LARS Contract Report 112678

Final Report

Vol. II Multispectral Scanner System Parameter Study and Analysis Software System Description

by **B. G. Mobasseri**
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Principal Investigator
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November 1978

Prepared for
National Aeronautics and Space Administration
Johnson Space Center
Earth Observation Division
Houston, Texas 77058
Contract No. NAS9-15466
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Submitted by
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16. Abstract

This report represents the culmination of three major pieces of research effort in defining and evaluating the performance of the next generation of multispectral scanners within the framework of a set of analytic analysis packages. The integration of the available methods provides the analyst with the Unified Scanner Analysis Package (USAP), the flexibility and versatility of which is superior to many previous integrated techniques. USAP consists of three main subsystems, (a) a spatial path, (b) a spectral path and (c) a set of analytic classification accuracy estimators which evaluate the system performance. The spatial path consists of satellite and/or aircraft data, data spatial correlation analyzer, scanner IFOV and random noise model. The output of the spatial path is fed into the Analytic Classification Accuracy Predictor (ACAP). The spectral path consists of laboratory and/or field spectral data, EXOSYS data retrieval, optimum spectral function calculation, data transformation and statistics calculation. The output of the spectral path is fed into the Stratified Posterior Performance Estimator (SPEST). A brief theoretical exposition of the USAP individual building blocks are presented and example outputs produced. References are provided for a more complete coverage of the algorithms. Each building block carries with it at least one software unit. The programming provides a complete input-output compatibility among these units. One test case starting from the raw data base is carried through the system and the performance figures in terms of a set of classification accuracies are produced. A listing of the underlying software is provided in Appendix I.

17. Key Words (Suggested by Author(s)) Multispectral scanner, USAP, spatial path, spectral path, classification accuracy estimators, IFOV model, optimum spectral function calculation, optimum band selection.	18. Distribution Statement

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C1. Multispectral Scanner System Parameter Study and Analysis Software System Description

1. INTRODUCTION

The utilization of sensors on earth orbiting platforms as the main element of an Earth Observational System has undergone substantial growth in recent years. ERTS-1 (Landsat-1) followed by Landsat-2 and -3 have proven exceptionally successful in collecting data to help monitor the Earth's resources.

The principal data collection unit aboard the first three Landsats is the multispectral scanner known as MSS. Although this scanner has been providing data with a quality which exceeded most prelaunch expectations, it has been clear from the beginning that MSS does not represent the ultimate in multispectral instruments; more advanced instruments providing greater detail would be needed as the user community begins to become familiar with the use of such space data.

The design of a multispectral scanner is a very complex matter; many different, interacting factors must be properly taken into account. Currently operational systems such as MSS have been designed primarily using subjective judgements based upon experience with experimental data. In designing a scanner the use of empirical methods, at least in part, is essential. Each of the large collections of scenes which a given scanner will be used upon is a very complex information source; not enough is known to make a simple (or even a complex) model of it by which to make the design of a scanner a simple straightforward exercise of a mathematical procedure.

And yet, more is known than when MSS was designed, and it is important to be able to carry out future designs on a more objective basis than in the past. Thus the purpose of the present work is the development of appropriate mathematical design machinery within a theoretical framework to allow: (a) formulation of an optimum multispectral scanner

* The work in this report was done under Task 2.2C1 Multisensor Parametric Evaluation and Radiometric Correction Model.

system according to defined conditions of optimality and (b) an ability for convenient manipulation of candidate system parameters so as to permit comparison of the theoretically optimum designs with that of practical approximations to it.

In order to deal with the complexity of the design situation, the first step is to determine a suitable set of parameters which adequately characterize it but is not so large as to be unmanageable. It has been observed [1] that there are five major categories of parameters which are significant to the representation of information in remotely sensed data. They are:

1. The spatial sampling scheme
2. The spectral sampling scheme
3. The signal-to-noise ratio
4. The ancillary data type and amount
5. The informational classes desired

Thus, it is necessary to have present in the design machinery, some means for evaluating the impact of change in parameter values in each of these five categories.

Such a scanner design tool has been assembled in the form of a software package for a general purpose computer. Each of the parts of this package, called Unified Scanner Analysis Package (USAP) has been carefully devised and the theory related to it fully documented [2, 3, 4, 5]. The goal of this report is to provide a documentation and description of the software. In constructing this documentation it was assumed that this package will be useful for sometime into the future, however it was also assumed that it will only be used by a small number of highly knowledgeable scientists.

Section 2 recaps the theoretical concepts behind some of the primary components of USAP. These are divided into (a) scanner spatial characteristics modeling and noise effects, (b) optimum spectral basis function calculations, (c) analytical classification accuracy predictions

(d) stratified posterior classification estimation and (e) an information theory approach to band selection. Although (e) is not a part of the USAP system, the results from this approach are helpful in understanding the scanner design problem.

Section 3 shows the integration of the above modules into the software system. Section 4 is the user's guide to USAP describing the required inputs and the available output products. A listing of all programs is provided in the appendix.

The work which led to USAP was immediately preceded by a simulation study of possible parametric values for the Thematic Mapper, a new scanner now being constructed for launch on Landsat-D in 1981. The purpose of this simulation was to compare the performance for several proposed sets of parameters. We will conclude this introductory section by briefly describing this work because it provides useful background and serves well to illustrate the problem. A more complete description of this simulation study is contained in [1, 6].

The general scheme used was to simulate the desired spaceborne scanner parameter sets by linearly combining pixels and bands from (higher resolution) airborne scanner data to form simulated pixels, adding noise as needed to simulate the desired S/N; the data so constructed was then classified using a Gaussian maximum likelihood classifier and the performance measured. The problem was viewed as a search of the five dimensional parameter space defined above with the study localized around the proposed Thematic Mapper parameters. The scope of the investigation was primarily limited to three parameters (a) spatial resolution, (b) noise level and (c) spectral bands. Probability of correct classification and per cent correct area proportion estimation for each class were the performance criteria used. The major conclusions from the study are as follows:

1. There was a very small but consistent increase in identification accuracy as the IFOV was enlarged. This is presumed to stem primarily from the small increase in signal-to-noise ratio with increasing IFOV, Figure 1.
2. There was a more significant decrease in the mensuration accuracy as the IFOV was enlarged, Figure 2.
3. The noise parameter study proved somewhat inconclusive due to the greater amount of noise present in the original data than desired. For example, viewing Figure 3 moving from right to left, it is seen that the classification performance continues to improve as the amount of noise added is decreased until the point is reached where the noise added approximately equals that already initially present.* Thus, it is difficult to say for what signal-to-noise ratio a point of diminishing return would have been reached had the initial noise not been present.
4. The result of the spectral band classification studies may also be clouded by the noise originally present in the data. The relative amount of that change in performance due to using different combinations of the .45-.52 μm, .74-.80 μm, .80-.91 μm and .74-.91 μm bands is slight but there appears to be a slight preference for the .45-.52 μm band. The performance improvement of the Thematic Mapper channels over those approximating Landsat-1 and -2 is clear however.
5. Using spectrometer data it was verified that the .74-.80 μm and .80-.91 μm bands are highly correlated.
6. Correlation studies also showed that the range from 1.0-1.3 μm is likely to be an important area in discriminating between earth surface features. Further, it is noted that the absolute calibration procedure described above results in a global atmosphere correction of a linear type in that assuming a uniform atmosphere over the test site, the calibration procedure permits a digital count number at the airborne scanner output to be related directly to the present reflectance of a scene element.

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* The noise level in the original A/C data was equivalent to about .005 $NE\Delta\rho$ on the abscissa. See Reference [1].

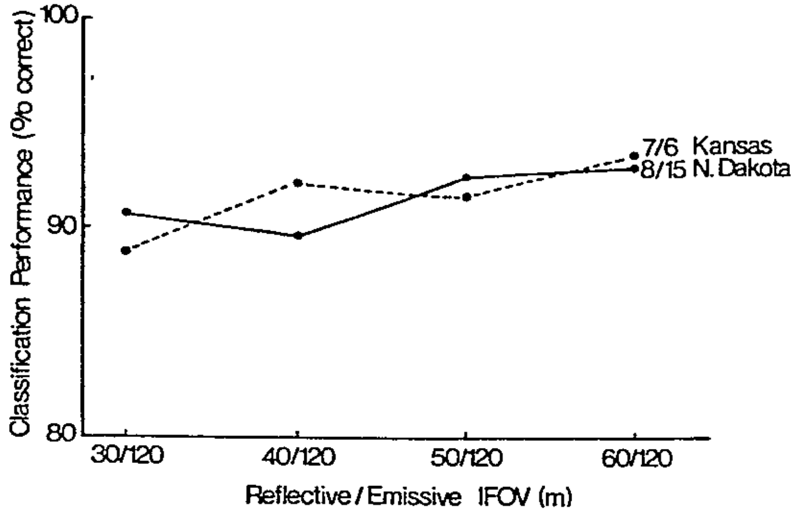


Figure 1. Classification Performance vs. Spatial Resolution.

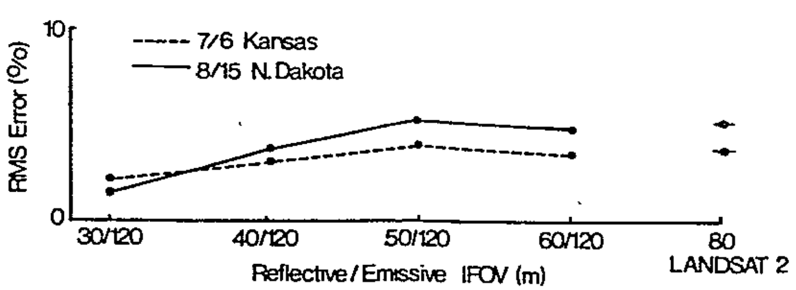
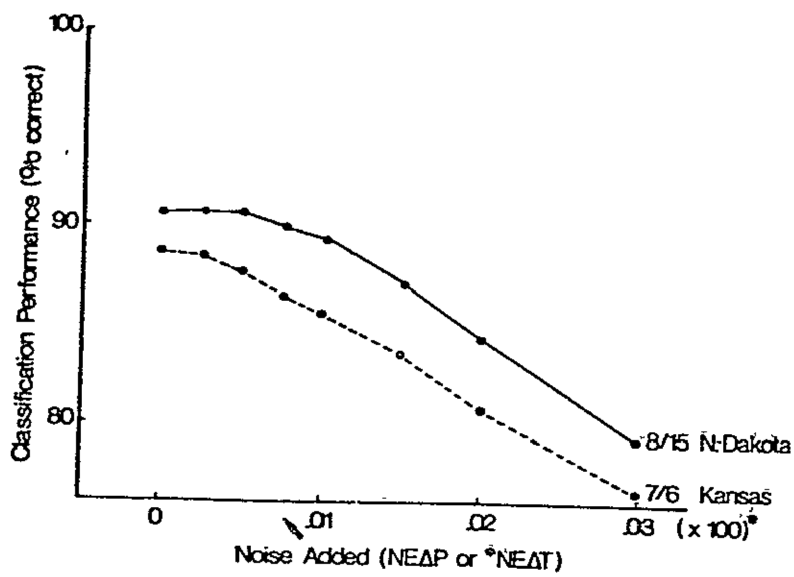


Figure 2. RMS Error of Proportion Estimates vs. Spatial Resolution.


100
90
80
Classification Performance (% correct)
0
.01
.02
.03
(x 100)*
Noise Added (NEΔP or *NEΔT)
8/15 N.Dakota
7/6 Kansas

2. SCANNER PARAMETERS ANALYSIS TECHNIQUES

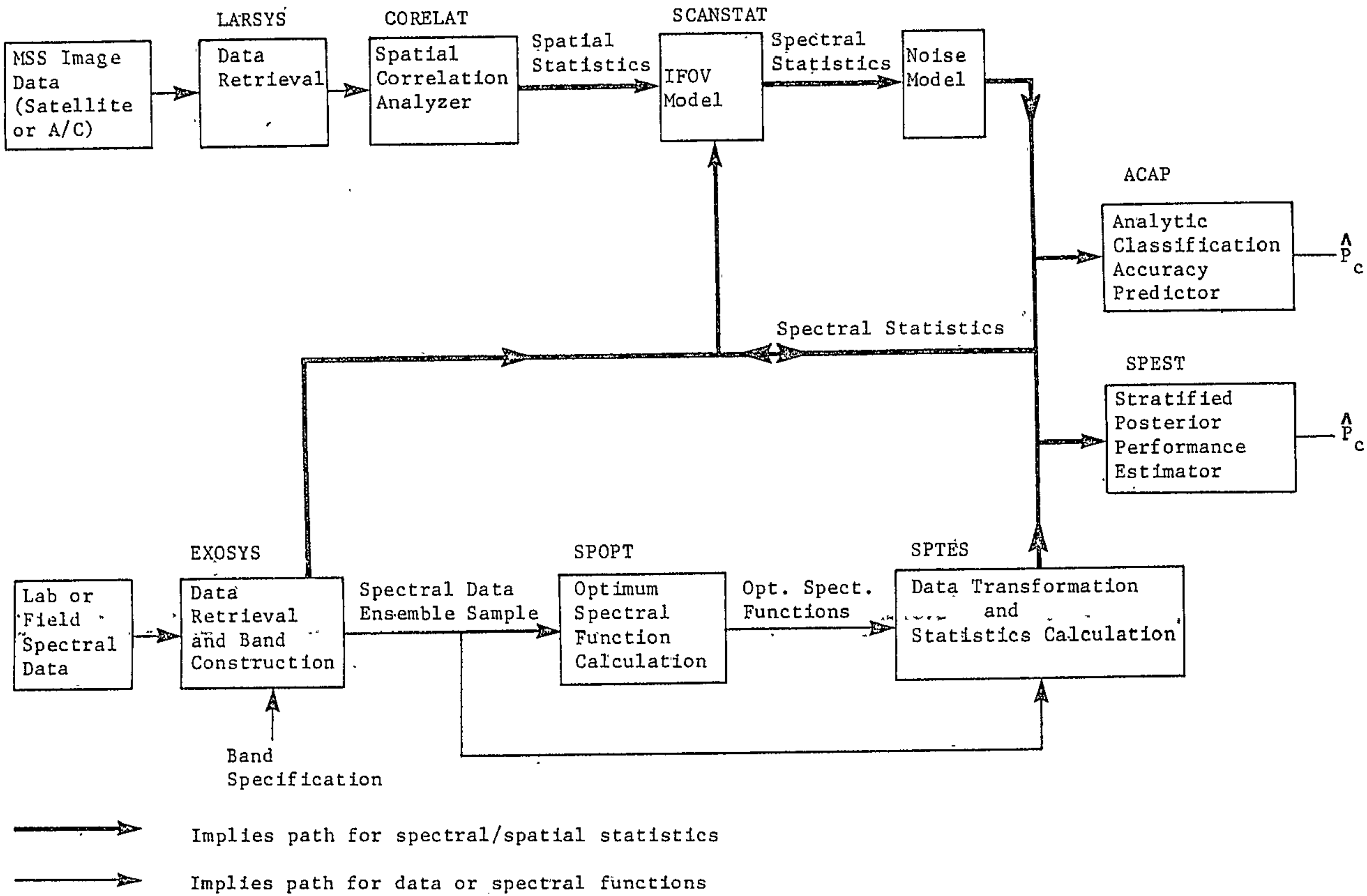
Based upon the parametric approach introduced above, the development of a parametric scanner model must give explicit concern for the spatial, spectral and noise characteristics of the systems. This is what has been done in the Unified Scanner Analysis Package (USAP) shown in Figure 4. USAP is composed of two distinct subsystems. The spatial aspect of it contains (a) a data spatial correlation analyzer, (b) a scanner IFOV model and (c) a random noise model. The spectral techniques are capable of producing an optimum spectral representation by modeling the scene as a random process as a function of wavelength, followed by the determination of optimum generalized spectral basis functions. Conventional spectral bands can also be generated. Also studied was an information theory approach using maximization of the mutual information between the reflected and received (noisy) energy. The effect of noise in the data can be simulated in the spectral and spatial characteristics. Two different data bases are used in the system. The spectral techniques require field spectral data while the spatial techniques require MSS generated data, aircraft and/or satellite. The system performance, defined in terms of the classification accuracy, is evaluated by two parametric algorithms. A detailed system description and user's guide is presented in Sections 3 and 4. In the following, the theoretical ideas behind the five major elements of USAP are discussed.

2.1 Analytical Classification Accuracy Prediction

Throughout the analysis of remotely sensed data, the probability of correct classification has ranked high among the set of performance indices available to the analyst. This is particularly true in a scanner system modeling where generally the optimization of various system parameters has as its prime objective the maximization of the classification accuracy of various classes present in the data set.

The estimation of the classification accuracy is fairly straightforward if Monte-Carlo type methods are employed. In system simulation and modeling however, such approaches are generally a handicap due to their

Figure 4. Block Diagram of the Unified Scanner Analysis Package (USAP).



heavy dependence on an experimental data base, the availability of which can be limited due to a variety of reasons. What is required, therefore, is a parametric classification accuracy estimator for a multiclass, multidimensional Gaussian Bayes classifier. This procedure should require the class statistics, mean vectors and covariance matrices, as its only input and produce a set of probabilities of correct classification. This technique has been developed, tested, implemented and comprehensively reported [2]. The following is a summary of the method and some results.

The probability of Error as an N-Tuple Integral.

The classification of a multidimensional observation vector into one of M populations is conceptually identical to the binary case. Let Ω, M and N be the feature space, number of classes and the dimensionality of Ω respectively. The procedure is to divide Ω into M mutually disjoint sets, Γ_i, and to assign each feature vector to a set in accordance with an appropriate rule. This is illustrated in Figure 5. Let Z_i, i = 1, 2, ..., M partition Ω in R^N. The Bayes risk is defined as

$$R = \sum_{i=1}^{M} \int_{Z_i} \sum_{J=1}^{M} P(\omega_j)\ C_{ij}\ f(\underline{X}|\omega_j)\, d\underline{X} \tag{1}$$

where C_{ij} is the cost of deciding ω_i where ω_j is true. In the case where $C_{ij}=0$ for $i=j$ and $C_{ij}=1$ for $i\neq j$. R is the probability of error.

Among all possible choices of Z_i the Bayes rule partitions Ω into $Z_i=Z_i{}^*$ such that $R=R^*$ is the the minimum probability of error. Assuming that the population statistics follows a multivariate normal law, the optimum Bayes rule is as follows [7].

$$\underline{X} \in \omega_i \text{ if } W_i < W_j\ \forall i \neq j = 1, 2, \ldots, M$$

where

$$W_i = (\underline{X} - \underline{\mu}_i)^T \underline{\Sigma}_i^{-1} (\underline{X} - \underline{\mu}_i) + \ln|\underline{\Sigma}_i| - 2\ln P(\omega_i)$$

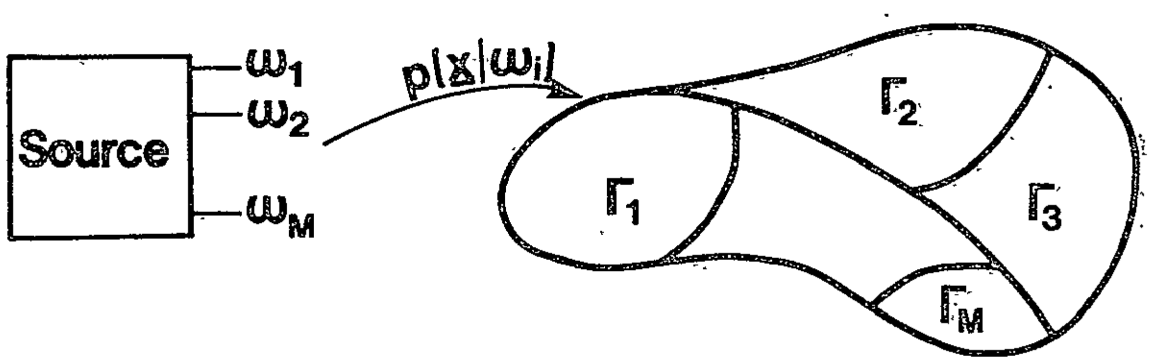


Figure 5. Allocation of a Measurement Vector $\underline{X}$ to an Appropriate Partition of the Feature Space.

with

$$\begin{aligned} \underline{X} &= \text{observation vector} \\ \underline{\mu}_i &= \text{mean vector for class } \omega_i \\ \underline{\Sigma}_i &= \text{covariance matrix for class } \omega_i \\ P(\omega_i) &= \text{apriori probability for } \omega_i \end{aligned} \tag{2}$$

The error estimate based on direct evaluation of Eq. (1) exhibits all the desired properties outlined previously.

The evaluation of multiple integrals bears little resemblance to their one dimensional counterparts, mainly due to the vastly different domains of integration. Whereas there are three distinct regions in one dimension; finite, singly infinite, and double infinite; in an N dimensional space there can potentially be an infinite variation of domains. The established one dimensional integration techniques, therefore, do not carry over to N dimensions in general. Hence, it is not surprising that no systematic technique exists for the evaluation of multivariate integrals except for the case of special integrands and domains [8]. The major complicating factor is the decision boundaries defined by Eq. (2). Γ_i is defined by a set of intersecting hyperquadratics. Any attempt to solve for the coordinates of intersection and their use as the integration limits will be frustrated if not due to the cumbersome mathematics, because of impractically complicated results.

In order to alleviate the need for the precise knowledge of boundary locations and reduce the dimensionality of the integral, a coordinate transformation followed by a feature space sampling technique is adopted. The purpose of the initial orthogonal transformation of the coordinates is an N to 1 dimensionality reduction such that the N-tuple integral is reduced to a product of N one dimensional integrals. Let the conditional classification accuracy estimate, $\hat{P}_{c|\omega_j}$ be the desired quantity. Then the transformed class ω_j statistics is given by

$$\underline{\mu}_j' = \underline{\mu}_j - \underline{\mu}_i$$
$$\underline{M}_j = \underline{\phi}_i^T \mu_j' \qquad j = 1, 2, \ldots, M \tag{3}$$
$$\underline{S}_j = \phi^T \underline{\Sigma}_j \underline{\phi}$$

where ϕ_j is the eigenvector matrix derived from $\underline{\Sigma}_i$. Naturally, in each transformed space, $T_i(\Omega)$, ω_i has a null mean vector and a diagonal covariance matrix.

The discrete feature space approach is capable of eliminating the need for the simultaneous solution of M quadratic forms. If Ω is the continuous probability space, a transformation T_i is required such that in $T_i(\Omega)$, Γ_i can be completely described in a nonparametric form, thereby bypassing the requirement for an algebraic representation of Γ_i. This desired transformation would sample Ω into a grid of N-dimensional hypercubes. Since the multispectral data is generally modeled by a multivariate normal random process, a discrete equivalent of normal random variables that would exhibit desirable limiting properties is required. Let y_n~Bi (n, p) be a binomial random variable with parameters n and p. The x_n defined by

$$x_n = \frac{y_n - np}{\sqrt{np(1-p)}} \qquad y_n = 0, 1, 2, \ldots, n \tag{4}$$

converges to x~N(0, 1) in distribution [9], i.e.,

$$\lim F_n\,(X) \longrightarrow F(x)$$

The convergence is most rapid for $p=\frac{1}{2}$, then

$$x_n = \frac{(y_n - n/2)2}{\sqrt{n}} \tag{5}$$

The variance of x_n is set equal to the eigenvalues of the transformed $\underline{\Sigma}_i$ by incorporating a multiplicative factor in Eq. (5).

The segmentation of Ω by a union of elementary hypercubes makes nonparametric representation of Γ_i and its contours feasible. Following the orthonormal transformation on ω_i and sampling of Ω accordingly, each cell's coordinate is assigned to an appropriate partition of Γ. This process is carried out exhaustively, therefore Γ_i can be defined as a set such that

$$\Gamma_i = \{U\underline{X}_n : \underline{X}_n \varepsilon \Gamma_i\} \tag{6}$$

once the exhaustive process of assignment is completed, the integral of $f(\underline{X}|\omega_i)$ over Γ_i is represented by the sum of hypervolumes over the elementary cells within Γ_i. The elementary unit of probability is given by

$$\int_{C_i} f\ (\underline{X}|\omega_i) = \int_{-\frac{\delta_1}{2}}^{\frac{\delta_1}{2}} f\ (x_1|\omega_i)\ dx_1 \int_{-\frac{\delta_2}{2}}^{\frac{\delta_2}{2}} f\ (x_2|\omega_i)\ dx_2 \text{ ---- } \int_{-\frac{\delta_N}{2}}^{\frac{\delta_N}{2}} f\ (x_N|\omega_i)\ dx_N \tag{7}$$

where C_i is the domain of a sampling cell centered at the origin and δ_i is the width of a cell along the ith feature axis. The conditional probability of correct classification is therefore given by

$$\hat{P}_c|\omega_i = \sum_{C\varepsilon\Omega} \int_{c_1-\frac{\delta_1}{2}}^{c_1+\frac{\delta_1}{2}} f\ (x_1|\omega_i)\ I_i\ (C)\ dx_1 \int_{c_2-\frac{\delta_2}{2}}^{c_2+\frac{\delta_2}{2}} f\ (x_2|\omega_i)\ I_i\ (C)\ dx_2 \tag{8}$$

$$\ldots \int_{c_n-\frac{\delta_n}{2}}^{c_n+\frac{\delta_n}{2}} f\ (x_N|\omega_i)\ I_i\ (C)\ dx_N$$

with overall classification accuracy given by

$$\hat{P}_c = \sum_{i=1}^{M} P(\omega_i)\, \hat{P}_{c|\omega_i} \tag{9}$$

where

$$I_i(C) = \begin{cases} 1 \text{ if } C\varepsilon\Gamma_i \\ 0 \text{ otherwise.} \end{cases} \tag{10}$$

C = The domain of an elementary cell

Figure 6 is a geometrical representation of Eq. (8).

Experimental Results.

