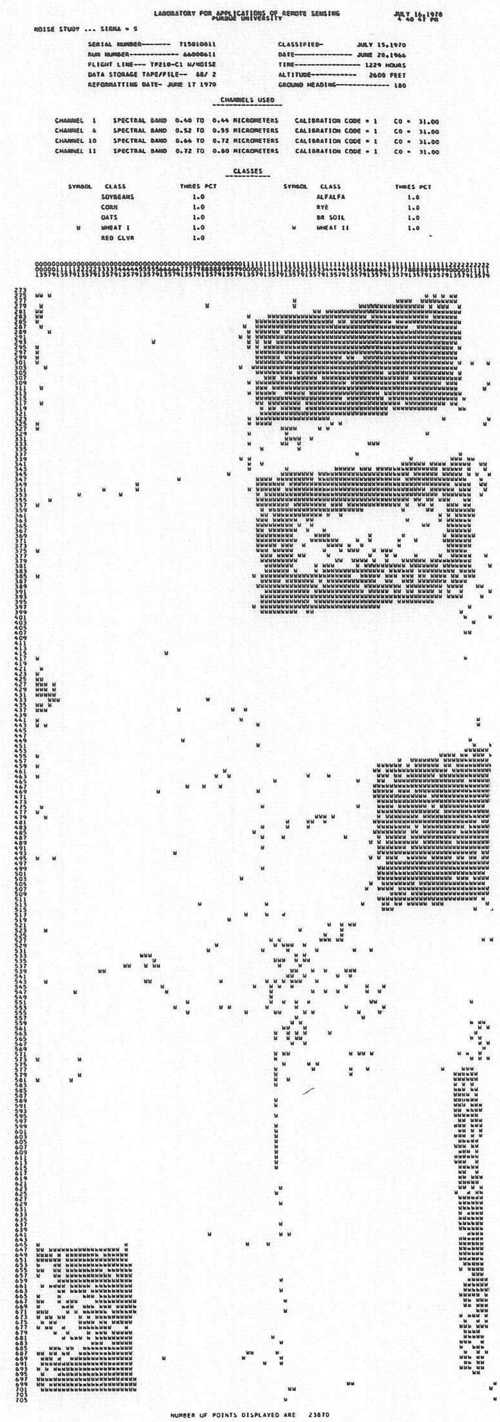
Sigma = 15

Classification Results for C-1
Test Fields are Outlined

Figure 8b



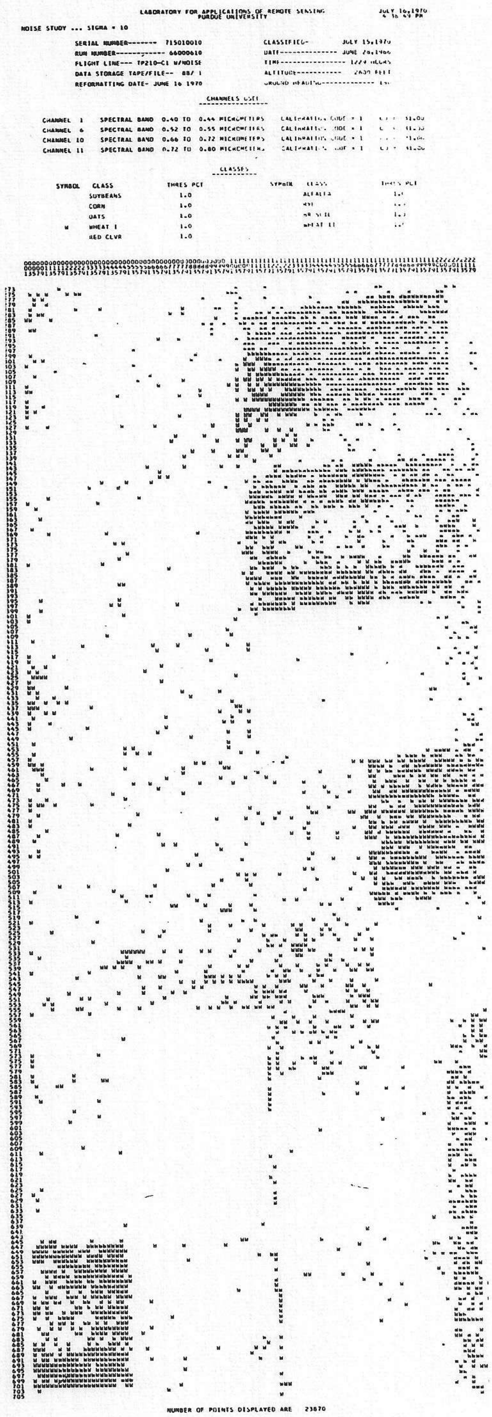
No Added Noise



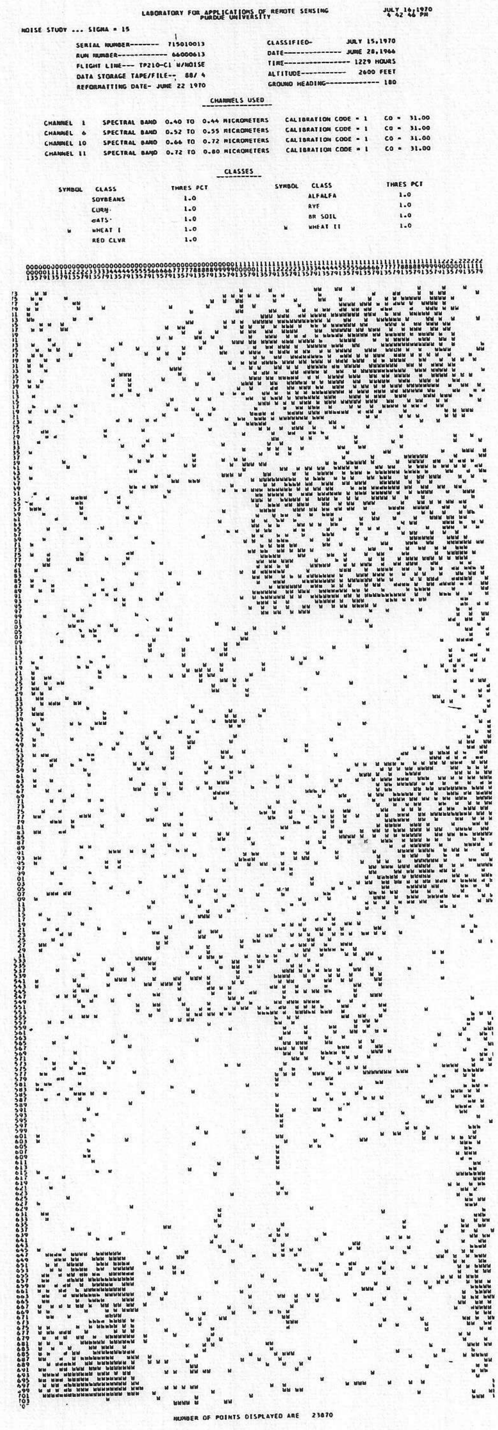
Sigma = 5

Classification Results for C-1
Wheat Only

Figure 9a