The analytic classification accuracy prediction (ACAP) has been thoroughly tested and documented [2]. Two examples are repeated here. The first experiment investigates the performance of the estimator vs. grid size i.e., number of cells per axis, n. Small to moderate range of n is required if computation time is to remain realistic. Figure 7 shows the variation of $\hat{P}_{c|\omega_i}$ vs. n for three classes having some hypothetical statistics in 3 dimensions. The main property of the estimator is its rapid convergence toward a steady state value thereby alleviating the need for excessively fine grids and hence high computation costs.

The data collected over Graham County, Kansas is used to perform a comparison between the ACAP algorithm and a ratio estimator such as LARSYS. The results are tabulated in Table 1.

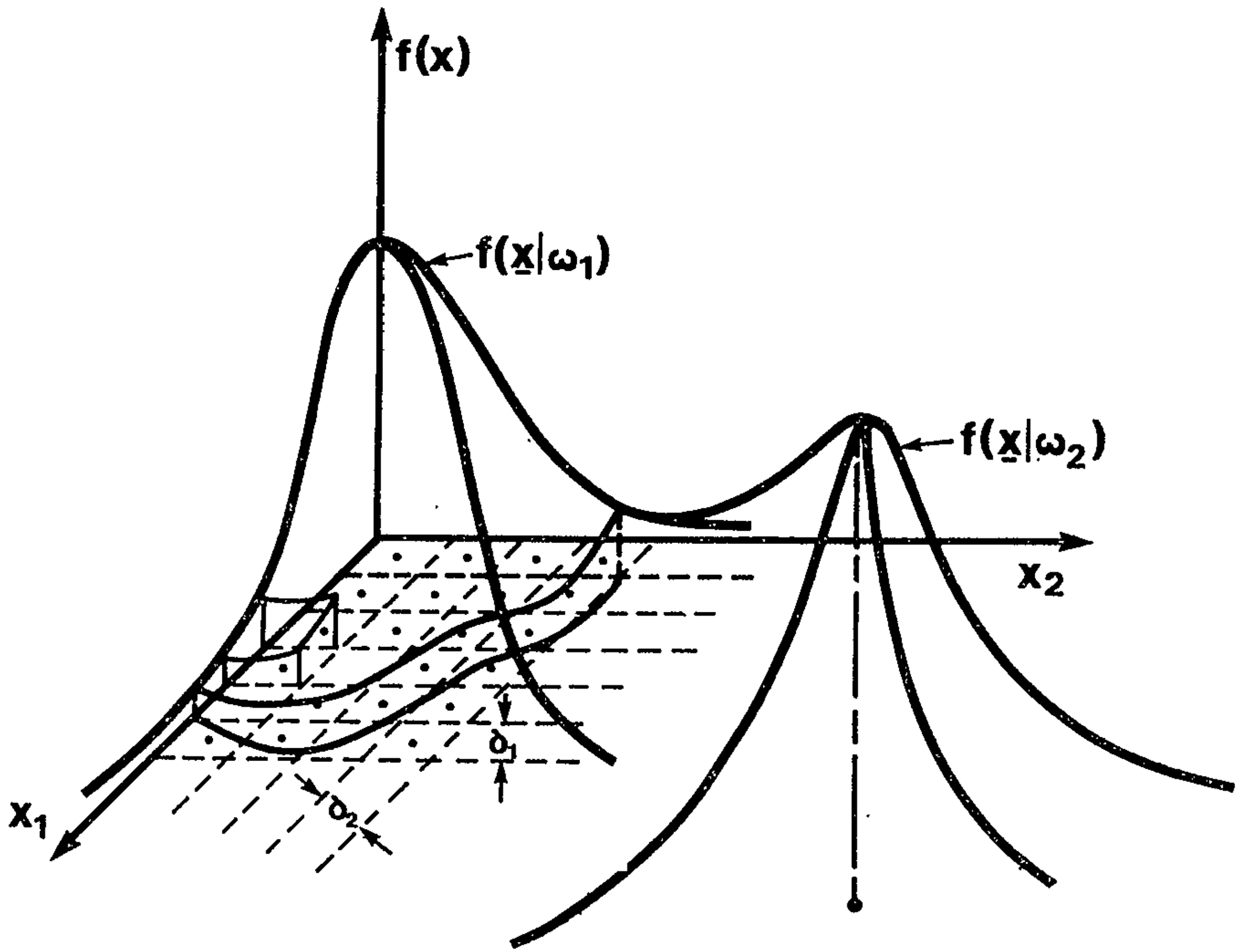


Figure 6. A Conceptual Illustration of the ACAP Error Estimation Technique Using Two Features.

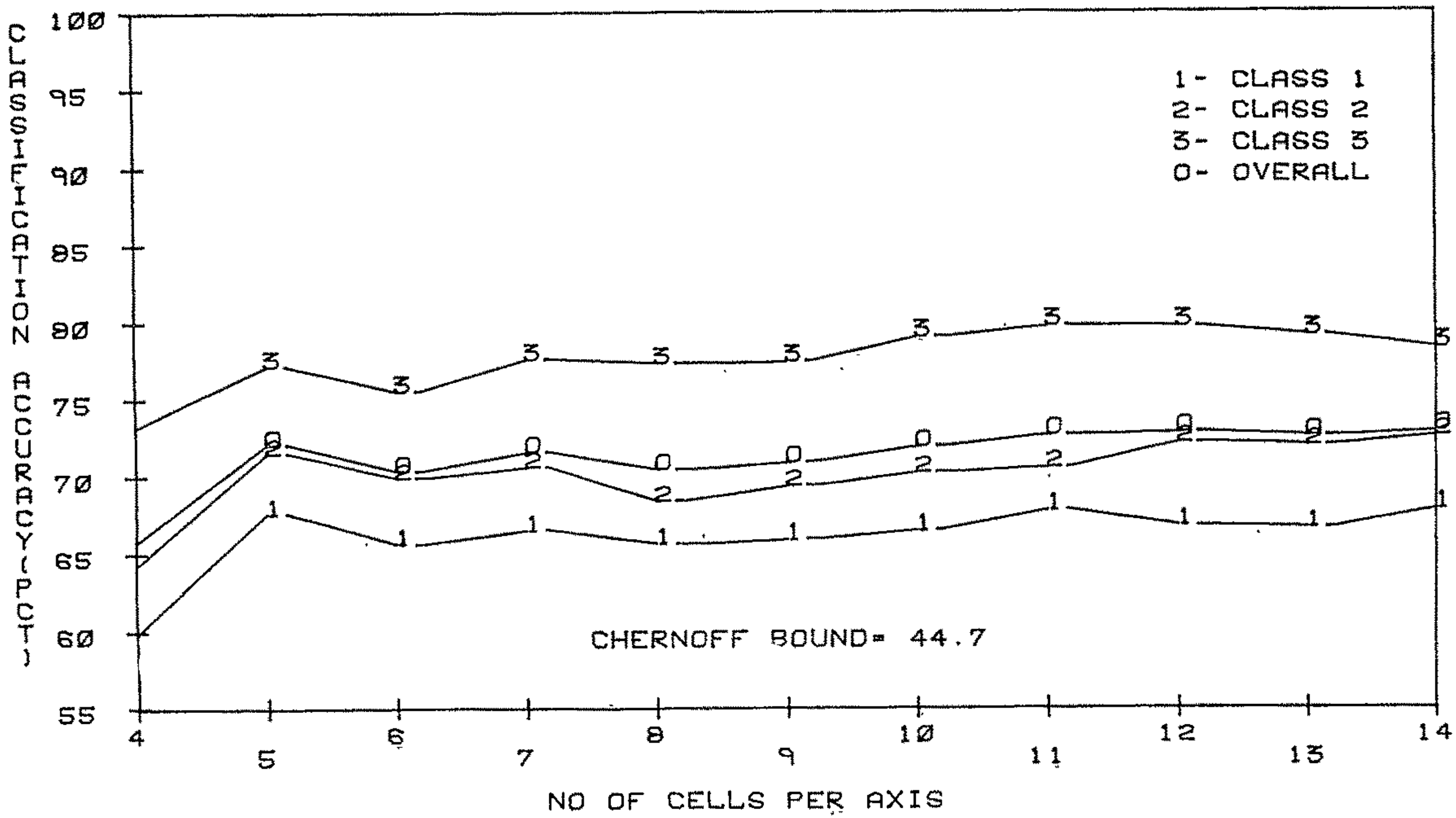


Figure 7. ACAP Classification Accuracy Estimate vs. Grid Size for the Test Population Data.

Table 1. ACAP and LARSYS Performance Comparison.

Class	No. of Points	LARSYS	ACAP
Bare Soil	443	65.9	78.3
Corn/Sorghum	99	89.9	91.0
Pasture	1376	98.4	95.0
Wheat	459	94.8	93.9
Overall	2377	87.2	89.6

The comparison of ACAP and LARSYS results from Table 1 is inconclusive. In some cases the difference is negligible (corn) and in some, significant (bare soil). Examination of the data statistics revealed that the assumption of normality does not hold throughout the populations statistics. This problem can be rectified by simulating random Gaussian data having identical statistics with the real data, hence assuring the normality assumption. Repeating the LARSYS and ACAP procedures produces a new set of classification accuracies, Table 2.

Table 2. ACAP and LARSYS Performance Comparison Simulated Data.

Class	LARSYS%	ACAP%	Accuracy Difference%
Bare Soil	77.8	78.3	0.5
Corn	91.2	91.0	0.2
Pasture	95.3	95.1	0.2
Wheat	94.2	93.9	0.2
Overall	89.6	89.6	0

The differential between ACAP and LARSYS results has been narrowed considerably, ranging from a high of 0.5% for bare soil to 0% for the overall classification accuracy. Two conclusions can be drawn from the results of this experiment. First, the ACAP and Monte Carlo type classifiers produce practically identical results if the underlying assumptions are satisfied (e.g., normality of the statistics). Second and more revealing is the fact that the results of the ACAP processor indicated an upper bound for the classifability of bare soil had its statistics been Gaussian. This result is a direct property of ACAP's data independence. Figure 8 is the ACAP estimator vs. n for Graham Co. data.

The above selected experiments and others reported in the bibliography establishes ACAP as a viable and necessary tool in any analytical remote sensing data collection system modeling and simulation when the performance index is defined as the probability of correct classification.

2.2 Stratified Posterior Classification Performance Estimator.

The second classification accuracy estimator to be presented here (SPEST) is based on the maximization of the aposteriori probability associated with each sample. This formulation is closely related to the maximum likelihood principle used in the ACAP. The distinction arises in the determination of integration domains. Where in ACAP a "deterministic" grid was set up to sample the feature space, SPEST uses an internally generated random data base and assigns the feature vector to the appropriate class via the maximum aposteriori principle. Due to the different approaches adopted, the statistical properties of the estimators could be substantially different although no major study has yet been carried out. It has been observed however, that the SPEST algorithm is somewhat faster than the ACAP in selected cases. The approach here is similar to that described in Moore, Whitsitt, and Landgrebe [10].

Let X be an observation from one of M classes ω_i, $i = 1, 2, 3, \ldots, M$, with a priori probabilities P_i. The maximum likelihood decision rule can be stated as follows: Assign X to the class ω_k if

$$P(\omega_k|X) = \max_i \{P(\omega_i|X)\}$$

This rule partitions the observation space Ω into subregions $\Gamma_1, \Gamma_2, \ldots, \Gamma_M$, corresponding to the classes $\omega_1, \omega_2, \ldots, \omega_M$, respectively. Define the indicator function as

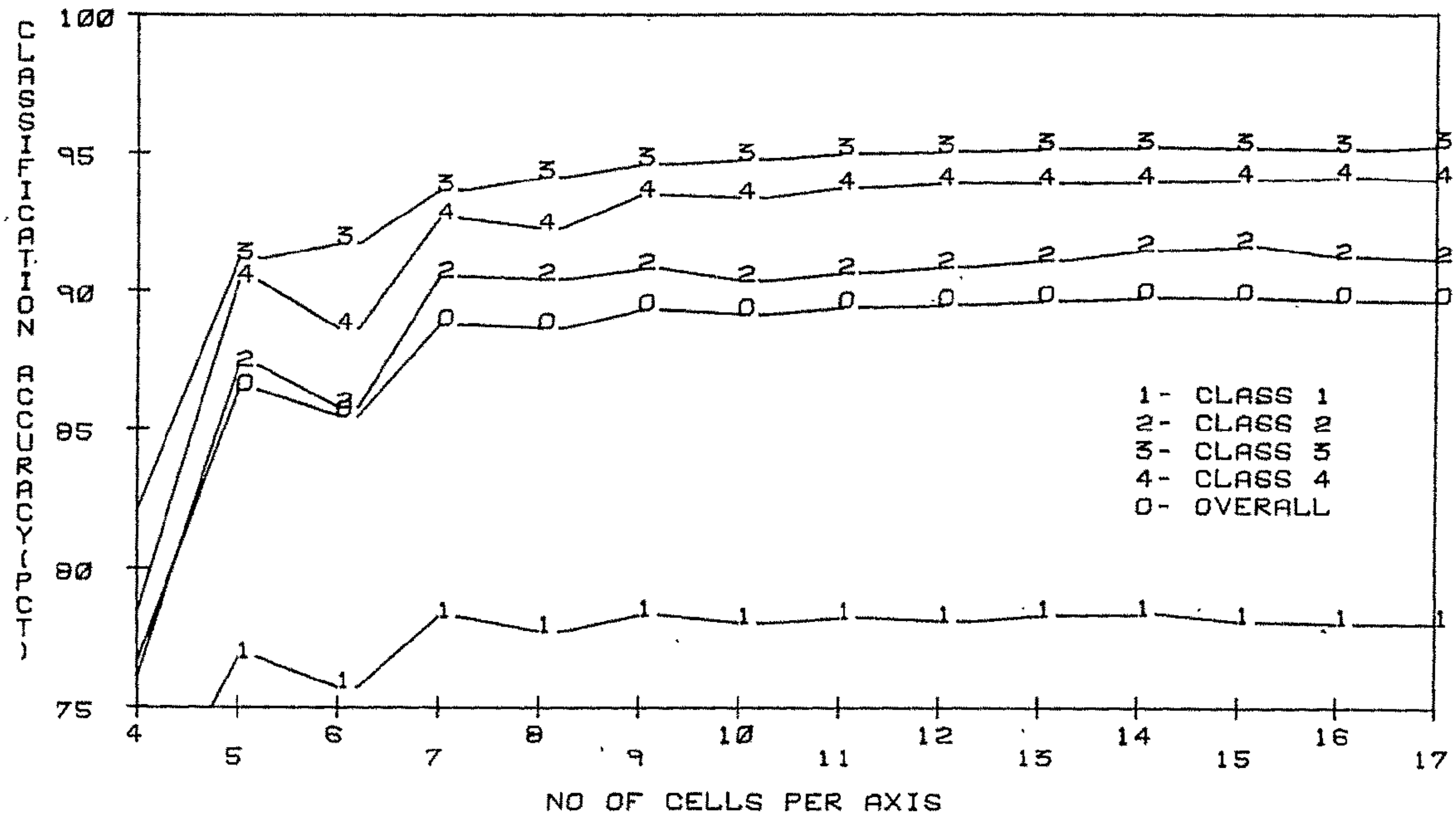


Figure 8. ACAP Classification Accuracy Estimate vs. Grid Size for Graham Co., Kansas Data.

$$I_i(x) = \begin{cases} 1 & x\varepsilon\Omega_i \\ 0 & x\not\varepsilon\Omega_i \end{cases}$$

The probability of correct classification integral is given by

$$P_c = \int_\Omega \sum_{i=1}^{M} P_i \; I_i(x) \; p(x|\omega_i) \; dx \tag{11}$$

It is desirable to evaluate the probability of correct classification for each class as well as the overall probability. The performance probability for the ith class is

$$P_{ci} = \int_\Omega I_i(x) \; p(x|\omega_i) \tag{12}$$

The overall performance, then, is

$$P_c = \sum_{i=1}^{M} P_i P_{ci} \tag{13}$$

From Bayes' rule

$$p(x|\omega_i) = \frac{P(\omega_i|x) \; p(x)}{P_i}$$

hence,

$$P_{ci} = \int_\Omega I_i(x) \; \frac{P(\omega_i|x)}{P_i} \; p(x) \; dx$$

p(x) is a mixture density

$$p(x) = \sum_{j=1}^{M} P_j \; p(x|\omega_j)$$

Therefore,

$$P_{ci} = \sum_{j=1}^{M} \frac{P_j}{P_i} \int_{\Omega} I_i(x)\ P(\omega_i|x)\ p(x|\omega_j)\ dx \tag{14}$$

Define

$$Q_i(x) = I_i(x)\ P(\omega_i|x)$$

Then

$$\int_{\Omega} Q_i(x)\ p(x|\omega_j)\ dx$$

is the conditional expected value of $Q_i(x)$ given that X comes from the class C_j. The estimate

$$\hat{P}_{ci} = \sum_{j=1}^{M} \frac{P_j}{P_i} \frac{1}{N_j} \sum_{k=1}^{N_j} Q_i(x_k) \tag{15}$$

is unbiased. This estimator is similar to the stratified posterior estimator described by Whitsitt [10].

To do this a pseudo-random sequence of uniformly distributed random digits is generated by the power-residue method and is transformed by the inverse cumulative-distribution-function method to obtain nearly Gaussian samples. These samples are used to fill the elements of the data vector $\underline{Y}$. Each vector $\underline{Y}$, then, has expected value $\underline{0}$ and covariance matrix $\underline{I}$.

By performing the transformation

$$\underline{X} = \Phi_j\ \underline{\Gamma}_j^{1/2}\ \underline{Y} + \underline{m}_j \tag{16}$$

on the vectors $\underline{Y}$, the random vectors for class j are obtained, where $\underline{\Phi}_j$ is the matrix of eigenvectors required to diagonalize the covariance matrix of class j, Γ_j is the diagonal matrix of eigenvalues and m_j is the desired mean vector. These random vectors are used to evaluate the estimators in Eqs. (15) and (13).

The term that must be evaluated from Eq. (14) is

$$P(C_j|x) = \frac{P_j \; p(x|\omega_j)}{\sum_k P_k \; p(x|\omega_k)}$$

To evaluate this probability compute $P_j \; p(x|\omega_j)$ for each class ω_j. Choose the largest value of the product $P_j \; p(x|\omega_j)$ and divide by the sum $\sum_k P_k \; p(x|\omega_k)$

$$p(x|\omega_k) = \frac{1}{(2\pi)^{N/2} |\underline{K}_k|^{\frac{1}{2}}} \exp \{-\tfrac{1}{2}(\underline{x}-\underline{m}_k)^T \underline{K}_k^{-1} (\underline{x} - \underline{m}_k)\} \tag{17}$$

$\underline{m}_k$ and $\underline{K}_k$ are the mean vector and covariance matrix respectively for class k. Substituting Eq. (16) into (17),

$$p(x|\omega_k) = \frac{1}{(2\pi)^{N/2} |\underline{K}_k|^{\frac{1}{2}}} \exp \{-\tfrac{1}{2} [\underline{Y}^T \underline{\Gamma}_j \underline{\Phi}_j^T \underline{K}_k^{-1} \underline{\Phi}_j \underline{\Gamma}_j^{\frac{1}{2}} \underline{Y} +$$

$$2\underline{Y}^T \underline{\Gamma}_j^{\frac{1}{2}} \underline{\Phi}_j^T \underline{K}_k^{-1} (\underline{m}_j-\underline{m}_k) + (\underline{m}_j-\underline{m}_k)^T \underline{K}_k^{-1} (\underline{m}_j-\underline{m}_k)]\} \tag{18}$$

In this form it is not necessary to perform the intermediate computational step of transforming the data. We need only to generate M sets of random vectors $\underline{Y}$ with mean vector $\underline{0}$ and covariance matrix $\underline{I}$ and use them in the Eq. (18).

Estimator Evaluation.

A subroutine program was written to evaluate classification performance by the above method. To test the method a three class problem was constructed. The mean vectors for the classes were

$$\underline{M}_1 = [-1, -1, \ldots, -1]^T$$
$$\underline{M}_2 = [0, 0, \ldots, 0]^T$$
$$\underline{M}_3 = [1, 1, \ldots, 1]^T$$

The covariance for each class was the identity matrix. The number of random vectors generated for each class was 1000. The exact classification accuracy as a function of the dimensionality can be evaluated for this case

$$P_{c1} = 1 - \text{erfc}\ (\sqrt{N}/2)$$
$$P_{c2} = 1 - 2\ \text{erfc}\ (\sqrt{N}/2)$$
$$P_{c3} = 1 - \text{erfc}\ (\sqrt{N}/2)$$
$$P_{c} = 1 - 4/3\ \text{erfc}\ (\sqrt{N}/2)$$

$$\text{where erfc}\ (a) = \int_a^\infty \frac{e^{-x^2/2}}{\sqrt{2\pi}}\ dx$$

and n is the number of dimensions. Table 3 contains the results of evaluating the class conditional performance and overall performance from one to ten dimensions.

To evaluate the variance of the estimates different starting values for the random number generator were used. Twenty trials were used to evaluate the maximum bound and the standard deviation from the true value. These results are presented in Table 4.

For the overall accuracy the estimate is within .005 of the true value. This is certainly sufficient for performance estimation. The

Table 3. Test of the SPEST Error Estimate.

	P_{c_1}	P_{c_2}	P_{c_3}	P_c	$\hat{P}_{c1}$	$\hat{P}_{c2}$	$\hat{P}_{c3}$	$\hat{P}_c$
1	0.6915	0.3829	0.6915	0.5886	0.6859	0.3793	0.7001	0.5884
2	0.7602	0.5205	0.7602	0.6803	0.7671	0.5116	0.7700	0.6829
3	0.8068	0.6135	0.8068	0.7423	0.8037	0.6202	0.8081	0.7440
4	0.8413	0.6827	0.8413	0.7885	0.8283	0.6852	0.8550	0.7895
5	0.8682	0.7364	0.8682	0.8243	0.8642	0.7425	0.8703	0.8256
6	0.8897	0.7793	0.8897	0.8529	0.8767	0.7939	0.8787	0.8498
7	0.9071	0.8141	0.9071	0.8761	0.8993	0.8242	0.9065	0.8766
8	0.9214	0.8427	0.9214	0.8951	0.9129	0.8472	0.9240	0.8947
9	0.9332	0.8664	0.9332	0.9109	0.9193	0.8809	0.9360	0.9120
10	0.9431	0.8862	0.9431	0.9241	0.9209	0.9012	0.9481	0.9234

Table 4. Variation of Estimates.

	P_{c1}	P_{c2}	P_{c3}	P_c	
1	.016 .033	.010 .019	.017 .049	.003 .005	σ Bound
2	.018 .036	.010 .018	.014 .027	.002 .005	σ Bound
3	.016 .046	.017 .031	.017 .055	.003 .007	σ Bound
4	.011 .025	.016 .029	.015 .029	.003 .005	σ Bound
5	.015 .031	.014 .033	.012 .026	.002 .004	σ Bound
6	.014 .026	.014 .023	.010 .022	.003 .006	σ Bound
7	.009 .027	.016 .033	.012 .027	.003 .005	σ Bound
8	.013 .025	.013 .036	.012 .023	.003 .006	σ Bound
9	.013 .026	.014 .031	.012 .021	.002 .004	σ Bound
10	.009 .016	.012 .024	.009 .019	.002 .005	σ Bound

σ = standard deviation

Bound = maximum difference between estimate and true value over 20 trials class conditional estimates are less reliable but are sufficient to observe trend in the performance due to individual classes.

2.3 Scanner Spatial Characteristics Modeling

The multispectral scanner represents the most important element in a remote sensing data gathering system. Therefore, an understanding of the signal flow through this subsystem is essential. As data is processed through the scanner, its statistical properties undergo a transformation. This in turn will alter the population separabilities and hence the classification accuracies. The comparison of this quantity at the scanner input and output and observation of its variation with the system parameters sheds considerable light on the overall system design. Since the Bayes spectral classifier depends solely on the population of spectral statistics, methods need to be developed that relate the scanner's input and output statistics. A complete derivation of such relationship is given in Appendix A of [2]. A summary follows:

Scanner Characteristic Function.

Figure 9 is a basic block diagram of the scanner spatial model where f_1 through f_N are N stochastic processes corresponding to N spectral bands and $h(x,y)$ is a two dimensional PSF. In particular where the Landsat scanner is concerned, the assumption of a Gaussian shaped IFOV has been widespread. Let $f(x,y)$, $g(x,y)$ and $h(x,y)$ denote the input and output random processes associated with any two matching bands and the scanner PSF respectively. It is well known that the above quantities are related by a convolution integral.

$$g(x,y) = \int\int f(x-\lambda_1, y-\lambda_2)\ h(\lambda_1,\lambda_2)\ d\lambda_1 d\lambda_2 \tag{19}$$

it follows that

$$S_{gg}(u,v) = S_{ff}(u,v)\,|H(u,v)|^2 \tag{20}$$

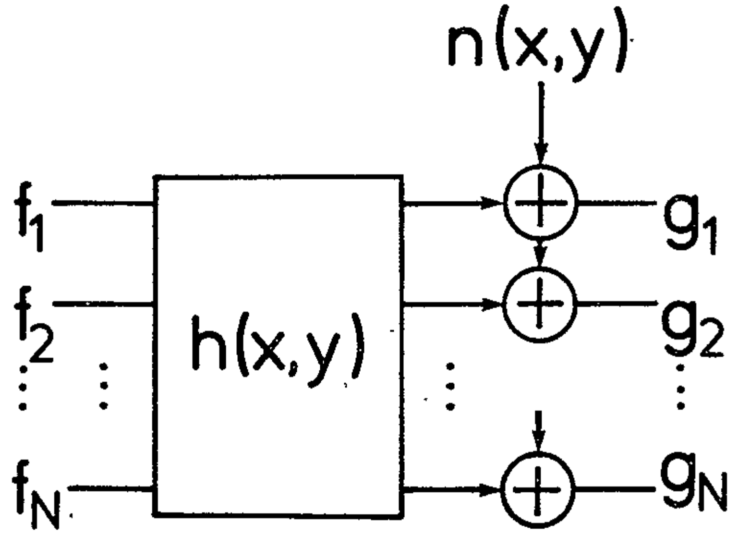


Figure 9. Scanner Spatial Model as a Linear System.

where $S(u,v)$ is the spectral density of the appropriate random process and $H(u,v)$ is the two dimensional Fourier transform of the scanner PSF. Let $M(u,v) = |H(u,v)|^2$, and $m(\tau,\eta)$ its inverse transform. Then the output, spatial autocorrelation function is given by

$$R_{gg}(\tau,\eta) = R_{ff}(\tau,\eta) * m(\tau,\eta) \tag{21}$$

In order to obtain specific results, the following assumptions are invoked; (a) exponential data spatial correlation, (b) Gaussian IFOV,

$$R_{ff}(\tau,\eta) = \rho_x^{|\tau|} \rho_y^{|\eta|} \qquad \tau,\eta = 0, 1, 2, \ldots$$

$$h(x,y) = c_1 \; e^{\frac{-x^2}{r_o^2}} \; e^{\frac{-y^2}{r_o^2}} \tag{22}$$

where $\rho_x = e^{-a}$ and $\rho_y = e^{-b}$ are the adjacent sample and line correlation coefficients respectively, r_o is the scanner PSF characteristic length in pixels and c_1 a constant providing unity filter gain. Using the separability property of the functions involved,

$$R_{gg}(\tau,\eta) = \int R_{ff}(\tau-x)\; m(x)\; dx \int R_{ff}(\eta-y)\; m(y)\; dy$$

where

$$m(\tau,\eta) = \frac{\pi c_1^2 r_o^2}{2} \; e^{-\frac{\tau^2}{2r_o^2}} \; e^{-\frac{\eta^2}{2r_o^2}} \tag{23}$$

carrying out the integration, the scanner characteristics function is given by

$$W_s(\tau,\eta,a,b) = \left[e^{\frac{a^2 r_o^2}{2} - a\tau} Q(ar_o - \frac{\tau}{r_o}) + e^{\frac{a^2 r_o^2}{2} + a\tau} Q(ar_o + \frac{\tau}{r_o}) \right] X$$

$$\left[e^{\frac{b^2 r_o^2}{2} - b\eta} Q(br_o - \frac{\eta}{r_o}) + e^{\frac{b^2 r_o^2}{2} + b\eta} Q(br_o + \frac{\eta}{r_o}) \right] \tag{24}$$

where

$$Q(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{\xi^2}{2}} d\xi$$

The spectral statistics is a subset of the spatial statistics hence $W_s(0,0,a,b)$ defines the ratio of the variance at the scanner output to the corresponding input quantity.

The output crosscovariance terms can be similarly derived. Let the crosscorrelation function between bands i and j be defined as

$$R_{f_i f_j}(\tau,\eta) = r_{f_i f_j} \sigma_{f_i} \sigma_{f_j} e^{-a_{ij}|\tau|} e^{-b_{ij}|\eta|} \tag{25}$$

where $r_{f_i f_j}$ is the spectral crosscorrelation coefficient at the input such that $|r_{f_i f_j}| \le 1$ · a_{ij} and b_{ij} are defined similar to a and b with the additional channel specification. Following the previous technique it follows that the crosscovariance term between channels i and j is given by

$$R_{g_i g_j}(0,0) = r_{f_i f_j} \sigma_{f_i f_j} \sigma_{f_i f_j} W_s(0,0,a_{ij},b_{ij}) \tag{26}$$

The corresponding crosscorelation coefficient follows:

$$r_{g_i g_j} = \frac{W_s(0,0,a_{ij},b_{ij})}{W_s^{\frac{1}{2}}(0,0,a_{ii},b_{ii})\ W_s^{\frac{1}{2}}(0,0,a_{jj},b_{jj})}\, r_{f_i f_j} \tag{27}$$

Therefore, the band-to-band correlation coefficients are identical at the scanner input and output provided the spatial auto and crosscorrelation function at the input are equivalent, i.e., $a_{ii}=a_{ij}$, $b_{ii}=b_{ij}$. A closed expression can also be obtained for a rectangular PSF defined by

$$h(x,y) = \begin{cases} \dfrac{1}{r_o^2} & |x|,\ |y| \le r_{o/2} \\ 0 & \text{elsewhere} \end{cases} \tag{28}$$

the corresponding characteristic function is given by

$$W_s(0,0,a_{ii},b_{ii}) = \frac{2}{a_{ii}r_o}\left(1 - \frac{1-e^{-a_{ii}r_o}}{a_{ii}r_o}\right) X$$

$$\frac{2}{b_{ii}r_o}\left(1 - \frac{1-e^{-b_{ii}r_o}}{b_{ii}r_o}\right) \tag{29}$$

Eq. (24) and Eq. (29) are plotted in Figures 10 through 13 for different scene correlations with the IFOV = r_o as a running parameter. The universal property exhibited by W_s of either PSF is the increasing output variance reduction as IFOV is increased. This property has been widely verified experimentally. Comparison of Figures 11 and 13 indicates that for the same IFOV, a Gaussian shaped IFOV causes a heavier variance reduction in a spectral band than that of a rectangular PSF. This property can potentially produce a higher separability among the populations as the signal is processed through the scanner electronics.

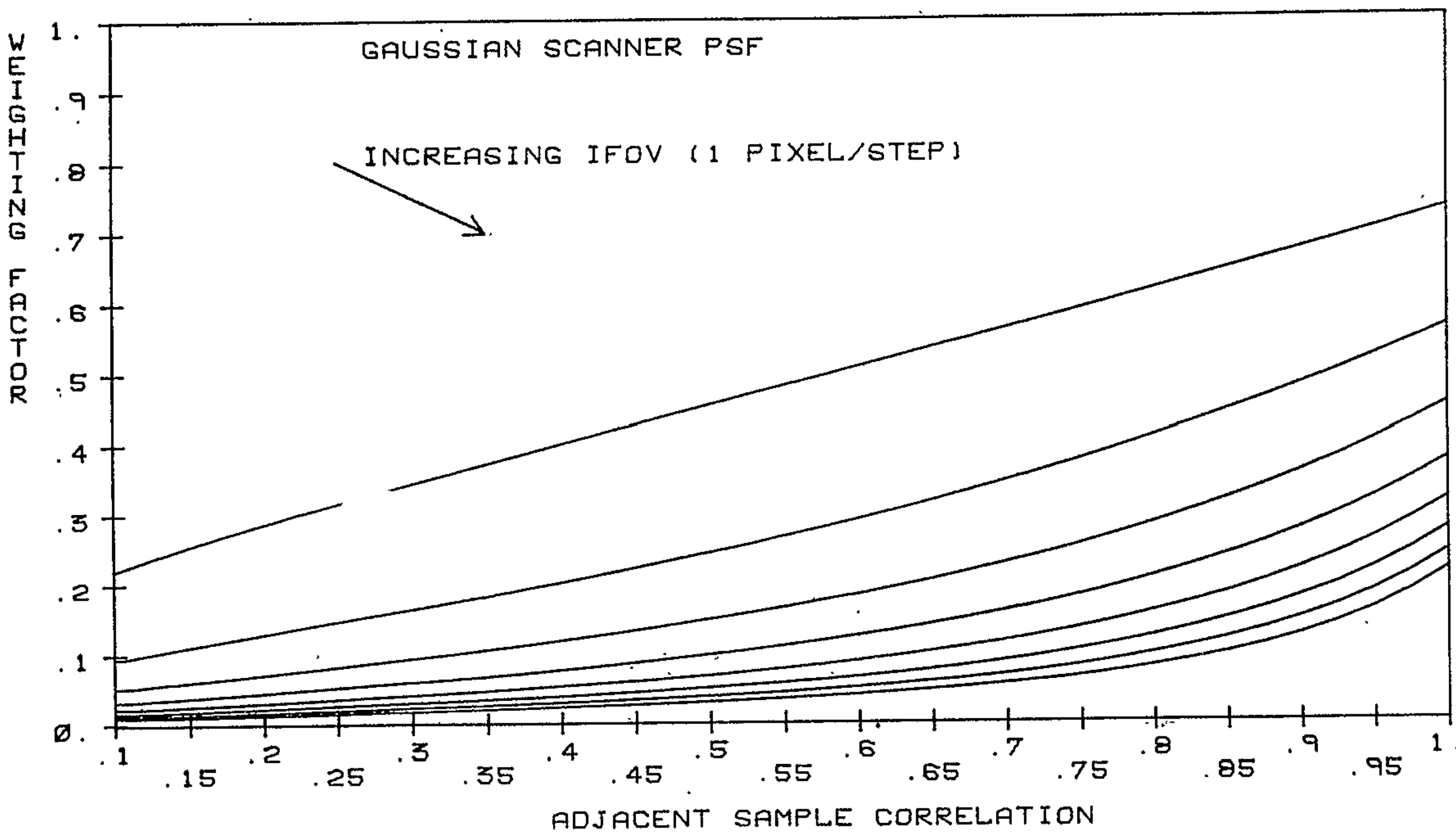


Figure 10. Scanner Characteristic Function vs. Scene Correlation. Adjacent Line Correlation = .65.

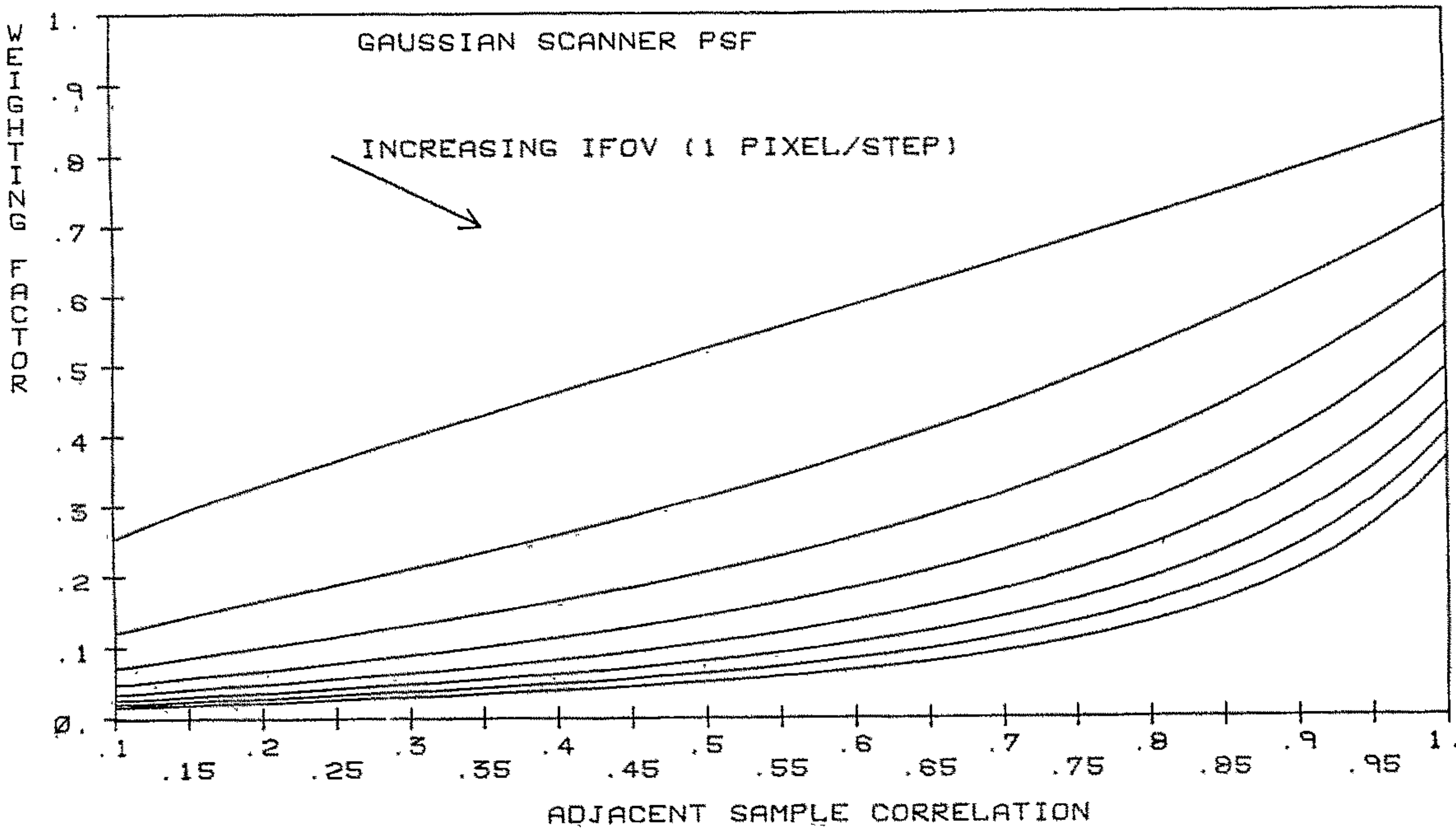


Figure 11. Scanner Characteristic Function vs. Scene Correlation. Adjacent Line Correlation = .8.

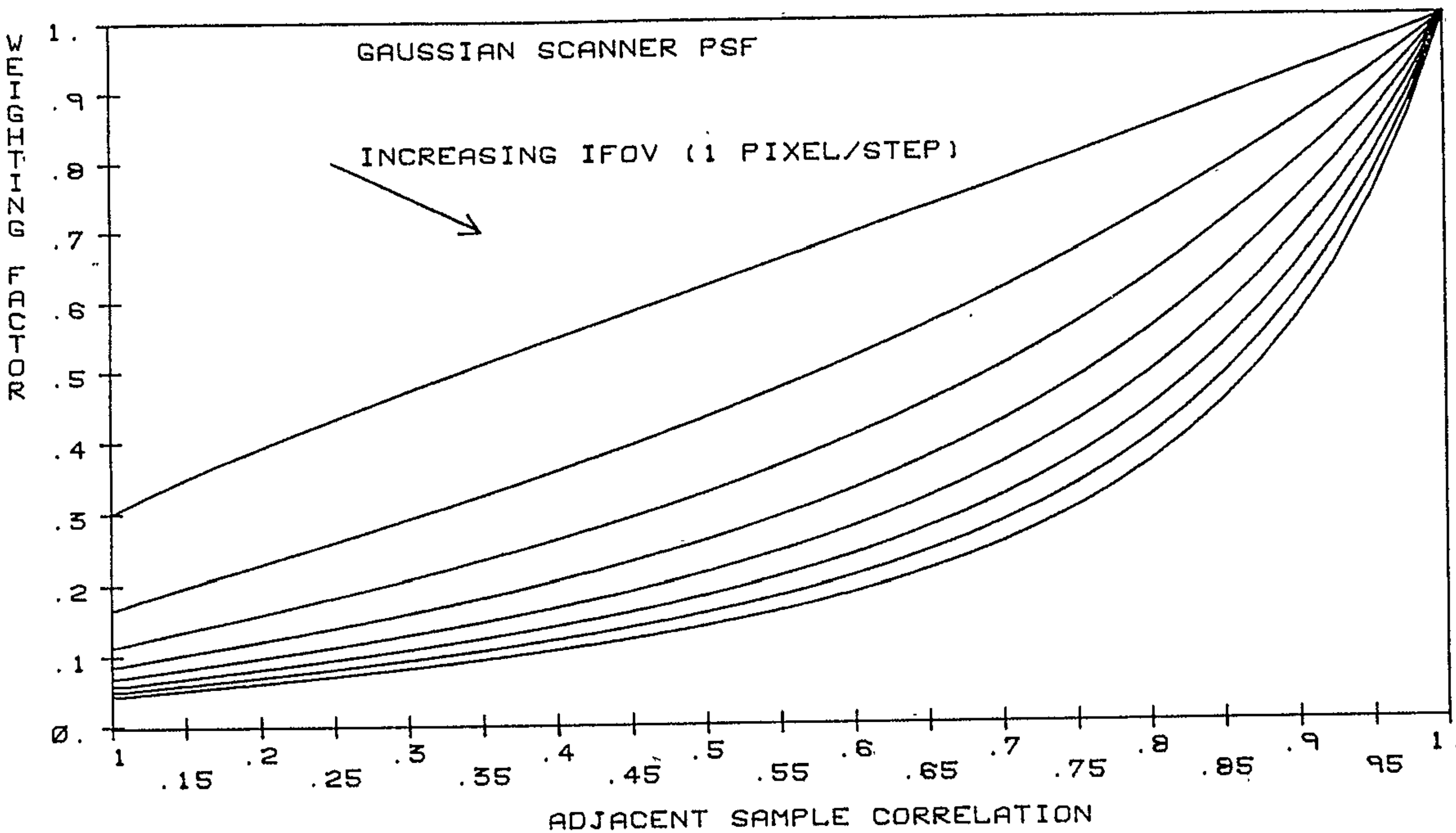


Figure 12. Scanner Characteristic Function vs. Scene Correlation.
Adjacent Line Correlation = 1.

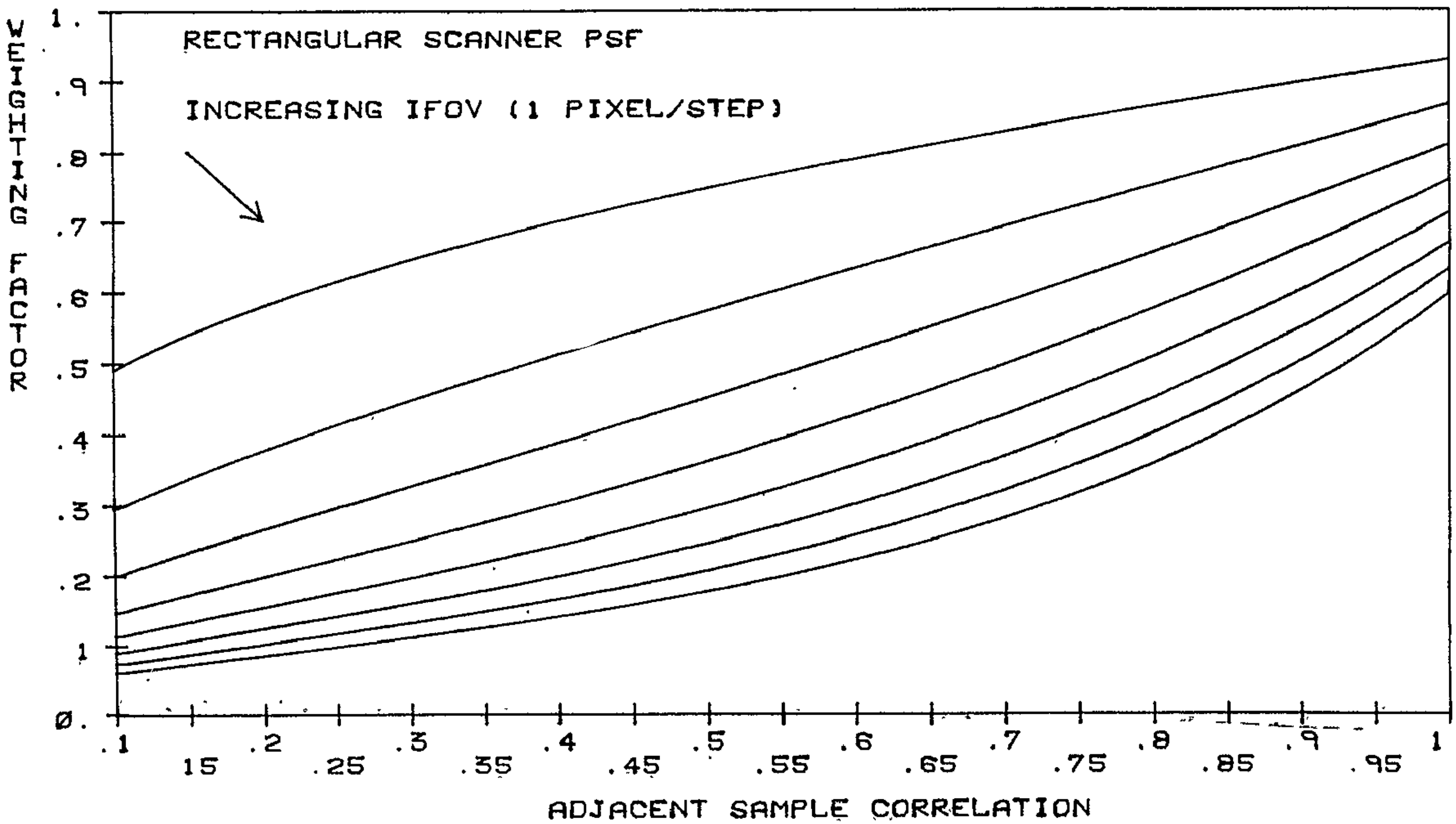


Figure 13. Scanner Characteristic Function vs. Scene Correlation.
Adjacent Line Correlation = .8.

The input-output statistical relationship just developed along with the analytic classification accuracy predictor, provide the two basic tools required for a parametric evaluation of the MSS performance under varying operating conditions. As an illustration, three hypothetical classes with some prescribed statistics are specified at the scanner input. Three different scenes with adjacent sample correlations of 0.5, 0.85 and 0.95 are considered. The scanner characteristic function produced a set of transformed statistics at the output followed by the estimation of the conditional classification accuracies using ACAP, for 8 different IFOV's. The results are plotted in Figures 14 through 16.

Two main properties stand out. First is the improvement in class separability as the IFOV is enlarged. This is true in all the cases. The rate of improvement however, is strongly correlated with the scene spatial correlation. The lower the input scene correlation, the higher the classification accuracy improvement per IFOV step. This property is brought about by the characteristics of W_s where one step increase in IFOV size produces a greater variance reduction for a low scene spatial correlation than a similar increase would cause in a highly correlated scene.

The scene spatial correlation plays a significant role in the overall system performance which is not readily obvious. One of the well known properties of linear systems with random inputs is the reduction of the output variance/input variance ratio as the point spread function (PSF) is widened. In this section it has been shown theoretically that a third factor in this reduction is the spatial correlation structure of the input stochastic process. Specifically, with everything else fixed, a process having a moderate scene correlation will undergo a tighter clustering around its mean than an otherwise identical process with a highly correlated spatial characteristic. On the extreme side of the correlation scale with a small pixel-to-pixel correlation, the ratio of the output variance to the corresponding input quantity is very negligible. Consider a bandlimited white noise process with a spectral density, shown in Figure 17, where W is the bandwidth and No/2 the two sided spec-

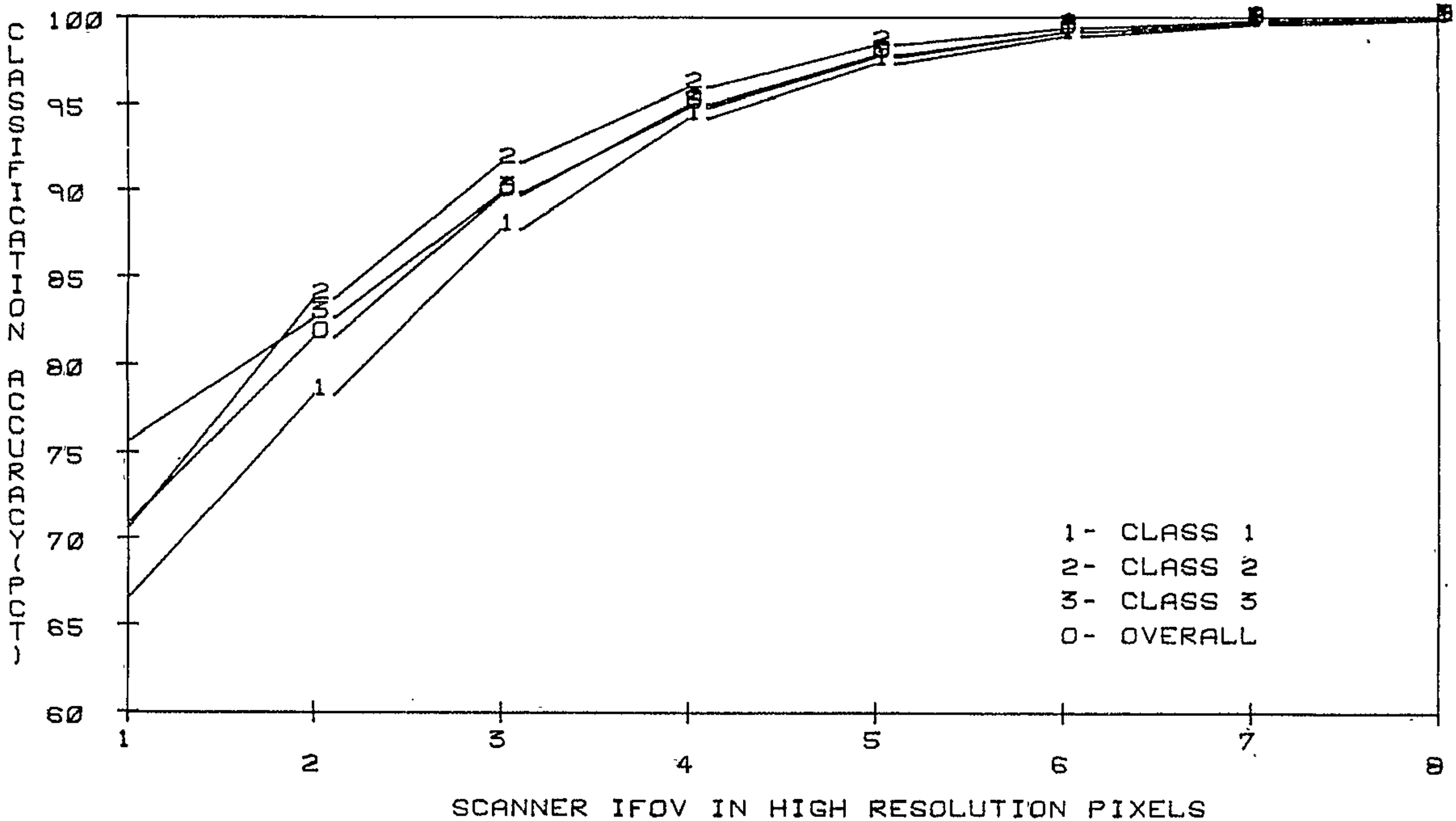


Figure 14. Scanner Output Classification Accuracy vs. IFOV. Adjacent Sample Correlation = .55.