Sigma = 10



Sigma = 15

Classification Results for C-1
Wheat Only

Figure 9b

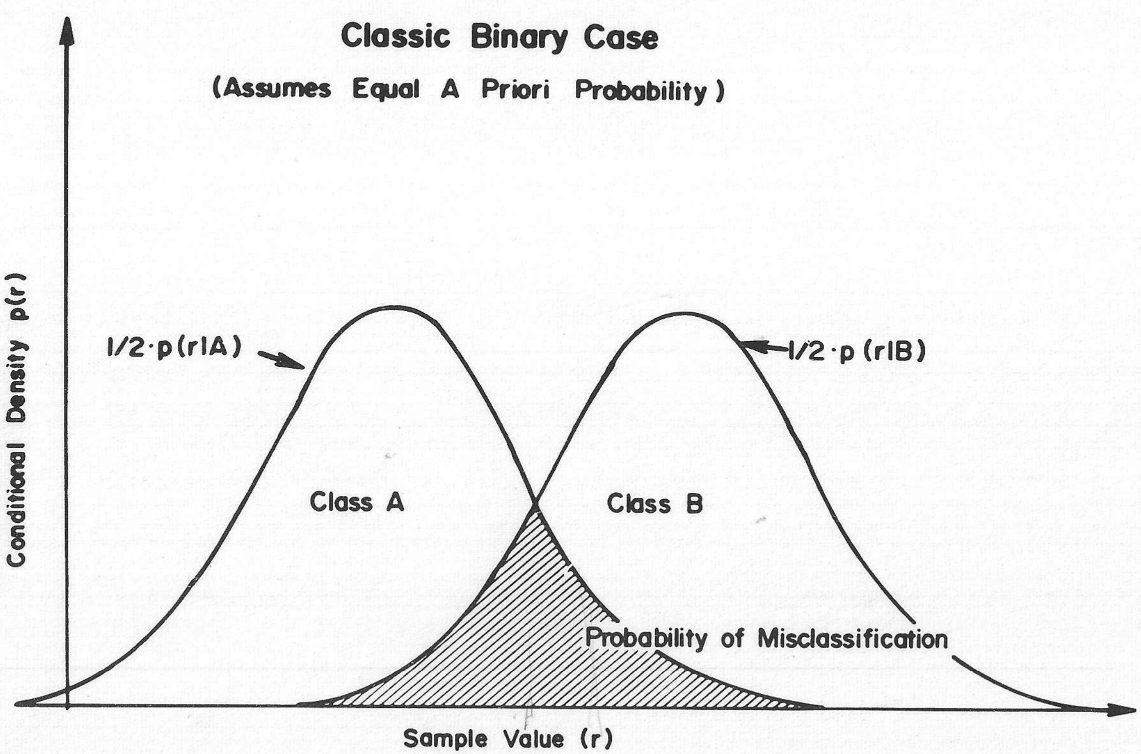

Classic Binary Case
(Assumes Equal A Priori Probability)
Conditional Density p(r)
1/2·p(r|A)
1/2·p(r|B)
Class A
Class B
Probability of Misclassification
Sample Value (r)

Figure 10