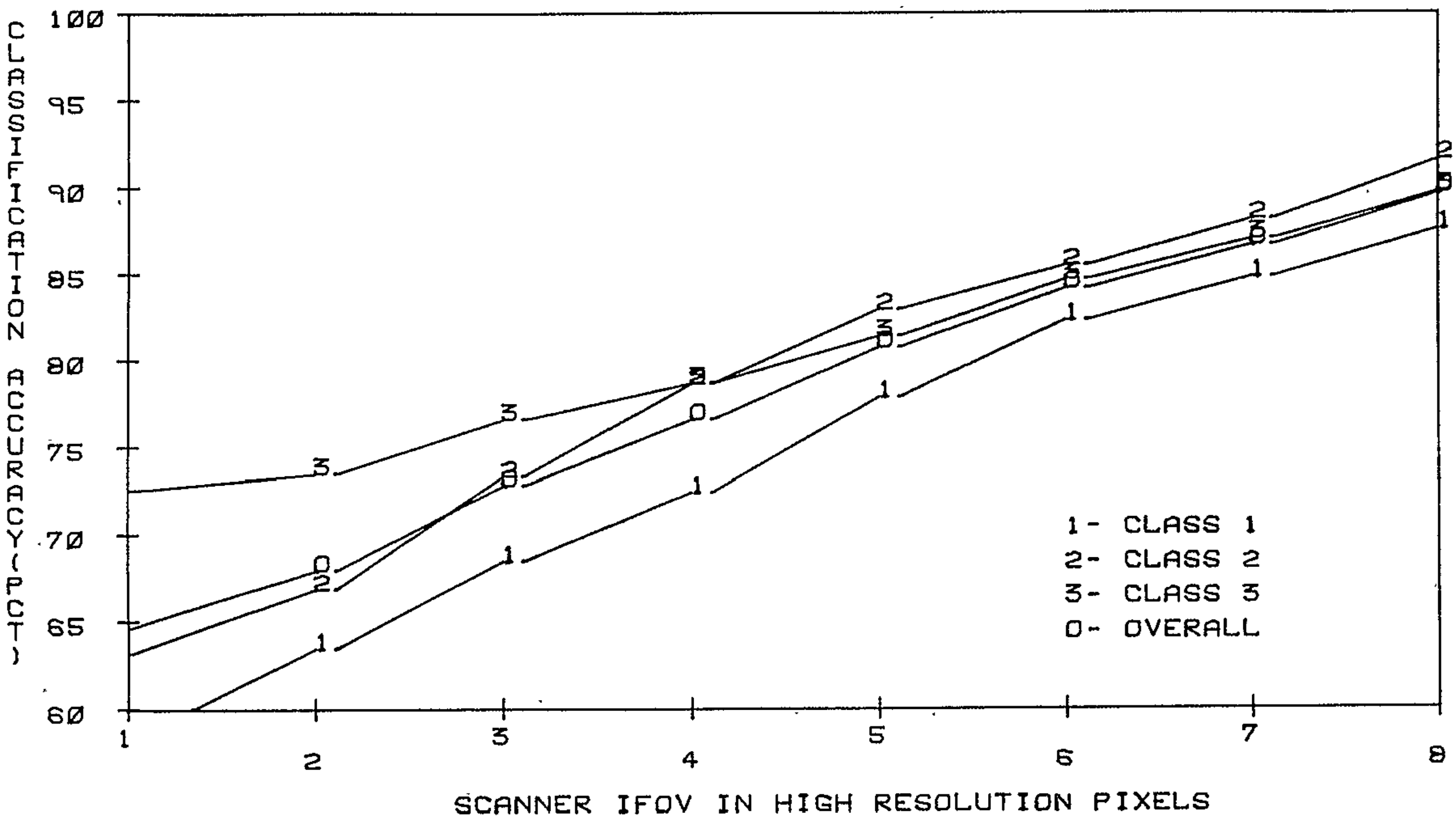


Figure 15. Scanner Output Classification Accuracy vs. IFOV.
Adjacent Sample Correlation = .8.

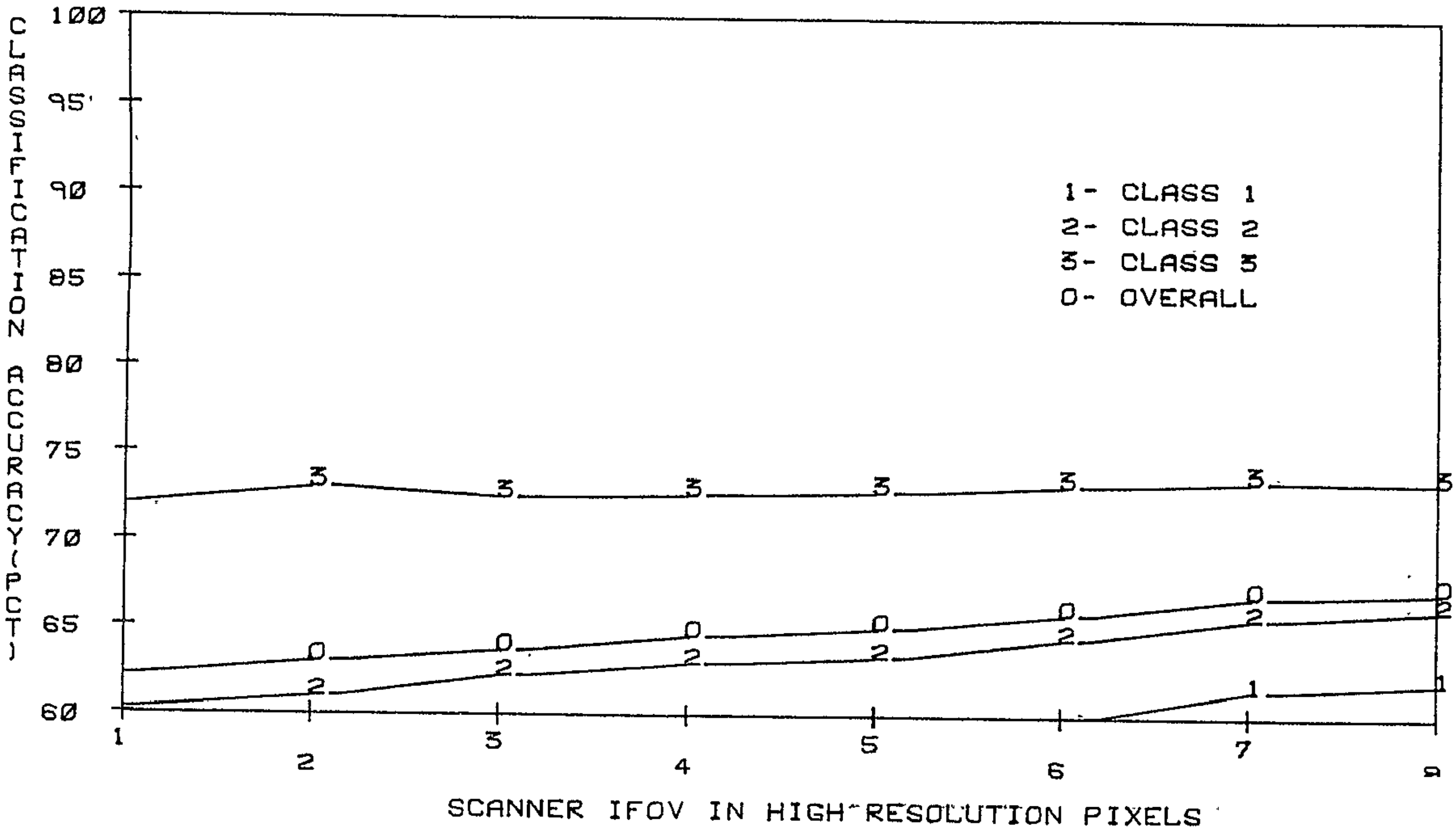


Figure 16. Scanner Output Classification Accuracy vs. IFOV.
Adjacent Sample Correlation = .95.

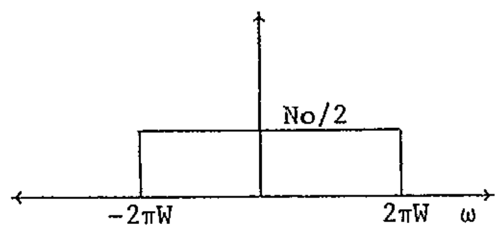


Figure 17. Bandlimited White Noise Spectral Density.

tral density. As W increases the adjacent pixel correlation in the scene decreases. The increase in W, however, is accompanied by a decrease in No/2 if this process is to remain physically realizable (finite energy). Under a finite energy constraint, therefore, as $W \longrightarrow \infty$, $No/2 \longrightarrow 0$. In the limit the energy content of the output random process will be nil.

Random Noise.

Additive random noise entered at various stages of a scanner system can degrade the overall system performance substantially. The noise can be classified into two broad categories: external and internal. A major source of external noise is atmospheric in nature mainly due to absorption (e.g., water vapor) and scattering. The detector and quantization noise comprise the major component of the internal noise sources. From a system analysis point of view, the latter represents a more tractable and better understood component of the random noise [11], while the former still awaits further experimental documentation. The purpose of this work is not so much the exploration of the various noise sources but the integration of its effect within an analytic analysis package once its characteristics and origin has been determined.

From the theoretical results obtained it can be stated that atmospheric noise, in the uplink path at least, has negligible degrading factor compared with the detector and quantization noise. Let $f(x,y)$, $N_f(x,y)$, $f'(x,y)$ and $N_f'(x,y)$ be the input random process, input additive white noise, the output random process and the noise component of the output signal respectively, then

$$f'(x,y) = f(x,y) * h(x,y) \tag{30}$$

$$N_{f'}(x,y) = N_f(x,y) * h(x,y) \tag{31}$$

Define

$$(SNR)_f = \text{Var}\,\{f(x,y)\}/\text{Var}\,\{N_f(x,y)\} \tag{32}$$

$$(SNR)_{f'} = \text{Var}\,\{f'(x,y)\}/\text{Var}\,\{N_{f'}(x,y)\} \tag{33}$$

Recalling the functional dependence of W_s on the input scene spatial correlation, it follows that the ratio of the variance of a white noise process at the scanner output to the corresponding input quantity is of the order of 5% to 10%, higher or lower depending on the IFOV size. Therefore,

$$\text{Var}\,\{f'(x,y)\} < \text{Var}\,\{f(x,y)\} \tag{34}$$

$$\text{Var}\,\{N_{f'}(x,y)\} << \text{Var}\,\{N_f(x,y)\} \tag{35}$$

hence

$$(SNR)_{f'} >> (SNR)_f \tag{36}$$

It then follows that the noise component of the output process prior to detector and quantization noise is negligible in most cases.

In order to observe the effect of noise on the scanner output class separability the test class statistics were modified to exhibit the effect of random noise. The assumed properties of the noise are additive, white and Gaussian. Let $f''(x,y)$ be the signal to be telemetered to Earth.

$$f''(x,y) = f'(x,y) + N_{f'}(x,y) \tag{37}$$

the statistics of $f''(x,y)$ and $f'(x,y)$ are related by

$$\sum_{f''} = \sum_{f'} + \sum_{N_{f'}} \tag{38}$$

the simple addition is due to the signal and noise independence. Assuming a zero mean $N_{f''}$ the mean vector are identical, i.e.,

$$E\{f''\} = E\{f'\}$$

Among the four assumptions about the noise, its Gaussian property is the weak link due to the Poisson distributed detector noise and uniformly distributed quantization noise. Relaxing the Gaussian noise assumption, however, would mean the design of an optimum classifier for non-normal classes and evaluation of its performance. A task that would complicate matters considerably. Due to the relatively insufficient documentation of the characteristics of random noise in multispectral data, the initial Gaussian assumption is adhered to.

Following the adopted SNR definition, three different noise levels are considered and the corresponding overall classification accuracies for the three previously used test classes are estimated. Figure 18 is the variation of $\hat{P}_c$ vs. IFOV with SNR as the running parameter. For a fixed IFOV, $\hat{P}_c$ increases with increasing SNR. For a fixed SNR, $\hat{P}_c$ increases with increasing IFOV size. These illustrations have shown that with a proper coupling between the ACAP and the scanner characteristic function, the progress of the population statistics through the system can be studied on an analytical and entirely parametric basis. The accompanying classification accuracies can measure the designer's success in selecting the spatial and/or spectral characteristics of a Multispectral Scanner System.

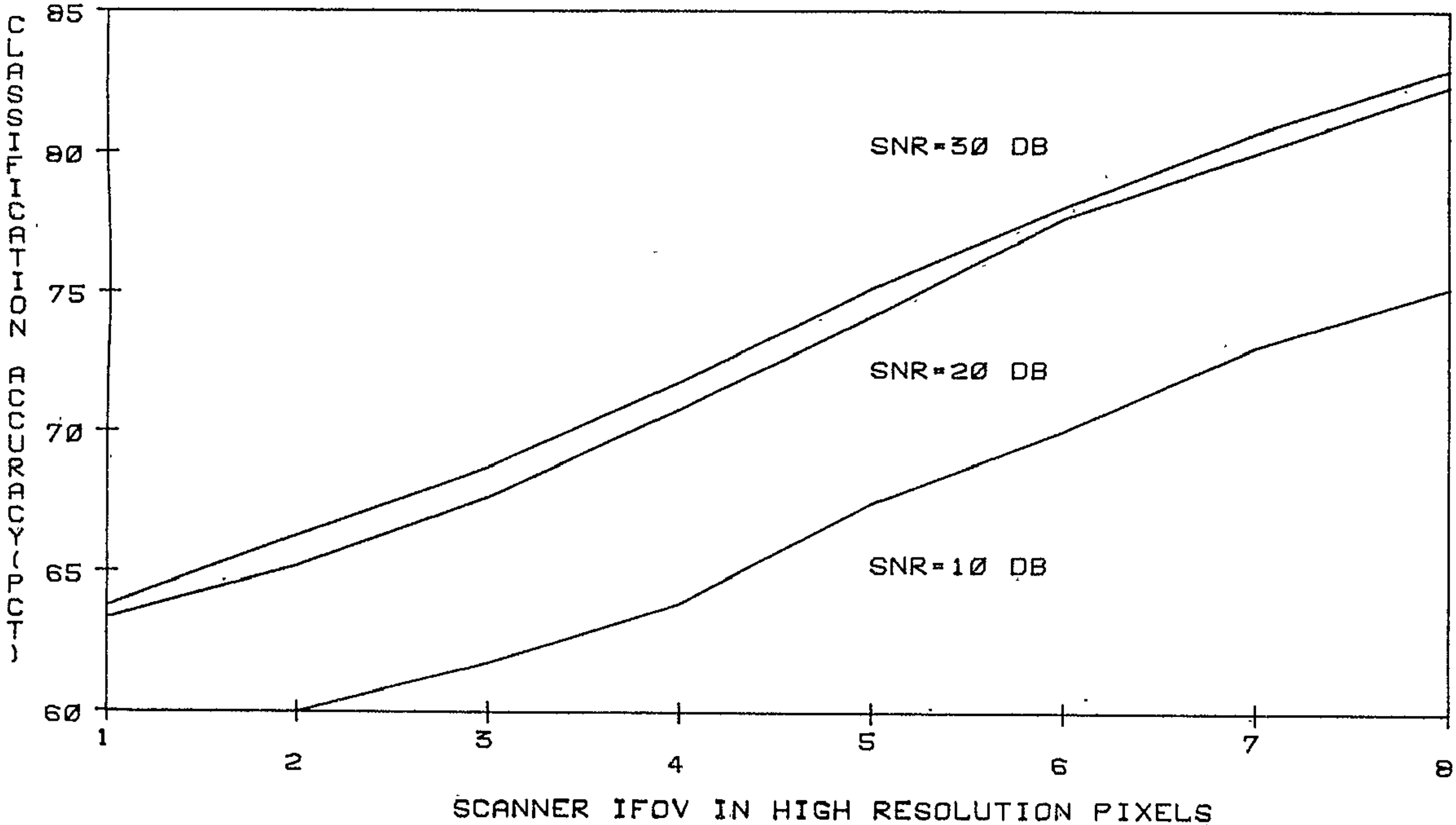


Figure 18. Overall Output Classification Accuracy Variation with Noise and IFOV.

2.4 Optimum Spectral Function Research

In earth observational remote sensing much work has been done with extracting information from the spectral variations in the electromagnetic energy incident on the sensor. Of primary importance for a multispectral sensor design is the specification of the spectral channels which sample the electromagnetic spectrum. An analytical technique is developed for designing a sensor which will be optimum for any well-defined remote sensing problem and against which candidate sensor systems may be compared.

Let the surface of the earth at a given time be divided in strata where each stratum is defined to be the largest region which can be classified by a single training of the classifier. Each point in the stratum is mapped into a spectral response function $X(\lambda)$ as in Figure 19. That is if one observes a point in the stratum with the sensor, then the function $X(\lambda)$ describes the response variations with respect to the wavelength, λ. The stratum together with its probabilistic description defines a random process, and the collection of all of such functions $X(\lambda)$ which may occur in the stratum is called an ensemble.

The general concept of a pattern recognition system in this application requires that if each $X(\lambda)$ is to be classified by a classification algorithm, this can be accomplished by first measuring a finite number of attributes of $X(\lambda)$, called features. This is the function of the sensor system as depicted in the upper left portion of Figure 20 where $X_1, X_2, \ldots, X_N$ are the values of N features for a given $X(\lambda)$. It may be viewed as a filtering operation on $X(\lambda)$.

For example, on the right portion of Figure 20 the function of MSS of the Landsat satellites is illustrated. In this case a number proportional to the average energy in a wavelength interval is reported out by the sensor for each of four wavelengths. Mathematically this may be expressed as

$$X_n = \int X(\lambda)\phi_n(\lambda)d\lambda \qquad n = 1, 2, 3, 4$$

Next we must consider what would constitute an optimum sensor. We first note that in general the sensor may be used over any part of the earth's surface, at anytime, and for many different applications (sets of classes). Therefore the sensor must be optimized with respect to the entire set of strata represented by these cases. As a result of the large size of this set and the fact that its statistical description is not known, we will optimize the sensor with respect to its signal representation characteristics. The $\{X(\lambda)\}$ each contain information useful to the classifier; we require of the sensor design that for a given N a maximum of this information which was in $X(\lambda)$ still be present in $\{X_n\}$. Since the specific nature of this information is not known a priori, we can only assure that this will be the case for any stratum if $X(\lambda)$ is recoverable from $\{X_n\}$.

Let $X(\lambda)$ be the result of attempting to reconstruct $X(\lambda)$ from $\{X_n\}$. A fidelity criterion which is useful in this instance is

$$\varepsilon = \int [X(\lambda) - \hat{X}(\lambda)]^2 \, d\lambda \tag{39}$$

the so called mean square error or mean square difference between $X(\lambda)$ and $\hat{X}(\lambda)$.

It is known [12] that a reconstruction scheme which minimizes ε for a given N is

$$X(\lambda) = X_1\phi_1(\lambda) + X_2\phi_2(\lambda) + \dots + X_N\phi_n(\lambda)$$

$$= \sum_{n=1}^{N} X_n\phi_n(\lambda) \tag{40}$$

provided that the $\{\phi_n(\lambda)\}$ are orthogonal over the wavelength interval of interest, i.e.,

$$\int \phi_m(\lambda) \; \phi_n(\lambda) \; d\lambda = 0 \qquad m \neq n \tag{41}$$

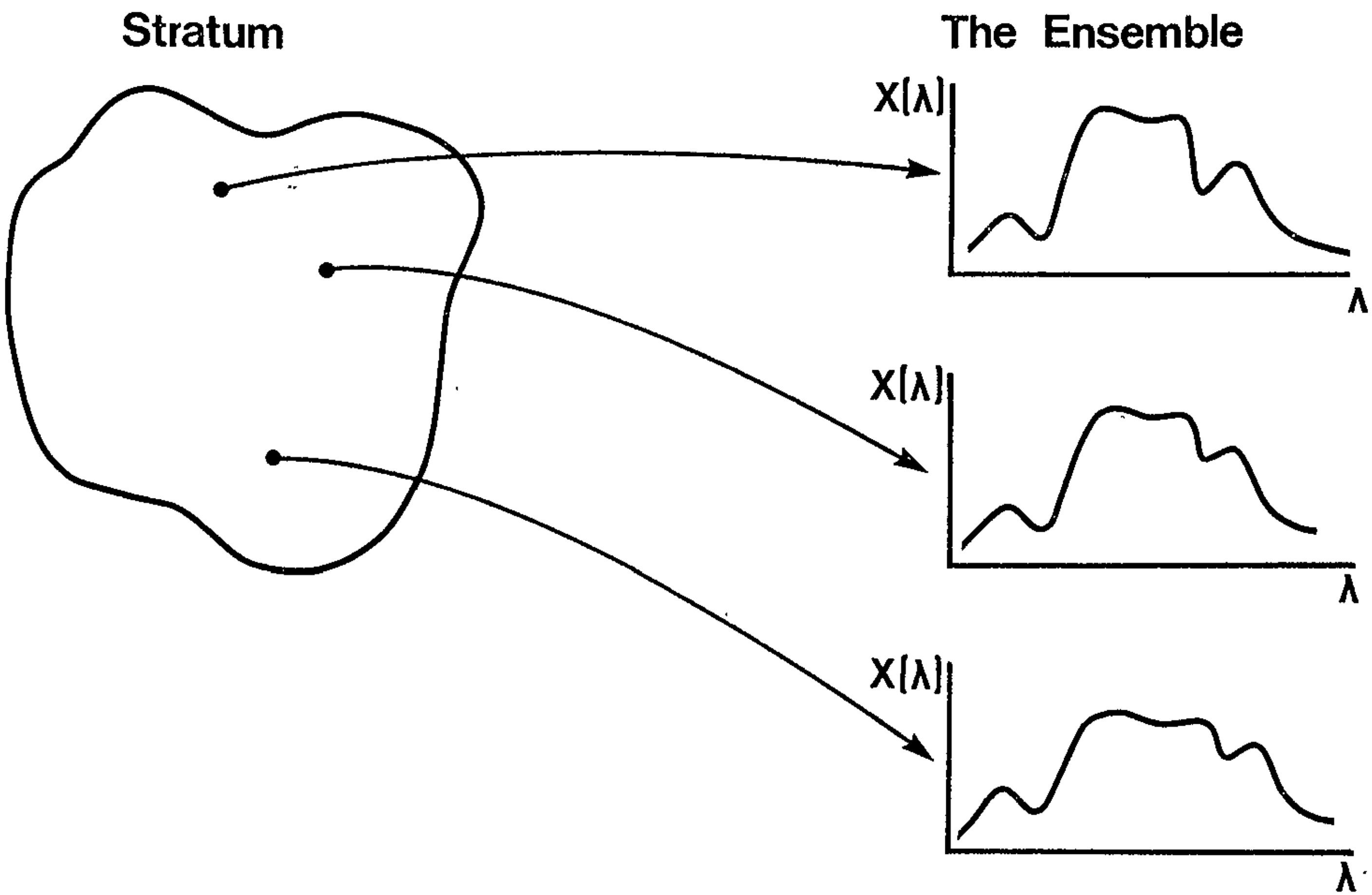
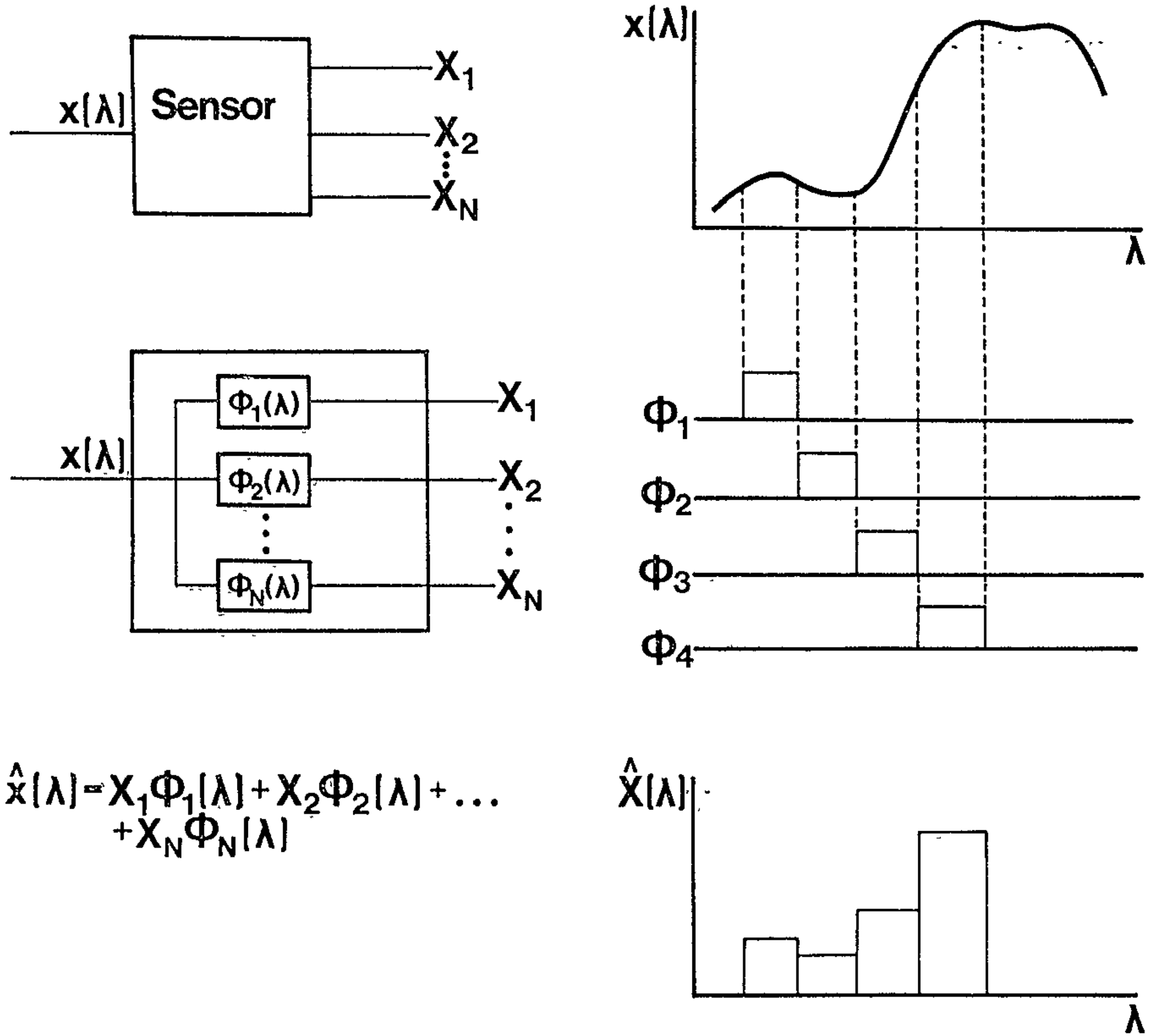


Figure 19. Realization of a Stratum as the Ensemble of Spectral Sample Functions.

Figure 20. Basis Function Expansion of a Random Process.



and the $\{X_n\}$ are calculated by

$$X_n = \int X(\lambda)\ \phi_n(\lambda)\ d\lambda \tag{42}$$

Note for example that the Landsat example of Figure 20 satisfies these conditions. In the lower right of Figure 20 is depicted the result of such a reconstruction for the Landsat example.

While use of Eq. (42) in the case does minimize ε with respect to the choice of values of $\{X_n\}$, a further improvement may by obtained by choosing a set of $\{\phi_i|\lambda|\}$ which minimizes ε. It can be shown [5, 12] that the set $\{\phi_n(\lambda)\}$ which accomplishes this must satisfy the equation

$$\sigma_i\phi_i(\lambda) = \int R(\lambda,\xi)\ \phi_i(\xi)\ d\xi \tag{43}$$

where

$$R(\lambda,\xi) = E\{[X(\lambda) - m(\lambda)]\ [X(\xi) - m(\xi)]\} \tag{44}$$

is the correlation function of the random process and $m(\lambda)$ is its mean value at λ.

Such a signal representation defined by Eqs. (40-44) is known as a Karhunen-Loève expansion [13]. It provides not only for the most rapid convergence of $\hat{X}(\lambda)$ to $X(\lambda)$ with respect to N but in addition the random variables $\{X_n\}$ are uncorrelated and since the random process is Gaussian they are statistically independent. Further the only statistic required of the ensemble is $R(\lambda,\xi)$. This representation of $\{X(\lambda)\}$ is therefore not only optimal, it is convenient.

A useful generalization of the Karhunen-Loève expansion can be made. Suppose a priori information concerning portions of the spectral interval are known and it is desired to incorporate this knowledge into the analysis. A weighting function $w(\lambda)$, is introduced which weights portions of the interval according to the a priori information. As an example, measurements were taken over the spectrum and it was observed that there was considerable variation in the signal in the water absorption bands around 1.4 and 1.8 micrometers. This variation was due to measurement and calibration difficulties rather than being a result of variations in the scene. Therefore, the weighting function was set to zero in these absorption bands. This generalization is referred to as the weighted Karhunen-Loève expansion [5]. Eqs. (40), (42) and (43) become

$$\hat{X}(\lambda) = \sum_{i=1}^{N} x_i \, \phi_{w_i}(\lambda) \qquad \lambda \varepsilon \Lambda \tag{45}$$

$$\sigma_{w_i} \, \phi_{w_i}(\lambda) = \int R(\lambda,\xi) \, w(\xi) \, \phi_{w_i}(\xi) \, d\xi \tag{46}$$

$$x_i = \int X(\lambda) \, w(\lambda) \, \phi_{w_i}(\lambda) \, d\lambda \tag{47}$$

where the eigenfunction, $\phi_{w_i}(\lambda)$ are solutions to the integral Eq. (46) with the weight $w(\lambda)$. The special case where $w(\lambda) = 1.0$ for all $\lambda \varepsilon \Lambda$ reduces the expansion to the original form in Eq. (40), (42) (43), and (44).

The results of having utilized this means of optimal basis function scheme on spectral data are contained in reference [5]. From them one can see the significant improvement in classification accuracy which decreased spectral representation error will provide. One can also determine the spectral resolution and band placement needed to achieve such classification accuracy improvement.

2.5 Information Theory Approach to Band Selection

The problem of selecting a set of "optimum" windows in the electromagnetic spectrum for observing the reflected sunlight has always been of considerable interest. Depending on the definition of the optimality different methods have been developed. One such approach was shown in Section 2.4 using K-L expansion to select an optimum set of basis functions. In this section an information theoretic definition of optimality developed in [3] is explored.

Mutual Information and Stochastic Modeling.

The reflected energy from the target is detected by the scanner and corrupted by various noise sources. If S is the "noise-free" signal, Y the observation and N a random disturbance, then

$$Y = S + N \tag{48}$$

the reduction of uncertainty about S obtained from Y is called the average or mutual information between the observation and original signal. Since the reconstruction of the reflected signal from the noisy observation is the highly desirable capability, the comparison of such average information and selection of these bands with the highest information content is chosen as a means of spectral band selection. Let

$$S_n = (s_1, s_2, \ldots, s_n)$$

and

$$Y_n = (y_1, y_2, \ldots, y_n)$$

where s_i and y_i are the coefficients of the orthonormal (K-L) expansion of Y and S, then the mutual information between Y and S is given by [3].

$$I(Y,S) = \tfrac{1}{2}\log \left[\frac{\det C_y}{\det C_n}\right] \tag{49}$$

where C_y and C_n are the covariance matrices of (y_i, i = 1, 2, ...) and (n_i = No/2, i = 1, 2, ...) and No/2 is the two sided spectral density of the additive white noise. Equivalently I(Y,S) can be represented in terms of the Wiener-Hopf optimum filter impulse response,

$$I(Y,S) = \tfrac{1}{2} \int_{\lambda_1}^{\lambda_2} h(\lambda,\lambda)\, d\lambda \tag{50}$$

$h(\lambda,\lambda)$ provides an estimate of S from Y with a minimum mean-square error. This relationship, however, is not a practical method of evaluating $I(Y,S)$ since the actual solution of the Wiener-Hopf integral itself is a nontrival task. This problem can be circumvented by a discrete state variable formulation, i.e.,

$$\underline{s}(k+1) = \underline{\phi s}(k) + \underline{\Gamma}W(k) \qquad k\varepsilon[\lambda_1,\lambda_2] \tag{51}$$

where

$$\underline{s}(k+1) = \begin{bmatrix} s_1(k+1) \\ s_2(k+1) \\ \cdot \\ \cdot \\ \cdot \\ s_n(k+1) \end{bmatrix}$$

$\underline{\phi}$ is an (nxn) matrix

$\underline{\Gamma}$ is an (nx1) vector

$W(k)$ = a descrete independent Gaussian zero mean random process

The formulation of the problem in the discrete domain provides a practical way of computing $h(\lambda,\lambda)$ through Kalman filtering techniques. The discrete version of Eq. (50) is given by

$$I(Y,S) = \tfrac{1}{2} \sum h(k,k)$$

$$k \ \varepsilon \ \{\lambda_1,\lambda_2\} \tag{52}$$

The discrete nature of this approach makes the evaluation of Eq. (52) considerably more practical than its continuous counterpart. This is due to the fact that the Wiener-Hopf equation is easily solved in only those cases for which the analytical form of $K_s(\lambda,u)$, the signal covariance function, is fairly simple, not likely for most random processes encountered in remote sensing. Since $h(k,k)$ is dependent on the parameters of Eq. (51) a concise representation of $\underline{s}(k+1)$ is needed.

The general form of Eq. (51) is given as an autoregressive (AR) model

$$\underline{s}(k) = \sum_{j=1}^{m_1} a_j s(k-j) + \sum_{j=1}^{m_2} b_j \psi(k-j) + W(k) \qquad (53)$$

$s(k)$ = The spectral response at the discrete wavelength k. It is a Gaussian random process.

$w(k)$ = zero mean independent Gaussian disturbance with variance ρ.

$\psi(k-j)$ = deterministic trend term used to account for certain characteristics of the empirical data.

a_j, b_j = are unknown constant coefficients to be determined.

m_1, m_2 = The order of the AR model.

The identification selection and validation of general AR models for the representation of a random process is a well developed technique [14,15]. The identification of an appropriate model provides the necessary parameters required for the evaluation of $I(Y,S)$ in Eq. (52). The model selection process for a selected number of ground covers has been carried out [3] leading to the ranking of a set of spectral bands according to the criterion outlined previously. A summary of the experimental results are given below.

Data Base and Model Selection.

Two different sets of empirical data are used to demonstrate the techniques developed here. The first set consists of observations of wheat scenes. The second set consists of several vegetation cover types such as oats, barley, grass, etc. For each scene the spectral responses, collected by the Exotech 20C field spectroradiometer, are averaged over the ensemble. It is thought the resultant average spectral response provides a relatively good data set for demonstration purposes. Figures 21 and 22 show the average response for both cover types.

In order that the study be carried out in a context that is relatively realistic for multispectral scanners, the spectral response of the two data sets is divided into a number of spectral bands. The division is fairly arbitrary, but each band must contain a sufficient number of data points to ensure accurate parameter estimation for model identification. The spectrum is divided into 9 bands from 0.45 μm to 2.38 μm with two gaps in the 1.34-1.45 μm and 1.82-1.96 μm range due to atmospheric absorption, see Tables 5 and 6.

Table 5. Spectral Bands for Wheat Scene.

Band Number	Spectral Wavelength Interval (μm)
1	0.45-0.54
2	0.54-0.62
3	0.62-0.71
4	0.71-0.85
5	0.85-0.99
6	0.99-0.13
7	1.13-1.34
8	1.45-1.82
9	1.96-2.38

Table 6. Spectral Bands for Combined Scene.

Band Number	Spectral Wavelength Interval (μm)
1	0.45-0.54
2	0.54-0.62
3	0.62-0.71
4	0.71-0.85
5	0.85-0.98
6	0.98-1.12
7	1.12-1.30
8	1.45-1.82
9	1.96-2.38

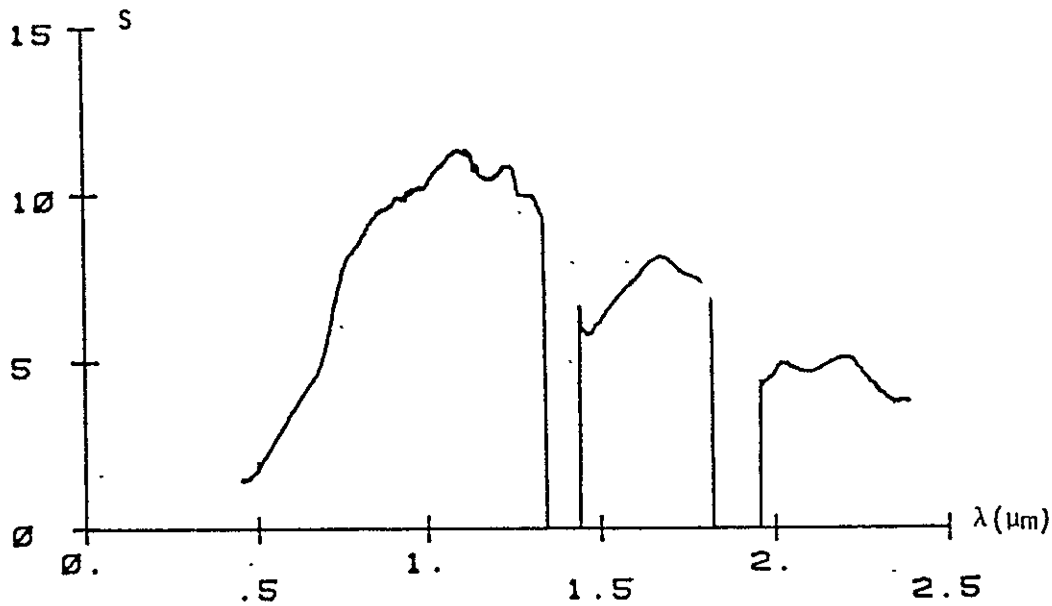


Figure 21. Average Spectral Response -- Wheat Scene.

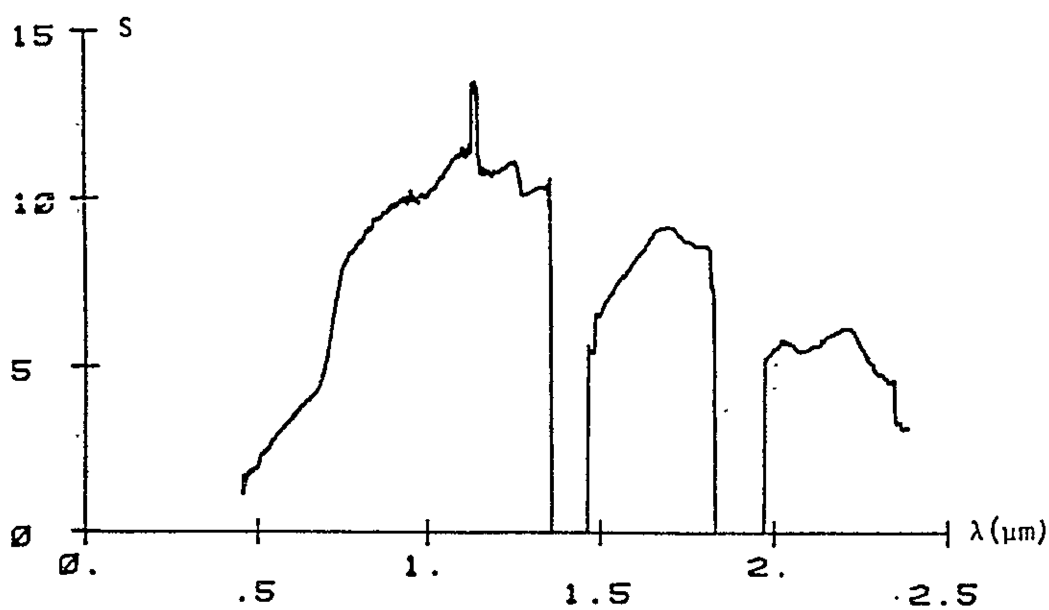


Figure 22. Average Spectral Response -- Combined Scene.

The next step is the identification and validation of models that would adequately describe the aforementioned spectral responses. Three different models were tested and compared, (a) autoregressive (AR), (b) autoregressive plus a constant trend (AR+C), (c) integrated autoregressive (IAR). Following the standard selection and validation techniques, one of the above 3 models is selected which describes the scene most satisfactorily. Tables 7 and 8 show the selected models for the wheat and combined scene respectively, IAR-2 in Table 8 in a second order IAR.

Table 7. Modeling of the Wheat Scene.

Band	Order of Model	Type of Model
1	7	AR
2	2	AR
3	11	IAR
4	1	AR+C
5	1	AR
6	2	AR+C
7	5	IAR
8	8	IAR
9	6	IAR

Table 8. Modeling of the Combined Scene.

Band	Order of Model	Type of Model
1	11	IAR-2
2	2	AR
3	11	IAR
4	1	AR+C
5	3	AR
6	1	AR
7	9	AR+C
8	8	IAR
9	1	AR

Spectral Band Selection.

It was initially stated that the information content of a set of spectral bands can be used in the selection of an optimum subset. Here, the preceding regression analysis will be used to evaluate the mutual information between the reflected energy and the observed signal in the 9 spectral bands under study.

The first step is the computation of the average information in y(k), the received spectral process, about s(k). The reflected spectral scene response as a function of spectral bandwidth for each band of both scene types. The average information is computed for several values of the noise variance, σ^2_N. Appropriate software is developed to carry out the calculation of I(Y,S). Figures 23 and 24 show the variation of I(Y,S) in nats for the wheat and combined scene in band 1. Similar plots are shown for the infrared band 7, Figures 25 and 26. Selecting a $\sigma_n^2 = 10^{-3}$ for demonstration purposes, the average information for wheat and combined scenes are tabulated in Tables 9 and 10 for 9 spectral bands.

Table 9. Average Information for Wheat Scene I

Band	I(Y,S) nats
1	34.50
2	10.52
3	20.35
4	30.00
5	44.96
6	37.20
7	60.31
8	34.80
9	50.10

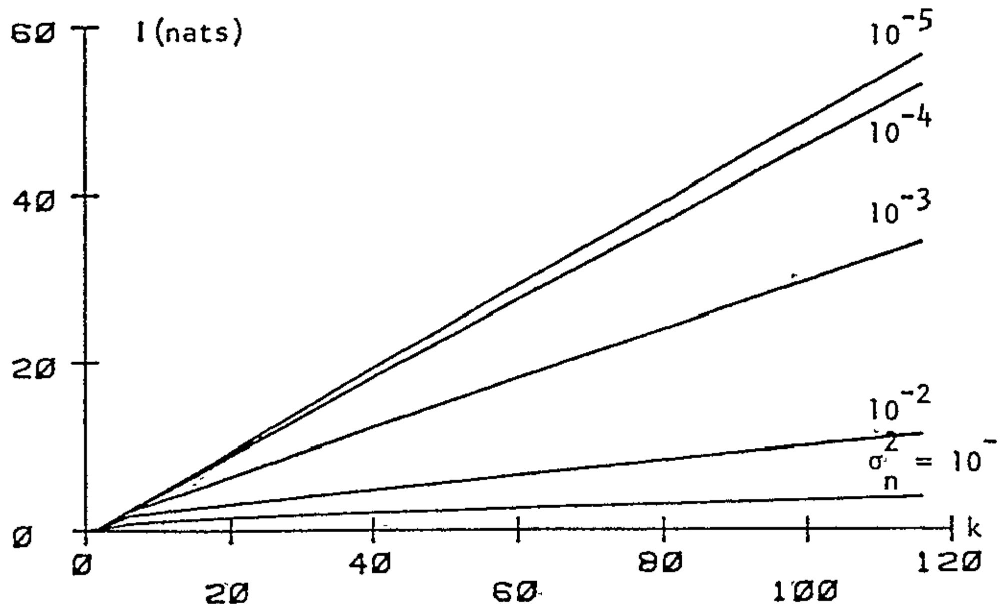


Figure 23. Average Information, Band 1, Wheat Scene.

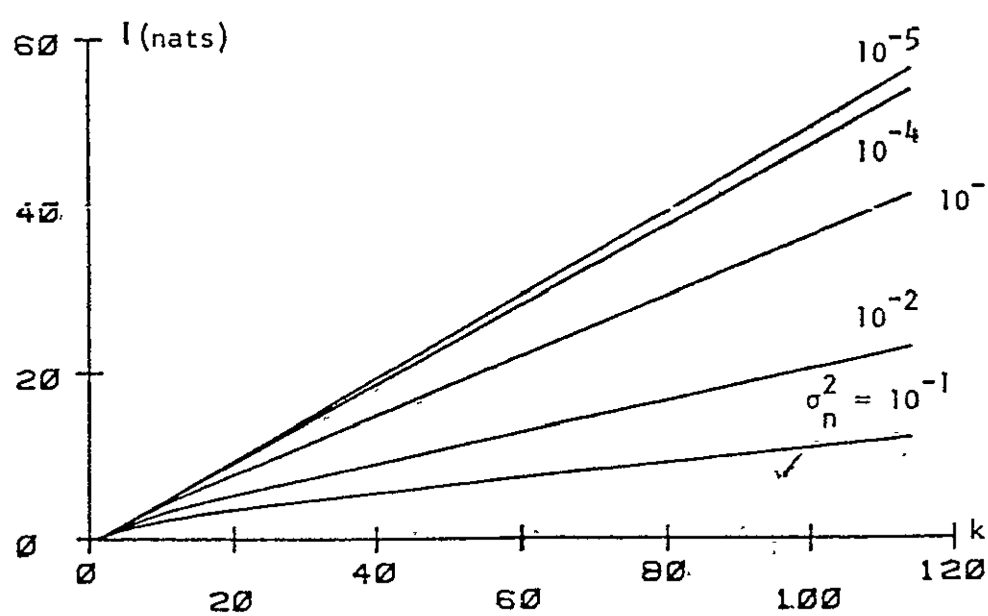


Figure 24. Average Information, Band 1, Combined Scene.

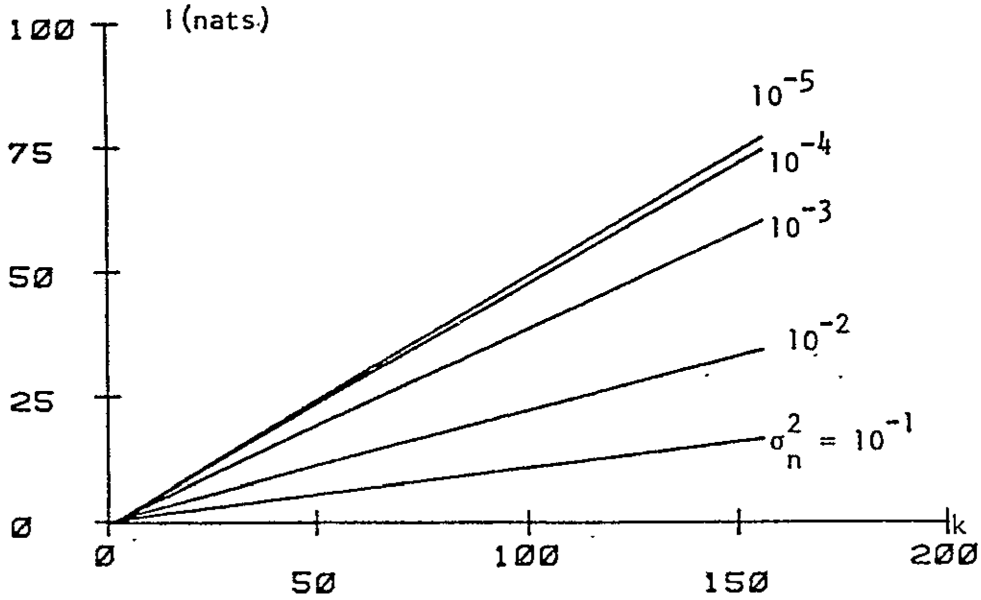


Figure 25. Average Information, Band 7, Wheat Scene.

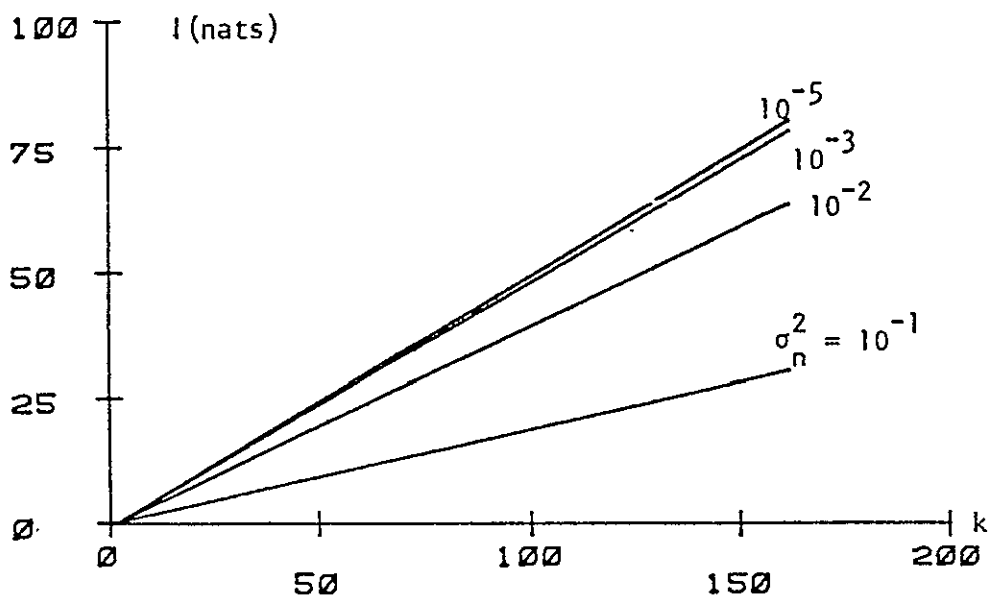


Figure 26. Average Information, Band 7, Combined Scene.

Table 10. Average Information for Combined Scene Band.

Band	$I(Y,S)$ nats
1	41.33
2	16.17
3	22.98
4	40.08
5	45.73
6	40.96
7	78.25
8	64.15
9	74.19

Using the information content of each band as an optimality criterion, the 9 spectral bands can be ranked, see Table 11.

Table 11. Order of Preference of Spectral Bands for the Wheat and Combined Scenes.

Rank	Wheat Scene Band	Combined Scene Band
1	7	7
2	9	9
3	5	8
4	6	5
5	8	1
6	1	6
7	4	4
8	3	3
9	2	2

The top 6 highest ranked bands, although ordered differently, are the same for both cover types. Moreover, other than band 1 which is in the visible portion, the remaining 5 are all in the infrared portion of the spectrum. Thus, relative to the average information criterion, the infrared portions of the spectrum is generally preferred to the visible portion since bands 2 and 3 are ranked lowest for both the wheat scene and combined scene.

The selection of a subset of the available spectral bands using the idea of their information content is a new approach in band selection and requires further investigation to evaluate its optimality in more concrete terms. One of the most useful optimality criterion is the selection of these bands that maximize the overall classification accuracy. No documented relationship exists between the average information contents of a set of bands and the subsequent class separability. It is true however, that such information measure is directly related to the optimum Weiner filter thereby providing a basis for the optimality of this ranking technique.

3. THE UNIFIED SCANNER ANALYSIS PACKAGE BLOCK DIAGRAM

The identification and development of a set of individual techniques and algorithms is only the first step toward a complete system simulation package. The usefulness of this package is fully realized only when the elementary modules are interconnected in a logical and clear fashion. The objective here is the integration of the available processors such that starting with a raw data base, the question of optimum spectral bands, IFOV size and the noise model can be answered with the classification accuracy as a primary performance index.

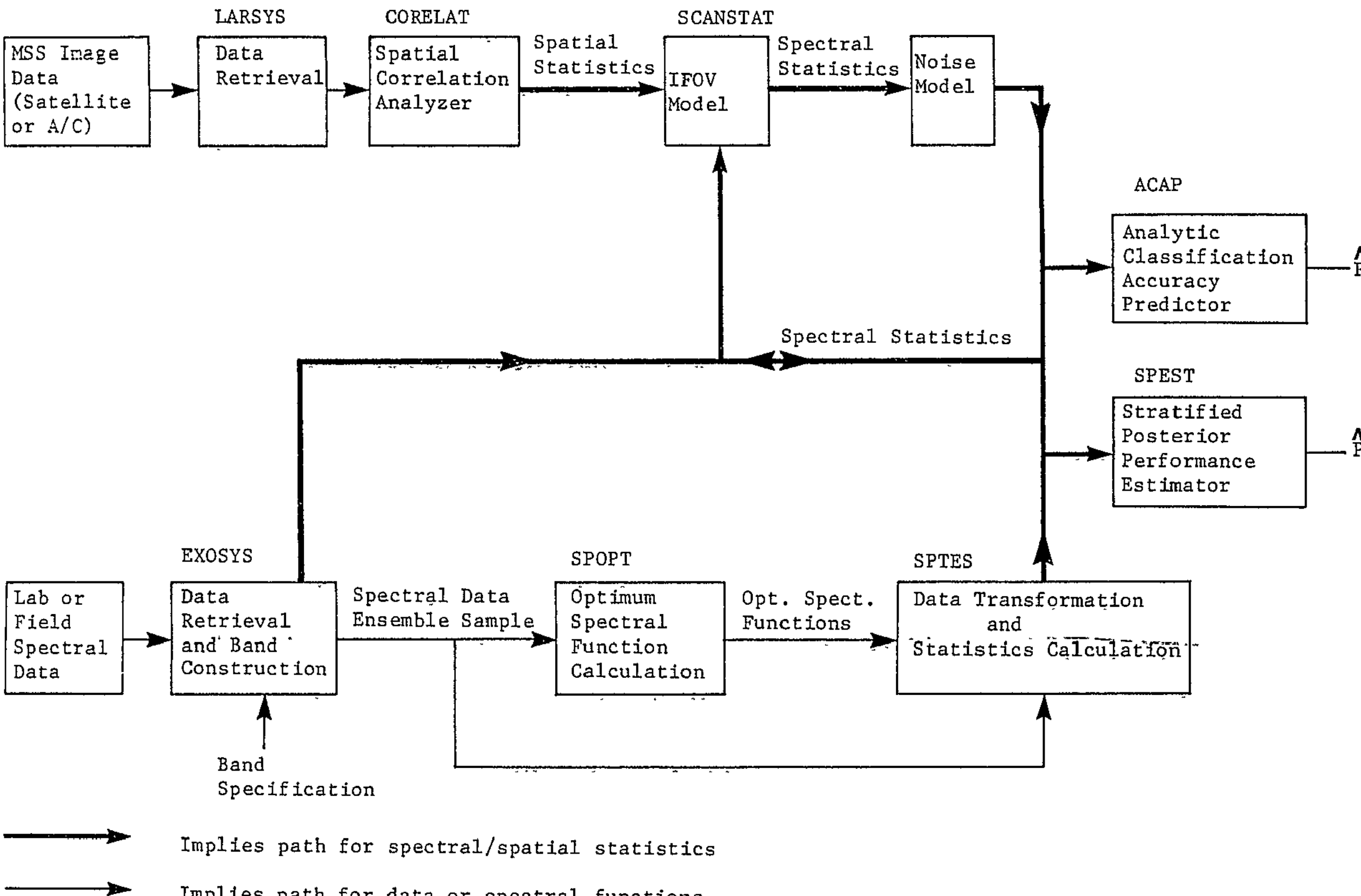
3.1 System Structure

One realization of such simulation model was shown in Figure 4 and is repeated here for convenience in Figure 27. USAP is basically composed of three distinct parts (a) a spatial path, (b) a spectral path and (c) a means for classification performance estimation. In the following individual software modules are discussed.

Classification Accuracy Estimators.

There are two classification performance estimators available (a) the analytic classification accuracy predictors and (b) the stratified posterior performance estimator.

Figure 27. The Block Diagram of the Unified Scanner Analysis Package (USAP).



Analytic Classification Accuracy Predictor. The ACAP algorithm discussed in Section 2.1 is the primary processor in evaluating the performance of a scanner system when the probability of correct classification is defined as the primary performance index. This piece of software, as shown in its theoretical development, requires one major input in the form of the population statistics. In order to facilitate the operations, the format of the statistics deck is chosen to be identical to the one produced by LARSYS statistics processor although it contains a considerable amount of redundancy such as field coordinates. These cards are skipped. Among other user-supplied information is the desired spectral bands to be used in the analysis and the sampling and grid fineness in the form of number of elementary cells per axis. There is obviously a trade-off between the estimator's variance, a decreasing function of the grid size, and the computation time. If N is the number of spectral bands and n the number of cells per axis, the per class number of cells to be tested in a set of M quadratic discriminant functions is n^N. This exponential relationship calls for a careful selection of n particularly for a high dimensional space. On the other hand a small variance is very much required property of any estimator.

The relationship between grid structure and the estimator's variance has been covered in detail [2]. It was shown that the classification accuracy obtained using ACAP exhibits a relative independence from n for $n \geq 12$. This property is preceded by a fairly rapid rise to a steady state value after which the magnitude of the $\hat{P}_c$ oscillations is within 0.5% of the true value or the Monte Carlo derived reference. The choice of n is ultimately decided by the user depending on his specific needs and after some experimentation. Initially, however, a default option of n = 9 cells per axis is considered to provide quick turn-around time while keeping the quality of the estimate high. The output, in addition to the classification accuracy estimate, contains information on the transformed class statistics, feature space and sampling grid structure.

Stratified Posterior Classification Performance Estimator. This is the software implementation of the algorithm discussed in Section 2.2. The maximum conditional aposteriori class probability is the criterion for classification and error estimation purposes. The program does not provide any options and the size of the internally generated random data is fixed. ACAP and SPEST produce different, but very close results.

Spatial Path.

Data Base. The input data to the spatial scanner model is via the multispectral image storage tape containing satellite or aircraft collected data. This tape has been reformatted and is compatible with any LARSYS processor.

Data Retrieval. The individual software units can access the available data base through various system support routines or any of the LARSYS processors.

Spatial Correlation Analyzer. The determination of the scanner characteristic function requires a knowledge of the spatial properties of the input data, therefore a class conditional estimate of the spatial auto and crosscorrelation functions is needed. Let $f_k(x,y)$ be a two dimensional image of size $N_o \times N_o$ pixels in the kth spectral band then the spatial autocorrelation function estimate is given by [16].

$$\hat{R}_{kk}(\tau,\eta) = C \sum_{i=1}^{N_o-\tau} \sum_{j=1}^{N_o-\eta} [f_k(i,j) - \mu_k] [f_k(i+\tau, j+\eta) - \mu_k]$$

$$\tau,\eta = 0, 1, \ldots, n_o-1 \tag{54}$$

where $\mu_k = E\{f_k(x,y)\}$. The multiplicative factor C can be chosen to be one of the following

$$C_1 = \frac{1}{(N_o-\tau)\ (N_o-\eta)} \tag{55}$$

$$C_2 = \frac{1}{N_o^2}$$

if μ_k is known and $C = C_1$ then $E\{R_{kk}(\tau,\eta)\} = R_{kk}(\tau,\eta)$. If $\mu_k=\hat{\mu}_k$ then neither selection of C_1 or C_2 will produce an unbiased estimate. The actual derivation of the mean and variance of $\hat{R}_{kk}$ when the mean is estimated is rather complicated. The bias of the estimate in one dimension is given by [16].

$$E\{\hat{R}_k(\tau) - R(\tau)\} = \frac{-|\tau|}{N_o}R(\tau) - \frac{N_o-|\tau|}{N_o} \text{ Var } \{\hat{\mu}_k\} + o(N_o^{-2}) \tag{56}$$

From Eq. (56) it follows that $\hat{R}_k(\tau)$ is asymptotically unbiased. This result can be extended to the two dimensional functions provided the autocorrelation function is separable along each spatial axis. In general the maximum lag, n_o, must be chosen such that $n_o << N_o$. As a rule of thumb it is desirable to keep the maximum lag less than one-tenth the sample size N_o. This will tend to avoid certain instabilities that can occur in autocorrelation function estimates. The across-band correlation function estimate is obtained using an identical relationship to Eq. (54).

The empirically obtained spatial correlation matrix needs further processing to be used in the scanner spatial model developed in Section 2.3. Specifically a Markov correlation model is fitted to the experimentally obtained $\hat{R}_{kk}(\tau,\eta)$. By invoking the separability assumption for small lag values,

$$\hat{R}(\tau;\eta) \simeq \hat{R}(\tau)\ \hat{R}(\eta) \tag{57}$$

where no subscript indicates either auto or crosscorrelation function. Table 12 shows the magnitude of the errors involved in carrying out this approximation on an aircraft data set. The error is expressed as a percentage of the experimental values. An exponentially dropping function is then fitted to the individual correlation functions along the sample and line directions. The fitting is accomplished using a

Table 12. Error Matrix for Cross Correlation Function Approximation Between Channels 2 and 8.

$$R_{28} = \begin{bmatrix} 1.00 & .92 & .81 & .69 & .59 & .50 & .44 \\ .93 & .88 & .78 & .67 & .56 & .48 & .41 \\ .73 & .71 & .64 & .54 & .44 & .36 & .3 \\ .48 & .47 & .43 & .36 & .28 & .21 & .16 \\ .30 & .31 & .29 & .24 & .18 & .12 & .08 \\ .23 & .25 & .24 & .21 & .16 & .12 & .08 \\ .22 & .24 & .24 & .22 & .19 & .15 & .12 \end{bmatrix}$$

$$\hat{R}_{28} = \begin{bmatrix} 1.00 & .92 & .81 & .69 & .59 & .50 & .44 \\ .93 & .86 & .75 & .64 & .55 & .46 & .4 \\ .73 & .67 & .6 & .5 & .43 & .36 & .32 \\ .48 & .44 & .38 & .33 & .28 & .24 & .71 \\ .30 & .27 & .24 & .2 & .17 & .15 & .13 \\ .23 & .21 & .18 & .16 & .13 & .11 & .1 \\ .22 & .2 & .18 & .15 & .13 & .11 & .1 \end{bmatrix}$$

$$\underline{E}_{28} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.2 & 3.8 & 4.5 & 1.8 & 4.16 & 2.43 \\ 0 & 5.6 & 6.2 & 7.4 & 2.3 & 0 & 6.75 \\ 0 & 6.4 & 11.6 & 8.3 & 0 & 17.5 & 23.8 \\ 0 & 13.0 & 17.2 & 16.6 & 5.5 & 20 & 38.4 \\ 0 & 16 & 25 & 23.8 & 18.7 & 8.3 & 20 \\ 0 & 16.6 & 25 & 31.8 & 31.6 & 26.6 & 16.6 \end{bmatrix}$$

weighted least square approach to the logarithm of $R(\tau)$ and $R(\eta)$. The slope of this linear fit determines the adjacent sample or line correlation coefficients. Specifically let

$$F(i) = \ln \hat{R}(i) \qquad i = 0, 1, \ldots, n_o-1$$

then the parameters of the linear fit, C_1+C_2x, are given by [17]

$$\underline{C} = (\underline{H}^T \underline{W}\, \underline{H})^{-1} \underline{H}^T \underline{W}\, \underline{F} \tag{58}$$

where

$$\begin{cases} H(i) = 1 & i = 0, 1, \ldots, n_o-1 \\ H(i) = 2n_o-1-i & i = n_o, n_o+1, \ldots, 2n_o-1 \end{cases} \tag{59}$$

and the diagonal weighting matrix, W

$$W(i) = \alpha^{(n_o-1-i)} \qquad i = 0, 1, \ldots, n_o-1 \tag{60}$$

with α as the weighting matrix diagonal base, $o \leq \alpha \leq 1$. The weighting matrix via the control parameter assigns a progressively smaller weight to $\hat{R}(\tau,\eta)$ for succeeding lag values. This weighting is necessary since the properties of the correlation functions show an increasing deviation from the underlying assumptions of separability and Markovian structure for higher lag values.

A complete specification of the spatial properties of the available spectral classes requires determination of

$$N + \frac{N!}{2!\ (N-2)!}$$

auto and crosscorrelation functions per spatial axis per class where N is the total number of spectral bands used in the analysis. The implementing software contains various default provisions in case the correlation properties of the input data differs considerably from the aforementioned assumptions.

The user specifies the area to be correlated by a run table entry run number followed by the field coordinates. In order to perform either the auto or crosscorrelation operations, appropriate spectral bands(s) need to be specified. The maximum lag, n_o, in computing $\hat{R}(\tau,\eta)$ is also a variable and is entered as a percentage of the image size in pixels. The value of n_o is dependent on the size of the area to be correlated. Since the magnitude of $\hat{R}(\tau,\eta)$ for Landsat data is generally negligible for more than 4 or 5 pixels lag, n_o as a percentage can take on small values for large fields and vice versa.

Scanner IFOV Model. The scanner IFOV software is the computer implementation of the scanner characteristic function discussed in Section 2.3. The input consists of (a) spectral covariance matrix, (b) spatial correlation matrix along the samples (c) spatial correlation matrix along the lines and (d) IFOV size in terms of the number of high resolution pixels. A standard LARSYS statistics deck produced by the statistics processor constitutes the first item. The spatial correlation information in entered through an N x N symmetric matrix the (i,j) element of which

$$\rho_{ij} = e^{-a_{ij}}$$

is the pixel-to-pixel correlation for bands i and j. a_{ij} is estimated by the spatial correlation software. The IFOV size is expressed on a relative scale in terms of the number of high resolution pixels within 1 IFOV of the scanner PSF, e.g., 1, 2, 3, etc. There are two choices available for the functional form of the PSF, Gaussian and rectangular.

The output generated by this software module is a spectral statistics deck which is the input class statistics transformed by the scanner characteristic function. This deck is used as input to the ACAP processor.

Additive Noise Model. By virtue of the parametric approach adopted here, the incorporation of the noise effect takes on a simple form. The noise statistics is characterized by a zero mean vector and a diagonal covariance matrix with zero off diagonal elements. This matrix is then simply added to each class covariance matrix,

$$\underline{\Sigma}_{f''} = \underline{\Sigma}_{f'} + \begin{bmatrix} \sigma^2_{n_1} & & & & \\ & \sigma^2_{n_2} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \sigma^2_{n_N} \end{bmatrix} \tag{62}$$

The diagonal elements of $\underline{\Sigma}_N$, $\sigma^2_{n_i}$, determines the SNR in each spectral band. By the appropriate selection of σ_{n_i}'s, different SNR can be specified for each band. Let $\sigma^2_{f_k}$ denote the variance of the noise-free signal at the scanneroutput, then the SNR in the kth spectral band is defined by

$$(SNR)_k = \sigma^2_{f_k} / \sigma^2_{n_k} \tag{63}$$

The choice of equal or unequal SNR in different bands based on experimental or theoretical results is at the analyst's discretion.

Spectral Path.

Data Base and Retrieval. EXOSYS is a software package which provides access to field measurement data taken with a variety of field instruments. A brief overview of the EXOSYS package will be given here, with more detail available in the EXOSYS manual [18]. Data is collected and stored on magnetic tape in field measurements format. During the reformatting procedure the data is calibrated and ancillary information such as weather readings soil conditions, and plant growth status is placed in the identification record for each run.

There are three processors in EXOSYS which are used to access the field data information - IDLIST, GSPEC, and DSEL. The IDLIST processor scans the tape and lists information from the identification record as required. One can use this information to select appropriate runs to represent informational classes.

The GSPEC processor provides a punched deck consisting of the numerical values of the spectral response function for all of the desired runs. One can select a set of run numbers as input and the output will consist of a punched deck. Options exist for plotting the spectral response functions for the desired runs.

The DSEL processor simulates rectangular spectral channels and uses data from the tape to evaluate the response in each channel for the ensemble. The inputs are the spectral band locations and the run numbers on the data tape. The output is a set of statistics for the specified channels.

Optimum Spectral Basis Function System. For the optimum spectral function calculation the output of the GSEC processor is required. The appropriate ensemble can be selected by specifying a set of identification parameters such as date of collection, scene type, run number, etc.

The cards containing the numerical values for the spectral response functions are used and stored on disk in a format which is compact and convenient for future processing by the program SPRDCT. The files that are stored on disk may be transferred to magnetic tape for future use to avoid repeating the procedure involving the EXOSYS package. SPRDCT requires some information to be entered at the terminal to provide ID information for the ensemble. A list of all runs used by run number is printed after the data has been stored on disk.

Optimum Basis Function Calculation. The calculation of the optimum set of basis functions for an ensemble is accomplished by solving the matrix equation

$$\phi\Gamma = KW\Phi \tag{64}$$

to get the eigenvalues $\sigma_1, \sigma_2, \ldots, \sigma_N$ and the eigenvectors $\phi_1, \phi_2, \ldots, \phi_N$. The matrix Φ is the matrix of eigenvectors $\Phi = [\phi_1, \phi_2, \ldots, \phi_N]$ and Γ is the diagonal matrix of eigenvalues

$$\Gamma = \begin{bmatrix} \sigma_1 & 0 & & \\ 0 & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \end{bmatrix} \tag{65}$$

The matrix W is a diagonal matrix of weights

$$W = \begin{bmatrix} W_1 & 0 & & \\ 0 & W_2 & & \\ & & \ddots & \\ & & & W_N \end{bmatrix} \tag{66}$$

R is the covariance matrix for the ensemble. Let the mean vector for the ensemble be $m = [m_1, m_2, \ldots, m_N]^T$ then

$$R_{ij} = E\,\{(x_i-m_i)\ (x_j-m_j)^T\} \tag{67}$$

The maximum likelihood estimate is

$$\hat{R}_{kj} = \frac{1}{N_S-1}\sum_{k=1}^{N_S} (x_{ik}-m_i)\ (x_{jk}-m_j)^T \tag{68}$$

where N_S is the number of sample functions in the ensemble.

If we let A=KW, then A is a real general matrix. An algorithm for solving for the eigenvalues and eigenvectors of a real general matrix is available [19] and is used here with only slight modification. The algorithm makes use of Householder's method and the QR double-step iterative process to compute the eigenvalues. The eigenvectors are obtained by the inverse iteration technique. A sorting routine was added to order the eigenvalues and the corresponding eigenvectors.

The required inputs are the data in the appropriate format and the weight function. The output of this processor is a set of N eigenvectors or basis functions punched onto cards. Also, the eigenvalues and means-square representation error are printed. The eigenvectors can be plotted using GCS subroutines.

Data Transformation and Statistics Calculation. The eigenvectors are used to perform a linear transformation on the original data vectors $\{\underline{X}\}$. The transformed vectors $\{\underline{Y}\}$ have the desired properties provided by the Karhunen-Loève expansion. Each element of the transformed vector is given by

$$y_i = \phi_{i1}x_1 + \phi_{i2}x_2 + \dots + \phi_{iN}x_n \tag{69}$$

where ϕ_{ij} is the jth element of the ith eigenvector.

The inputs to this processor are the eigenvectors and the data set stored on the disk. The output is the set of statistics for each class. The statistics are printed and punched on a deck of cards for future processing.

4. USER'S GUIDE TO USAP

The block diagram of the scanner parameter study, Figure 27, is made operational by a collection of compatible software packages. Each module is individually compatible with the LARSYS environment facilitating incorporation of LARSYS processor in the overall system performance evaluation. This section provides a guide for the acquisition and execution of each program available on the LARS IBM 370/148 computer system.

Prior to a discussion of the individual modules some general remarks are in order. The access to this program library is simplified by the allocation of two special disk storage devices designated by DHSYS and DHDSK. The former contains the text version of the software while the latter holds the source. These devices can be accessed using the appropriate GETDISK commands.

GETDISK DHSYS

and

GETDISH DHDSK

these commands will establish the proper links in a Read-Only mode and USAP initialization is complete. In the following subsections the required input and necessary steps to run each program are discussed.

4.1 The Classification Accuracy Estimators

There are two parametric classification accuracy estimators available to the USAP user, (a) the analytic classification accuracy predictor (ACAP) and (b) the stratified posterior classification accuracy estimator (SPEST). The theoretical aspects of these processors have been discussed in Section 2. Here is a guide to their software implementation.

Analytic Classification Accuracy Predictor

This program evaluates the performance of a Bayes classifier when the populations statistics are multivariate normal. The following control cards are required.

Control Word	Description
*ACAP	This card specifies the particular processor requested.
CHANNELS	The desired subset of the available channels is given here. Note that the numbers appearing on this card are the order of the selected channels not their actual number. For example, if the available channels are 8, 9, 12, 14 and channels 8, 9 and 14 are requested CHANNEL card should read 1, 2, 4.
CLASSES	This card specifies the name of each class. Each name must be placed in a field 7 characters long followed by a blank. The continuation card, if required, must have the word "CLASSES" at the beginning.
GRID	This quantity controls the quality of the estimate. The higher the number the closer the estimate is to the true Bayes error rate. (See "estimated CPU time" for more details.)
END	This card signals the end of the control card. Stat deck follows immediately.

Remarks. The ACAP processor in its current form is capable of handling up to 20 classes and 8 spectral bands. The extensions of these parameters presents no conceptual difficulty. The required statistics deck is a standard LARSYS produced deck with no modifications. It must be punched in the character format, however.

How to Run the Program. Make sure the DHSYS disk has been accessed properly. One reader file consisting of the control cards followed by a statistics deck is required. Type ACAP in the CMS environment. Appropriate terminal and printer output is produced.

Example of control card set up

```
*ACAP
CHANNELS 1,3,4
CLASSES SOYBEAN ALFALFA WHEAT
END
```

Since GRID card is not included, its default value (9) is selected.

Estimated CPU Time. The execution time is quite sensitive to the GRID card specifying the number of cells per axis. For the default grid size and a 4-dimensional space it takes approximately 2 minutes of CPU time per class to provide the requested classification accuracy estimates. The CPU time is most sensitive to the dimensionality of the feature space. Hence if the number of spectral bands is limited (less than 4) considerable increase in GRID number is possible. The default number of cells per axis is considered to be the minimum while still providing acceptable performance. Increasing the parameter improves the quality of the estimate somewhat at the expense of higher CPU time. The choice is left at the user's discretion.

Stratified Posterior Error Estimator.

This program is identical in purpose but different in approach to the ACAP processor. Given a set of multivariate normal populations, SPEST provides the classification accuracies associated with each class using an internally generated random data base. The different estimation procedures between the two methods is transparent to the user.

Description of the Control Cards

Control Word	Description
*SPEST	This card specifies the particular processor requested.
CHANNELS	The desired subset of the available channels is given here. Note that the numbers appearing on this card are the order of the selected channels not their actual number.
CLASSES	This card specifies the name of each class. Each name must be placed in a field 7 characters long followed by a blank. The continuation card, if required, must have the control word "CLASSES" in the beginning followed by the rest of the names.
END	End of the control cards.

Remarks. In usage, this program is identical to *ACAP. The standard LARSYS statistics deck follows the control and disk immediately. Printer output contains the estimated conditional classification accuracies. By virtue of their separate approaches, *ACAP and *SPEST provide different, but very close, estimates of the correct classification accuracies.

How to Run the Program. The reader file contains the control cards followed by the LARSYS statistics deck. A sample control card deck follows:

```
*SPEST
CHANNELS 1,2,4
CLASSES ALFALFA SOYBEAN WHEAT
END
```

4.2 Spatial Path

The spatial path in USAP consists of two main software units. The spatial correlation analyzer, CORELAT and the SCANNER IFOV model, SCANSTAT.

Spatial Correlation Modeling. This program is a 2-dimensional spatial correlator the primary output of which is a normalized spatial auto (cross) correlation matrix for any specified area. The user specifies the coordinates of the desired segment in the form of an initial and final line and column along with the appropriate spectral bands(s). Following the estimation of the correlation matrix, the exponential fit option, if invoked, will fit an exponentially dropping function to the experimental values of $R_{kk}(\tau)$ or $R_{kk}(\eta)$ using a weighted linear least squares technique.

Description of the Control Cards

Control Word	Control Parameter	Description
*CORRELATE		This card specifies the particular processor requested.
INPUT	RUN(.)	Run number of the desired area.
	TAPE(.)	Tape number of the desired area.
	FILE(.)	File number of the desired area.
BLOCK	LINE(.,.)	Initial and final lines.
	COLUMN(.,.)	Initial and final columns.
FUNCTION	AUTO	Autocorrelation function requested.
	CROSS	Crosscorrelation function requested.
CHANNELS		Channels used for correlation operation.
SAMPLELAG†		Maximum cross track lag used as a percentage of the total number of samples.
LINELAG†		Same as SAMPLELAG except for along track lag.
EXPOFIT†		If included exponential fitting operation is carried out.
END		End of control cards.

Remarks. This program is capable of processing areas containing up to 2400 pixels. The maximum lag default is set at 20 percent of the total number of lines and columns. Both quantities can be altered by user supplied control cards. The exponential fit option provides a pixel-to-pixel correlation coefficient for the channel(s) specified. This number is computed from the estimated parameters of the exponential correlation model.

How to Run the Program. The only required reader file is the control card deck, an example of which follows:

---

† optional

```
*CORRELATE
INPUT RUN (74028500), TAPE (2689), FILE (3)
BLOCK LINE (1,25), COLUMN (1,25)
FUNCTION AUTO
CHANNELS 2
SAMPLELAG† 25
LINELAG† 25
EXPOFIT†
END
```

after DHSYS disk has been properly linked to, type CORELAT to start execution. Appropriate terminal and printer output is generated.

Scanner IFOV Model.

This program computes the spectral statistics of a population at the output of a multispectral scanner provided the data spatial correlation approximately follows a Markov model. The scanner IFOV shape is limited to either a Gaussian or rectangular shape. No assumptions are made or indeed required about the type of the population statistical distribution.

Description of the Control Cards

Control Word	Description
*SCANSTAT	This card specifies the particular processor requested.
CHANNELS	The desired subset of the available channels is given here. Note that the numbers appearing on this card are the order of the selected channels not their actual number.
CLASSES	This card specifies the name of each class. Each name must be placed in a field 7 characters long followed by a blank. The continuation card, if required, must have the control work "CLASSES" in the beginning followed by the rest of the names.
IFOV	This card specifies the IFOV size of the scanner in terms of high resolution input pixels.

† optional

APERTURE	The choices here are "GAUSSIAN" or "RECTANGULAR."
SNR†	Output signal energy to noise energy in dB.
PUNCH†	The output statistics is punched out in an ACAP/SPEST format. Redundancies are added to replace field description cards.
END	End of control cards.

Remarks. This program is limited to 20 classes and 8 spectral bands. Execution time is quite short and extension of those parameters is straightforward. The input data immediately following the control cards consists of 3 parts:

1. Standard LARSYS statistics deck in character format.
2. Spatial correlation parameters (cross track) are entered via a NXN symmetric matrix where N is the number of channels. The (i,j) element of this matrix is the adjacent sample correlation between channels i and j. The lower triangular part of this matrix is punched in a 5 (E13.7,1X) format.
3. Spatial correlation parameter matrix except for along track pixels.

The above decks follow the control cards in the order listed. The signal-to-noise ratio is defined as the ratio of the output signal energy in a particular channel (diagonal element of the class covariance matrix) to the noise energy in the same bands expressed in dB and defined by

$$(\mathrm{SNR})_k = 10 \log_{10} \sigma^2_{s_k} / \sigma^2_{n_k}$$

where k refers to the particular spectral band.

---

† optional

How to Run the Program. One reader file consisting of 4 consecutive decks and appropriate link to DHSYS disk is required before the program execution. An example of a control card set up follows:

```
*SCANSTAT
CHANNELS 1,2,4
CLASSES ALFALFA SOYBEAN WHEAT
IFOV 2
APERTURE GAUSSIAN
SNR† 10
PUNCH†
END†
```

The output consists of an ACAP/SPEST compatible statistics deck for the modified population. This deck can be used in the ACAP/SPEST processors to obtain the new set of classification accuracy estimates.

4.3 Spectral Path

The spectral path in USAP consists of three main pieces of software (a) data retrieval through EXOSYS processor (b) optimum spectral function calculation and (c) data transformation and statistics calculation.

Procedure for Computing and Evaluating Optimum Basis Functions.

Data Retrieval. The data retrieval system is stored on EXOSYSDV and it is necessary to define storage as 768K. A card file containing the data points for each run will be constructed on a temporary disk (25 cylinders). It is desirable to make the temporary disk a P-disk and the permanent user disk a T-disk.

In CMS

```
RELEASE 191 P
LOGIN 191 T
GETDISK TEMP MODE P25CYL NO-UFD
```

† optional

In CP

```
I EXOSYSDV
CCINPUT TERMINAL
RUN EXOSYSDV
```

The control cards will be entered through the terminal. The tape on which the data is stored must be specified as well as the cover type and the collection date. A typical sequence of cards is as follows:

```
$ TAPE 4896
$ GSPEC
     GRAPH SPEC (SPRING WHEAT), DACO (770508)
     LIST NO LIST
     OPTIONS PUNCH, NOGRAPH
     END
$ END
$ EXIT
```

The runs taken over Williams Co., North Dakota for May 8, 1977 are on tape 4896. The crop species is spring wheat and the collection date is May 8, 1977. The output is a deck of cards with one hundred data values for each run punched onto the cards. Header information must be flushed at the line printer. Return the system to CMS and read the cards onto a disk file SPR100 DATA. The number of records in the file is equal to the number of runs. This number should be recorded as it will be needed later. This procedure is repeated for the second class. The cards from GSPEC are read onto the file INPUT DATA and the number of records recorded. The two files are combined by typing (in CMS)

```
COMBINE SPR100 DATA P1 SPR 100 DATA P1 INPUT DATA P1
ERASE INPUT DATA
```

This procedure is repeated until all classes have been included in the file SPR 100 DATA. The crop species SUMMER FALLOW and PASTURE are used in GRAPH SPEC(.) to complete the data set.

At this point the program SPRDCT is loaded and run. A disk file will be created using DSRN of 2 and file type FUNC. The following information is requested at the terminal by SPRDCT

Experiment Number			100
Number of Classes			3
Number of Sample Points per Run (Dimensions)			100
Class Name	Wheat	Fallow	Pasture
Number of Samples per Class	664	437	164

The information is requested and is entered between the slash marks, right justified.

At this point the data is ready to be used by the system. It is a good idea to store the file on tape for future use. Type

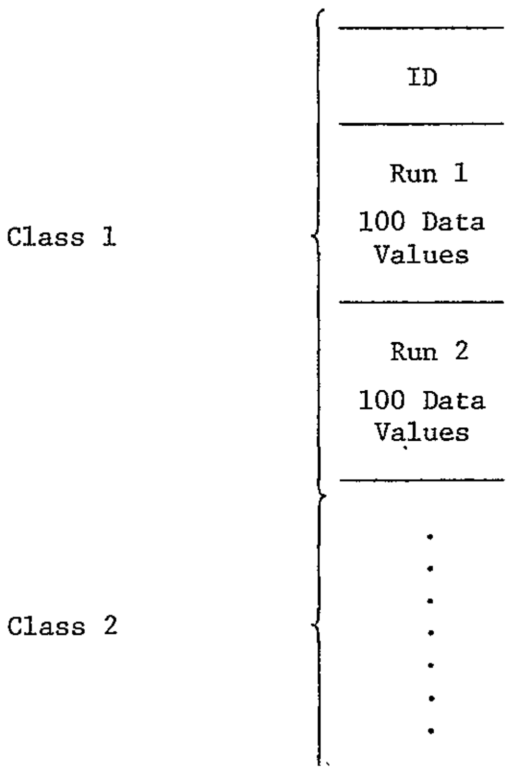
```
TAPMOUNT 156 TAP2 RI
T DUMP SPR 100 FUNC P1
```

The tape on which the data is stored is 156 in this example. To recall the data to the disk type

```
TAPMOUNT 156 TAP2 RO (If not already mounted)
T SLOAD SPR 100 FUNC
```

Note that the P-disk must be a large fairly empty disk (10 cyl)

The format for data storage is



ID Information Record

Item	Words
Date	1-15
Exp. Number	16
Number of Classes	17
Number of Dimensions	18
Number of Samples for Class 1	21
Number of Samples for Class 2	22
Number of Samples for Class 3	23
Number of Samples for Class 4	24
Number of Samples for Class 5	25
Number of Samples for Class 6	26
Number of Samples for Class 7	27
Name of Class 1	30-39
Name of Class 2	40-49
Name of Class 3	50-59
Name of Class 4	60-69
Name of Class 5	70-79

Name of Class 6	80-89
Name of Class 7	90-99

Optimum Spectral Functions Calculations.

Once the data set is on disk it is necessary to issue the following CMS commands to compute the eigenvectors.

```
FILEDEF 2 DSK-P4 SPR100 FUNC RECFM VS LRECL 400 BLKSIZE
400 (PERM
CP SP PUN TO USERID
LOAD SPOPTM (XEQ
```

4.4 Example Outputs

This section presents a sample output for the individual software units used in USAP. The example set consists of sample outputs from the ACAP and SPEST processors. CORELAT and SCANSTAT in the spatial path and SPOPT and SPTES in the spectral path. Graham Co., Kansas is used as the test site.

Classification Accuracy Estimators.

The following control card set up is used for the ACAP processor and output is shown in Table 13.

```
*ACAP
CHANNELS 1,2,3,4
CLASSES BARESOI CORN SOYBEAN WHEAT
END
```

The required control cards for the SPEST processor are

```
*SPEST
CHANNELS 1,2,3,4
CLASSES BARESOI CORN SOYBEAN WHEAT
END
```

the output is shown in Table 14.

Table 13. *ACAP Sample Output.

ANALYTIC CLASSIFICATION ACCURACY PREDICTION - A C A P

CLASS WHEAT

SAMPLING GRID CHARACTERISTICSO

GRID SIZE= 9 CELLS PER DIMENSION

TOTAL NO OF CELLS IN THE GRID= 6561

TRANSFORMED FEATURE SPACE CHARACTERISTICSO

EIGENVALUESO

2.9182E 01	1.1413E 01	1.3430E 00	7.6125E-01

EIGENVECTORSO

2.1045E-01	2.0478E-01	8.2319E-01	4.8593E-01
4.5844E-01	8.2659E-01	-1.5239E-01	-2.8873E-01
-6.7334E-03	1.9331E-01	-5.3271E-01	8.2390E-01
8.6342E-01	-4.8728E-01	-1.2389E-01	4.1283E-02

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TRANSFORMED MEAN VECTORSO

1.1654E 01	2.3928E 01	3.8200E-01	-5.4750E-02
-2.2766E 00	1.5248E 01	1.3871E-01	1.0838E-01
1.3111E 01	5.1333E 00	-5.9814E-01	1.0702E 00
0.0	0.0	0.0	0.0

P R O B A B I L I T Y O F C O R R E C T C L A S S I F I C A T I O N = 93.486 (

*****TOTAL PROB OF CORRECT CLASSIFICATION*****= 89.287 PERCENT

Table 14. *SPEST Sample Output.

```
STRATIFIED POSTERIOR ERROR ESTIMATOR

                CLASS BARESOI

P R O B A B I L I T Y   O F   C O R R E C T   C L A S S I F I C A T I O N =  79.218(

                CLASS CORN

P R O B A B I L I T Y   O F   C O R R E C T   C L A S S I F I C A T I O N =  92.721(

                CLASS SOYBEAN

P R O B A B I L I T Y   O F   C O R R E C T   C L A S S I F I C A T I O N =  96.359(

                CLASS WHEAT

P R O B A B I L I T Y   O F   C O R R E C T   C L A S S I F I C A T I O N =  92.614(

   OVERALL PROBABILITY OF CORRECT RECOGNITION =  90.228
```

Spatial Path.

The control card set up for CORELAT is as follows:

```
*CORRELAT
INPUT RUN(74028500), TAPE(2689), FILE(3)
BLOCK LINE(50,98), COLUMN(50,98)
FUNCTION AUTO
CHANNELS 1
EXPOFIT
END
```

The sample output is shown in Table 15.

The control card set up for SCANSTAT is as follows:

```
*SCANSTAT
CHANNELS 1,2,3,4
CLASSES BARESOI CORN SOYBEAN WHEAT
IFOV 2
APERTURE GAUSSIAN
END
```

The sample output is shown in Table 16.

Spectral Path.

The optimum basis function calculation and computation of the transformed data statistics comprises the main spectral processors. The sequence of required commands has been shown in Section 4.3. The following example is a weighted basis function calculation.

The weighting function $w(\lambda)$ is zero for the water absorption bands near 1.4 and 1.8 micrometer and zero elsewhere on the interval (.4-2.4 μm). The printer output is shown in Table 17, listing the first 30 eigenvalues. The first 4 eigenvectors are sent in card format to the reader. They can be stored on the disk by issuing the command.

```
0 READ EIG100 DATA T1
```

Table 15. *CORRELAT Sample Output.

```
TWO DIMENSIONAL SPATIAL CORRELATION ANALYSIS

CHANNELSO  1  1


2-D SPATIAL CORRELATION MATRIX

 1.00  0.75  0.50  0.36  0.28  0.23  0.19  0.13  0.08
 0.70  0.60  0.44  0.33  0.26  0.20  0.17  0.12  0.07
 0.50  0.45  0.36  0.27  0.21  0.17  0.14  0.10  0.05
 0.38  0.37  0.31  0.24  0.17  0.13  0.11  0.08  0.03
 0.31  0.32  0.27  0.21  0.14  0.11  0.10  0.08  0.04
 0.25  0.26  0.23  0.16  0.11  0.09  0.07  0.05  0.02
 0.20  0.20  0.18  0.13  0.10  0.07  0.06  0.05  0.03
 0.14  0.15  0.14  0.10  0.07  0.05  0.04  0.04  0.02
 0.10  0.11  0.12  0.11  0.09  0.06  0.03  0.02  0.00

        WEIGHTED LEAST SQUARES FIT INFORMATION

WEIGHTING MATRIX DIAGONAL BASE=0.40
WEIGHTED LSF ERROR (CROSS TRACK)= 0.1037569E-02
ADJACENT SAMPLE CORRELATION= 0.7193716E 00
WEIGHTED LSF ERROR (ALONG TRACK)= 0.6734634E-05
ADJACENT LINE CORRELATION    = 0.7026460E 00
```

Table 16. *SCANSTAT Sample Output.

SCANNER OUTPUT STATISTICS

APERTURED GAUSSIAN

IFOV SIZED 2 HIGH RESOLUTION PIXELS

CLASS CORN

INPUT COVARIANCE MATRIX

9.29			
12.26	19.79		
10.63	16.37	16.09	
4.43	7.14	6.24	3.45

OUTPUT COVARIANCE MATRIX

4.86			
4.61	10.37		
4.00	6.15	8.43	
1.67	2.68	2.34	1.81

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LABORATORY FOR APPLICATIONS OF REMOTE SENSING
PURDUE UNIVERSITY
SAMPLE FUNCTION INFORMATION 10 JULY, 1978

EXP. NO..............................100
NUMBER OF CLASSES.................. 3
CLASS...............................WHEAT
NUMBER OF SAMPLE FUNCTIONS..........664
CLASS...............................FALLOW
NUMBER OF SAMPLE FUNCTIONS..........437
CLASS...............................PASTURE
NUMBER OF SAMPLE FUNCTIONS..........164

WEIGHTING FUNCTION NUMBER 3

N	EIGENVALUE	VAR(GAM)	VAR(PHI)	MEAN-SQUARE ERROR
1	311.4133	307.0752	0.0086	299.156749
2	229.3520	166.5619	0.0067	69.804748
3	21.1702	1.4191	0.0104	46.634554
4	15.4660	0.7574	0.0126	33.168544
5	8.8838	0.2499	0.0106	24.284784
6	5.7765	0.1057	0.0129	18.508269
7	3.5611	0.0402	0.0237	14.947187
8	2.6887	0.0229	0.0578	12.256489
9	2.3128	0.0169	0.0635	9.945677
10	1.8363	0.0107	0.0444	8.109393
11	1.4195	0.0064	0.0663	6.689860
12	1.2304	0.0048	0.0643	5.459494
13	0.9300	0.0027	0.0336	4.529464
14	0.6806	0.0015	0.0338	3.848828
15	0.5217	0.0009	0.0351	3.327085
16	0.3517	0.0004	0.1206	2.975436
17	0.3138	0.0003	0.3414	2.601599
18	0.2945	0.0003	0.4782	2.307111
19	0.2771	0.0002	0.3430	2.090019
20	0.2336	0.0002	0.2196	1.856411
21	0.2102	0.0001	35.7262	1.646236
22	0.2092	0.0001	35.7220	1.437049
23	0.1792	0.0001	0.1469	1.257806
24	0.1514	0.0001	0.5298	1.106359
25	0.1452	0.0001	0.5206	0.961120
26	0.1137	0.0000	0.0943	0.847458
27	0.0976	0.0000	0.1201	0.749854
28	0.0851	0.0000	0.1318	0.664785
29	0.0744	0.0000	0.1153	0.590391
30	0.0583	0.0000	0.9213	0.532112

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Table 17. Eigenvalue and Mean-Square Representation Error for the Data Set.

A GCS routine was used to plot the graphs of the eigenvectors the first of which are displayed in Figures 28 through 31.

The statistics for the first 4 terms are computed by using the same FILEDEF command as above plus

```
CP SP PUN TO USERID
O PUNCH EIG100 DATA TI
LOAD SPTES (XEQ)
```

The program will ask 'NUMBER OF TERMS?', to get all 4 terms type '4.'

The output will be a statistics deck with the following format:

Card	
1	Number of Classes, Number of Terms
2	Apriori Probabilities for each Class = 1/Number of Classes
	Mean Class 1 [Format (20A4)]
	Covariance Matrix Class 2
	Mean Class 2
	.
	.
	.

The covariance are in upper triangular form. This statistics deck can be used as the input to the classification error estimator algorithms. Table 18 is a sample of the statistics obtained from the data set using the first 4 optimum basis functions. Also, the statistics were used as input to the classification performance estimator *SPEST to get an estimate of the probability of correct classification.

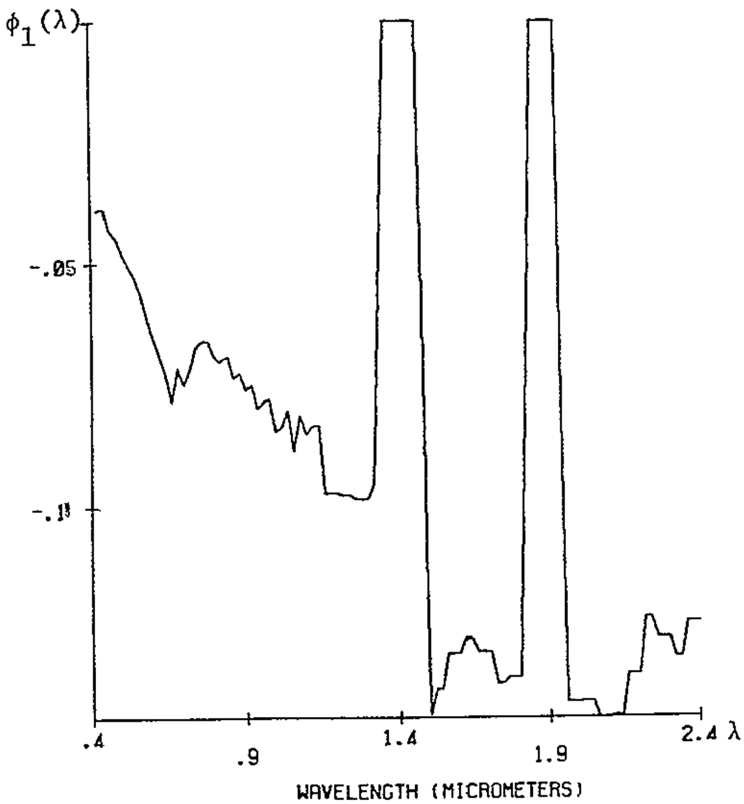

Figure 28. Eigenvector 1.

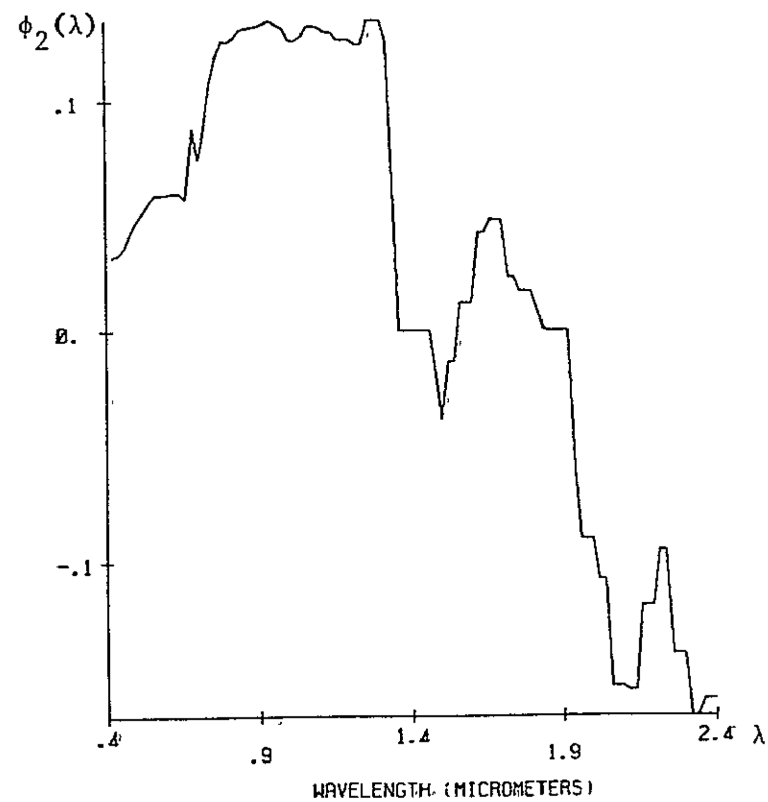

Figure 29. Eigenvector 2.

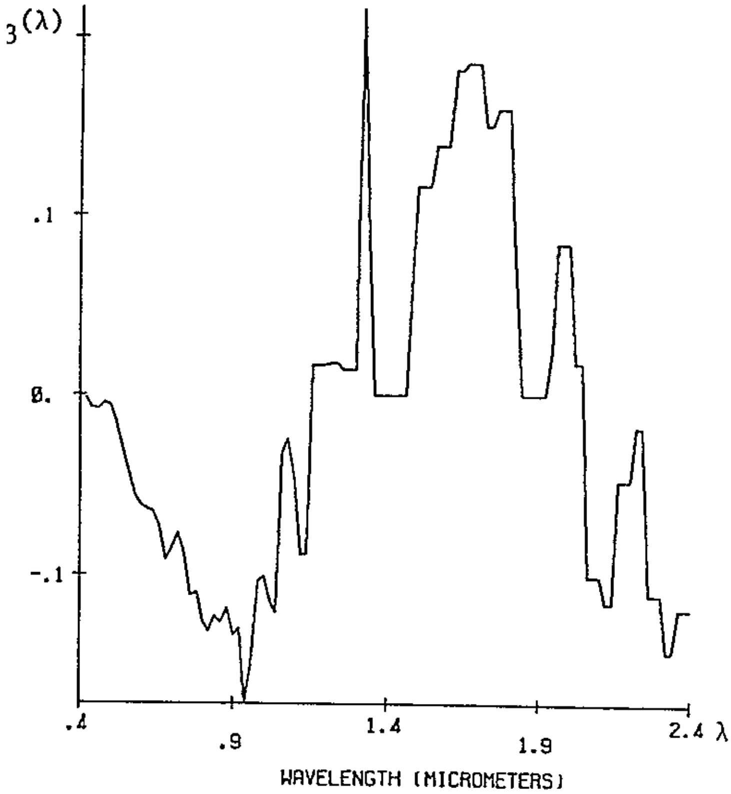

Figure 30. Eigenvector 3.

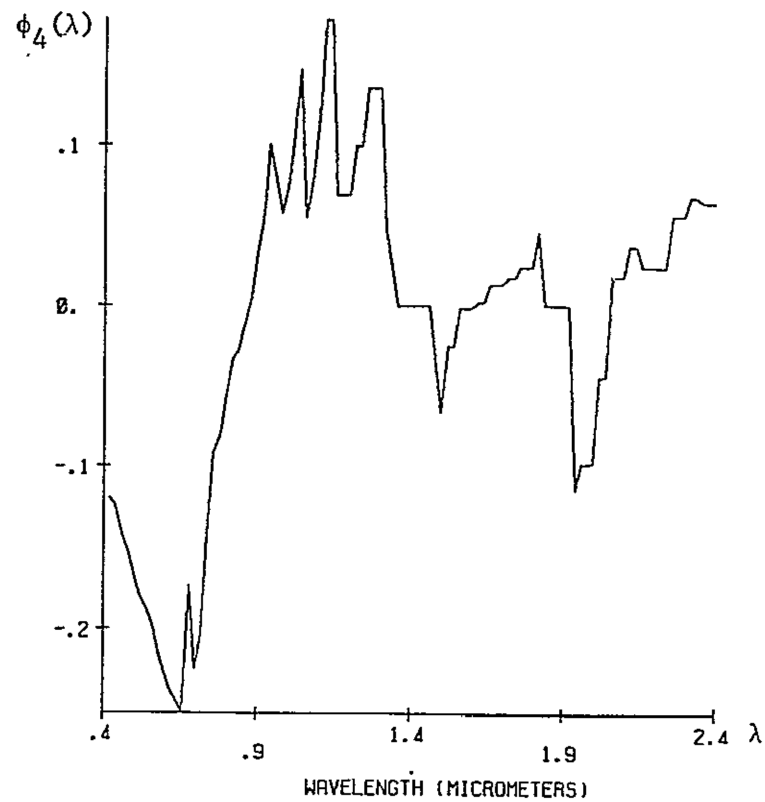

Figure 31. Eigenvector 4.

MEAN VECTOR

-200.3751	22.0529	4.5466	18.8553

COVARIANCE MATRIX

312.5391			
-24.4102	62.8445		
8.4233	-20.7412	14.0594	
-5.2539	-8.6057	4.5237	12.2612

MEAN VECTOR

-202.1029	35.2806	5.7945	16.3977

COVARIANCE MATRIX

244.7227			
-69.3594	152.5959		
-15.8333	10.3513	24.0876	
-2.2070	1.3667	-0.7130	12.3877

MEAN VECTOR

-187.5431	54.8315	8.2578	19.0705

COVARIANCE MATRIX

286.1719			
-1.2813	168.4688		
-19.2971	-47.7388	29.0520	
10.3206	54.3401	-16.4006	26.1763

PROBABILITY OF CORRECT CLASSIFICATION FOR CLASS 1 = 0.9187

PROBABILITY OF CORRECT CLASSIFICATION FOR CLASS 2 = 0.6624

PROBABILITY OF CORRECT CLASSIFICATION FOR CLASS 3 = 0.9003

OVERALL PROBABILITY OF CORRECT RECOGNITION = 0.8270

Table 18. SPTES and SPEST Sample Output Using the First 4 Eigenvectors and Estimates of the Classification Accuracy.

5. SUMMARY

The task of evaluating the performance of a multispectral scanner system while incorporating every spatial, spectral, electronic and telemetric parameter is exceedingly complicated. The primary objective of this project has been the investigation of the two important, and most relevant in remote sensing applications, of scanner parameters, namely spatial and spectral. The development of analytic techniques for system performance evaluation differentiates the approach adopted here from other experimental methods. This property provides an ease of parameter manipulation not available through some heavily data dependent algorithms. Although the development of individual components of such systems is fundamental to the overall system operation, it is the logical and proper integration of individual modules that determines its ultimate processing capabilities.

The Unified Scanner Analysis Package (USAP) is a fully integrated system with complete input-output compatibility of software units. It consists of a spatial and spectral path plus a shared unit providing the desired performance index in the form of probabilities of correct classification.

5.1 Classification Error Estimators

The primary performance index throughout this study is defined as the probability of correct classification of the various populations present in a data set. In keeping with the underlying requirement of a parametric approach, the available LARSYS and other classification accuracy estimators using a randomly generated data base were deemed less than satisfactory. It is well known that the exact probability of correct classification is a multiple integral over an appropriate domain. The direct evaluation of such integral in a continuous N dimensional space is a complicated and mathematically cumbersome task. This problem is circumvented by a deterministic sampling algorithm of the feature space preceded by an orthogonal transformation. This transformation when applied to the Gaussian probability density function of a particular class under consideration would conditionally decouple the feature space and hence reduce the N dimensional error integral to a product of N one dimensional integrals each of which is a widely tabulated quantity. This algorithm requires the population statistics as the only major input and provides classification error estimates of high quality without excessively fine feature space quantization. The second classification accuracy estimator uses the maximum a posteriori principle coupled with a Monte-Carlo type integration technique. Although this algorithm is in a way dependent on a simulated data base, from a userspoint of view the difference between ACAP and SPEST are essentially transparent since both methods require the spectral statistics of the populations as their primary input. The aforementioned classification accuracy estimation techniques provide the basic tools for the scanner system performance evaluation.

5.2 Scanner Spatial Parameters Selection

The scanner spatial modeling algorithm and software consists of one main plus two supporting routines, i.e., IFOV modeling (SCANSTAT), spatial correlation analyzer (CORELAT) and classification error estimator (ACAP). The objectives of an analytical representation of scanner IFOV model is the establishment of a parametric relationship between the system's input and output statistics in terms of the class conditional mean vectors and covariance matrices. This relationship is established using linear system analysis techniques extended to a 2 dimensional space. In order to derive any specific results two basic characteristics need to be specified: (a) scanner PSF and (b) ground scene spatial correlation model.

The choice of a Gaussian shaped PSF has been widespread in the field of image processing as applied to the Landsat data. This model closely approximates the averaging property of the scanner aperture. An added feature of a Gaussian shaped PSF is the simplification of an otherwise intractable and cumbersome mathematics. Generally speaking the amount of information available about the spatial correlation properties of remotely sensed data is sparse. It has been frequently observed however, that the ground scene spatial correlation model approximately follows an exponentially dropping function [2]. On the basis of previous experimental evidence and the mathematical simplicity afforded by these assumptions, a Gaussian PSF and a Markov scene spatial correlation model is adopted. Like many other instances, the choice of the problem assumptions does not necessarily rest on their strict validity but also on the tractiability of the ensuing algebra. It is entirely conceivable that much more elaborate scene correlation models and PSF shapes can be envisioned. This approach, however, could and would complicate the underlying mathematics to the point where the gains initially expected from the more accurate model are balanced out. For simulation purposes the entire analysis is repeated for a rectangular shaped PSF although no currently operational Landsat is equipped with such a scanner system.

Based on the foregoing discussion the scanner characteristics function is derived in a closed form. This function relates the input and output statistics as a function of the IFOV size and pixel-to-pixel correlation. SCANSTAT is the software implementation of this linear transformation. The auxilliary program, CORELAT, estimates the class conditional correlation functions and provides the best exponential curve fit to the experimental data using a weighted least-squares fit algorithm. The resulting output statistics is modified by additional random noise the power of which is computed from the specified SNR. The ACAP or SPEST processors provide the new classification accuracy sets. The probabilities of correct classification at the scanner output provide the basic information needed to evaluate the system performance under various operating conditions. For test purposes a hypothetical set consisting of 3 populations is selected and their statistics (mean vectors and covariance matrices) specified. The scanner output statistics and associated classification accuracies are computed for various IFOV sizes and scene correlations. The results are in close agreement with the numerically oriented experiments. For any fixed scene correlation, the population separability and hence the overall classification accuracy increases monotonically with IFOV size. The rate of increase, however, is a function of the scene spatial correlation. The classification accuracy increase per IFOV step is small for a highly correlated scene compared to a scene with a small adjacent sample correlation.

This property stems from the features of the scanner characteristic function and its particular weighting process. The addition of white Gaussian noise predictably degrades the output separability. The experimental results show that for a fixed IFOV size, SNR and classification accuracy increase monotonically. Same relationship exists between the classification accuracy and IFOV size when SNR is fixed.

5.3 Scanner Spectral Parameters Selection

The task of information extraction from remotely sensed data here primarily deals with the development of methods and techniques to select a set of spectral bands to enhance population separability. The first criterion employed in selecting a set of optimum spectral channels is the Karhunen-Loéve expansion of the ensemble of spectral responses associated with a cover type. This expansion provides a set of optimum basis functions, a linear combination of which reconstructs the original stochastic process with a minimum mean square error. These basis functions in effect define a set of optimum windows in the electromagnetic spectrum. The associated software consists of EXOSYS data retrieval package, SPOPT spectral function claculation and SPTES data transformation and statistics calculation. The classification accuracy estimates used to check the resulting separability is obtained using either the ACAP or SPEST processors.

The second approach employes an information theoretic concept for the specification of the optimal spectral bands. The observed spectral random process is modeled as the sum of a noise free signal plus an additive random noise component. For a candidate set of channels the quantity of interest, mutual information between the reflected and observed energy, is computed. The method consists of representing each random process as an autoregressive model. This type of representation facilitates the evaluation of the mutual information when expressed in terms of the Wiener-Hopf filter PSF. Experimental results consists of selecting a wheat scene and dividing the continuum of electro magnetic spectrum into 9 distinct bands. In each band a proper autoregressive model is fitted to the particular random process. Following the estimation of the parameters of the regression models, the average information content in each band is computed and on this basis spectral channels are ranked. Therefore on the basis of maximum mutual information optimality criterion, the top N bands represent the N "best" choice. The significant result obtained from the ranking method is that out of the 6 top ranked channels 5 lie in the infrared portion of the spectrum thus future scanner systems used in remote sensing application should contain more infrared spectral bands according to this analysis.

5.4 Conclusions

This report has presented a brief description of the algorithms and results of the Scanner Parameter Selection culminating in the development of a Unified Scanner Analysis Package by proper integration of the available software modules. Although this report is the final document in this project, it actually represents the first step toward a well coordinated scanner system parameter study technique. The current structure of USAP basically represents a skeleton of the future analysis packages. There exists a considerable software and theory development potential. The software by and large can take most of the streamlining to further facilitate their usage. Specific topics include extended diagnostic handling and error recovery capability, accelerated algorithms to further reduce execution times, etc.

An overall evaluation of the methods and results presented in this report shows that the objectives initially outlined have been successfully met. The resulting analysis package, starting from a data base, produces specific guidelines on the selection of spatial and spectral parameters of a multispectral scanner system and it does so on an entirely analytic basis. In closing it should be pointed out that USAP can have a pivotal role in any follow up project providing by far the widest and most economical parameter manipulation scope, fully complimenting any numerical or experimental scanner analysis techniques.

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APPENDIX I

```
FILE. . .      ACAP      FORTRAN  B1
C
C***********************************************************************
C
C      SUPERVISOR FOR THE ANALYTIC CLASSIFICATION ACCURACY PREDICTION
C
C               WRITTEN 08/16/78      BIJAN G. MOBASSERI
C
C***********************************************************************
C
C      DESCRIPTION AND PURPOSE
C
C      THE ANALYTIC CLASSIFICATION ACCURACY PREDICTION  - ACAP - IS A
C     SOFTWARE PACKAGE, THE PRIMARY OUTPUT OF WHICH IS A SET OF
C     PROBABILITIES OF CORRECT CLASSIFICATION FOR VARICUS NUMBER OF
C     CLASSES IDENTIFIED IN THE DATA SET.
C
C      THE ALGORITHM IMPLEMENTED HERE DIFFERS FROM THE DATA DEPENDENT
C     TECHNIQUES AND RATIO ESTIMATORS SUCH AS *CLASSIFYPOINT OF LARSYS.
C     WHEREAS LARSYS RETURNS TO THE DATA BASE FOLLOWING THE ESTIMATION
C     OF THE CLASS STATISTICS IN ORDER TO PROVIDE THE CLASSIFICATION
C     ACCURACIES, ACAP BYPASSES THIS STEP AND PROVIDES THE SAME
C     QUANTITY DIRECTLY FROM THE STATISTICS DECK GENERATED BY *STAT. TH
C     METHOD EMPLOYED IS A MATHEMATICAL ONE (AS OPPOSED TO STATISTICAL)
C     AND INVOLVES PERFORMING A MULTIDIMENSIONAL INTEGRATION OF EACH
C     PROBAILITY DENSITY FUNCTION OVER THE HYPERVOLUME OF CORRECT
C     DECISION DOMAIN.
C
C      DESCRIPTION OF CONTROL CARDS
C
C       *ACAP
C
C       THIS CARD SPECIFIES THE PARTICULAR PROCESSOR REQUESTED
C
C
C     CHANNELS
C
C       THE DESIRED SUBSET OF THE AVAILABLE CHANNELS IS GIVEN HERE.
C       IT IS IMPORTANT TO REMEMBER THAT THE NUMBERS APPEARING ON THIS
C       CARD IS THE ORDER OF THE SELECTED CHANNELS NOT THEIR ACTUAL
C       NUMBER. FOR EXAMPLE IF THE AVAILBLE CHANNELS ARE 8,9,12,14
C       AND CHANNELS 8,9 AND 14 ARE REQUESTED, SUBCHANNELS CARD SHOULD
C       READ 1,2,4.
C
C
C       CLASSES
C
C       THIS CARD SPECIFIES THE NAME OF EACH CLASS. EACH NAME MUST BE
C       PLACED IN A FIELD 7 CHARACTERS LONG FOLLOWED BY A BLANK. THE
C       CONTIUEATION CARD ,IF REQIURED , MUST HAVE THE WORD 'CLASSES'
C       IN THE BEGINNING FOLLOWED BY THE REST OF THE NAMES.
C
C      GRID
C
C      THIS QUANTITY CONTROLS THE QUALITY OF THE ESTIMATE. THE HIGHER
C      THE NUMBER THE BETTER THE ESTIMATE (ON A MACRO SCALE). THE CPU
C      TIME ,HOWEVER IS VERY SENSITIVE TO THIS QUANTITY AND FOR
C      INITIAL USAGE IT IS STRONGLY RECOMMENDED TO LEAVE THIS TO ITS
C      DEFAULT VALE OF 9 CELLS/AXIS UNTIL THE USER HAS ACQUIRED A
C      FEEL FOR THE PROCESS.
C
C      END
C
C      THIS CARD SIGNALS THE END OF THE CONTROL CARDS. STAT DECK
C      FOLLOWS IMMEDIATELY.
C
C     *** STAT DECK MUST BE PUNCHED IN CHARACTER FORMAT ***
C
C      REMARKS
C
C      THE PROGRAM IS CURRENTLY CAPABLE OF HANDLING 20 CLASSES AND
C      8 SPECTRAL BANDS. THE EXTENSION OF THESE PARAMETERS IS ONLY LIMI
C      BY COMPUTATION TIME. THE USE OF MORE THAN  5 FEATURES IS NOT
C      RECOMMENDED UNLESS THE GRID SIZE IS REDUCED. TRIAL AND ERROR IS
C      PROBABLY THE QUICKEST WAY OF FINDING THE PROPER TRADE OFF.
C
C      HOW TO RUN THE PROGRAM
C
C      THE FOLLOWING EXEC FILE ,NAMED ACAP, MUST RESIDE ON THE USER'S
```

```
FILE. . .      ACAP      FORTRAN  B1
C      PRIMARY DISK
C
C      &TYPEOUT OFF
C      GLOBAL TXTLIB SYSLIB CMSLIB SSPLIB
C      GETDISK LARSYS
C      CP C C
C      CP SP RDR HOLD
C      LOAD ACAP BCDVAL (CLEAR NOMAP XEQ)
C      &EXIT
C
C      AFTER THE READER FILE IS LOADED , TYPE ACAP IN CMS ENVIRONMENT.
C      THIS COMMAND INITIATES THE EXECUTION FOLLOWED BY APPROPRIATE
C      TERMINAL MESSAGES AND PRINTER OUTPUT.
C
C      EXAMPLE OF  THE CONTROL CARD SET UP
C
C      *ACAP
C      CHANNELS 1,3,4
C      CLASSES BARESOI CORN    PASTURE WHEAT
C      GRID 9
C      END
C
C      DEFAULT VALUE OF GRID IS SELECTED IF GRID CARD IS NOT
C      INCLUDED.
C
C     ESTIMATED CPU TIME
C
C      WITH THE DEFAULT GRID SIZE AND USING 4 FEATURES, IT TAKES
C      APPROXIMATELY 2 MINUTES OF CPU TIME PER CLASS TO PROVIDE THE
C      REQUESTED CLASSIFICATION ACCURACY ESTIMATES.
C      THE CPU TIME IS MOST SENSITIVE TO THE DIMENSIONALITY OF
C      THE FEATRE SPACE. HENCE IF THE NO OF SPECTRAL BANDS IS
C      LIMITED (LESS THAN 4) CONSIDERABLE INCREASE IN GRID NUMBER
C      IS FEASIBLE. EXPERIENCE HAS SHOWN HOWEVER, THAT THERE IS ONLY
C      A MARGINAL (FRACTION OF PERCENT) IMPPROVEMENT IN THE
C      CLASSIFICATION ACCURACY ESTIMATE BY INCREASING THE GRID
C      SIZE BEYOND ITS DEFAULT VALUE.
C
C***********************************************************************
C
      REAL*4 A(720),AA(720),SIGMA(1280),MA(160),MAA(160),M(160),SD(8),
     1       DELTA(8),P(160),PC(20),V(20),WV1(8),WV2(8),XPRIM(64),
     2       X(64),Q(64),PR(13),EV(8),LL(8),UL(8),W(20),Z(104),DET(20)
     3      ,EMA(160),EA(720)
      REAL*4 ICSET(90)
      INTEGER*4 NADR(720),IC(20),INDX(20),HEAD(20),SDIM
      INTEGER*4 LIST(5)/'*ACA','CHAN','CLAS','GRID','END '/,IVEC(1),
     1          ICARD(20)
      INTEGER*4 BLANK/'    '/,FSTCRD/'LARS'/
      INTEGER*2 ICSEL(30),NC(30),NCC(30)
      LOGICAL*1 FLAG(5)/5*.FALSE./
C
      COMMON /CAPCOM/ ND,NDIM,NS,NCLS,NP,ISIZE,KNTR
      COMMON /ACUCOM/ NTS,NTM,NSS,NSA,NSM,SDIM,NSIZE,ISKP
C
11    FORMAT(//)
1010  FORMAT(1X,'ERROR IN CTLWRD. EXECUTION TERMINATED.')
1011  FORMAT(1X,'ERROR IN IVAL. EXECUTION TERMINATED.')
1012  FORMAT(1X,'ERROR IN CHANEL. EXECUTION TERMINATED.')
1013  FORMAT(1X,'MISSING CONTROL WORD. EXECUTION TERMINATED.')
1014  FORMAT(1X,'ACAP PROCESSING STARTED')
1015  FORMAT(1X,'ALL CONTROL CARDS HAVE BEEN READ')
C
C
C***********************************************************************
C
C                      DECODE CONTROL CARDS
C
C***********************************************************************
C
C      SET THE DEFAULT GRID SIZE
C
      NS=9
      DO 777 I=1,30
      ICSEL(I)=0
      NC(I)=0
777   CONTINUE
```

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```
C
      DO 778 I=1,90
778   ICSET(I)=-50000.0
C
      FLAG(4)=.TRUE.
C
100   CONTINUE
      LSZ=5
      IER=0
      INRD=5
C
      CALL CTLWRD (ICARD,ICOL,LIST,LSZ,ICODE,INRD,IER)
      IF(IER.NE.0) GO TO 1001
      GO TO (99,101,102,103,104), ICODE
99    FLAG(1)=.TRUE.
      GO TO 100
C
C
C     CHANNELS CARD
C
101   CALL CHANEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,900)
      FLAG(2)=.TRUE.
      NDIM=NCR
      GO TO 100
C
C     CLASS NAMES CARD
C
102   DO 10 I=1,20
10    HEAD(I)=ICARD(I)
      FLAG(3)=.TRUE.
      GO TO 100
C
C
C     GRID SIZE CARD
C
103   LSZ=1
      CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
      NS=IVEC(1)
      GO TO 100
C
C     END CARD
C
104   LSZ=1
      CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
      FLAG(5)=.TRUE.
      GO TO 201
C
1002  WRITE(6,1011)
      WRITE(16,1011)
      GO TO 999
C
201   CONTINUE
C
C     CHECK IF ALL CONTROL CARDS HAVE BEEN READ
C
      DO 250 I=1,5
      IF(.NOT.FLAG(I)) GO TO 321
      GO TO 250
321   WRITE(16,1013)
      WRITE( 6,1013)
      GO TO 999
250   CONTINUE
C
      WRITE(16,11)
      WRITE( 6,11)
      WRITE(16,1014)
      WRITE( 6,1014)
C
      WRITE(16,1015)
      WRITE( 6,1015)
C
      GO TO 680
1001  WRITE(16,1010)
      WRITE( 6,1010)
      GO TO 999
680   CONTINUE
```



```
900   WRITE(6,1012)
      WRITE(16,1012)
      GO TO 999
720   CONTINUE
C
C*******************************************************************
C
C     READ THE TOTAL NO OF CHANNELS AND CLASSES FROM THE STAT DECK
C
C*******************************************************************
C
502   READ(5,501) ICRD
501   FORMAT(A4)
      ICRDSQ=ICRDSQ+1
C
      IF(ICRD.EQ.BLANK) GO TO 503
      GO TO 502
503   CONTINUE
C
      REWIND 5
602   READ (5,501) ICRD
      IF(ICRD.EQ.FSTCRD) GO TO 601
      GO TO 602
601   CONTINUE
C
      NUM=ICRDSQ-2
      DO 506 I=1,NUM
      READ(5,507)
507   FORMAT(18A4,I8)
506   CONTINUE
C
C
C
C
      READ(5,508) NCLS,NFLD,SDIM
508   FORMAT(I5,6X,I5,6X,I5)
C
C
      NUM2=SDIM+1
      DO 509 I=1,NUM2
      READ(5,507)
509   CONTINUE
C
C     FIND THE CHANNEL SET THAT IS NOT REQUESTED
C
      DO 611 I=1,SDIM
      DO 612 J=1,NDIM
C
         IF(I.EQ.NCC(J)) GO TO 611
C
612   CONTINUE
C
         K=K+1
         NC(K)=I
C
611   CONTINUE
C
C
C
      ND=NDIM**2
      NP=NS-1
      ISIZE=NDIM*(NDIM+1)/2
      NSIZE=SDIM*(SDIM+1)/2
C
      NTS=NCLS*ISIZE
      NTM=NCLS*NDIM
      NSS=NCLS*ND
      NSA=NCLS*NSIZE
      NSM=NCLS*SDIM
20    CONTINUE
C
      CALL ACUTST (A,EA,AA,SIGMA,EMA,MA,MAA,M,SD,DELTA,P,PC,V,WV1,WV2,
     1             XPRIM,R,Q,PR,EV,LL,UL,W,DET,Z,IQ,INDX,NADR,NC,NCC,
     2             HEAD)
C
999   STOP
      END
C
C...................................................................
```

```
C
C       SUBROUTINE ACUTST
C
C     PURPOSE
C
C          PERFORM THE INITIAL TRANSFORMATIONS OF THE COORDINATES PRIOR
C          TO CLASSIFICATION ERROR ESTIMATION
C
C     DESCRIPTION OF PARAMETERS
C
C          A      - COVARIANE MATRIX ARRAY FOR THE SPECIFIED SUB CHANNE
C          AA     - DIRECT COPY OF A USED IN INITIALIZATION
C          DELTA  - ARRAY OF CELL WIDTHS ALONG EACH FEATURE AXIS
C          DET    - ARRAY OF DETERMINANTS OF EACH TRANSFORMED COVARIANC
C                   MATRIX
C          EV     - ARRAY OF EIGENVALUES OF THE CURRENT CLASS
C          EA     - COVARIANCE MATRIX ARRAY FOR ALL THE CHANNELS
C          EMA    - MEAN VECTOR FOR ALL THE CHANNELS
C          HEAD   - ARRAY DISPLAYING TITLES
C          INDX   - POINTER ARRAY
C          IQ     - ARRAY DETERMINING THE CYCLIC STRUCTURE OF INDX
C          LL     - ARRAY OF THE LOWER COORDINATES OF A CELL
C          MA     - ORIGINAL MEAN VECTORS FOR THE SPECIFIEC SUB CHANNEL
C          MAA    - COPY OF MA FOR INITIALIZATION
C          M      - TRANSFORMED MEAN VECTORS
C          NADR   - UNWANTED  COVARIANCE MATRIX ENTRIES
C          NC     - COMPLEMENT OF NCC
C          NCC    - DESIRED CHANNELS(SUBSET OF THE TOTAL)
C          P      - ARRAY REPRESENTING ONE CELL CENTER COORDINATES
C          PC     - ARRAY OF CLASSIFICATION ACCURACIES
C          PR     - ONE DIMENSIONAL PROBABILITY ARRAY
C          Q      - WORK VECTOR
C          R      - VECTOR OF EIGENVALUES
C          SD     - STANDARD DEVIATION ARRAY FOR THE CURRENT CLASS
C          UL     - UPPER COORDINATE OF A SAMPLING CELL
C          V      - ARRAY COMPRISING PART OF W
C          W      - THE RESULTING DISCRIMINANT FUNCTION
C          WV1    - WORK VECTOR
C          WV2    - WORK VECTOR
C          XPRIM  - WORK VECTOR
C          Z      - ARRAY CONTAINING THE COORDINATES OF EACH SAMPLING
C                   CELL
C
C     REMARKS
C
C          NONE
C
C     SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C          GMPRD, ADRES,PMC,EIGEN,DIAG,GMTRA
C
C     METHOD
C
C          EACH CLASS IS PROCESSED SEQUENTIALLY. THE ORTHOGONAL
C          TRANSFORMATION THAT DECOUPLES THE FEATURE SPACE IS
C          DERIVED BY OBTAINING THE EIGENVECTOR OF THE CLASS UNDER
C          CONCIDERATION. THE ENTIRE SET OF STATISTICS,MEAN VECTORS
C          AND COVARIANCE MATRICES, ARE TRANSFORMED RESULTING IN A
C          DIAGONAL COVARIANCE MATRIX FOR THE CURRENT CLASS AND GENER
C          FORM FOR OTHERS.THE MEAN OF THE NEW COORDINATE SYSTEM IS
C          AT THE CURRENT MEAN. THIS PROCESS IS REPEATED UNTIL THE
C          ENTIRE SET OF POPULATIONS IS EXHAUSTED.
C
C..................................................................
C
C
      SUBROUTINE ACUTST (A,EA,AA,SIGMA,EMA,MA,MAA,M,SD,DELTA,P,PC,V,WV
     1                   WV2,XPRIM,R,Q,PR,EV,LL,UL,W,DET,Z,IQ,INDX,
     2                   NADR,NC,NCC,HEAD)
C
C******************************************************************
C
C      LOCAL VARIABLES DEFINITION
C
C******************************************************************
C
      REAL*4 A(NTS),AA(NTS),SIGMA(NSS),MA(NTM),MAA(NTM),M(NTM),SD(NDIM
     1       DELTA(NDIM),P(NTM),PC(NCLS),V(NCLS),WV1(NDIM),WV2(NDIM),
```



```
     2       XPRIM(ND),R(ND),Q(ND),PR(NS),EV(NDIM),LL(NDIM),UL(NDIM),
     3       W(NCLS),EA(NSA),EMA(NSM),Z(NDIM,NS),DET(NCLS)
C
      INTEGER*4 IQ(NDIM),INDX(NDIM),NADR(NSA),HEAD(1),SDIM
      INTEGER*2 NC(SDIM),NCC(SDIM)
      INTEGER*4 FMT1(6) /'    (','    3','(1PE','12.4',',3X)',')'/
      INTEGER*4 FMT2(4) /'(2X,','    0','E14.','7)'/
      INTEGER*4 FMT3(4) /'(2X,','    0','E14.','7)'/
      INTEGER*4 N1 /'    0'/,N2 /'    0'/,N3 /'    0'/
C
C
C
      COMMON /CAPCOM/ ND,NDIM,NS,NCLS,NP,ISIZE,KNTR
      COMMON /ACUCOM/ NTS,NTM,NSS,NSA,NSM,SDIM,NSIZE,ISKP
C
C******************************************************************
C
C       COMMON BLOCK VARIABLES DESCRIPTION
C
C       ISIZE - STORAGE SPACE REQUIRED FOR EACH COVARIANCE MATRIX
C       KNTR  - COUNTER FOR THE CURRENT CLASS
C       NCLS  - NO OF CLASSES
C       ND    - ENTIRE ENTRIES OF A COVARIANCE MATRIX
C       NDIM  - NO OF DIMENSIONS (FEATURES)
C       NS    - NO OF CELLS PER AXIS IN THE SAMPLING GRID
C       NSA   - CORE SPACE ALLOCATED TO ALL COV MATRICES USING ALL CHAN
C       NSIZE - CORE SPACE ALLOCATED FOR ONE COV MAT USING SUBCHAN
C       NSM   - CORE SPACE ALLOCATED TO ALL MEAN VEC USUNG ALL CHAN
C       NSS   - CORE SPACE ALLOCATED TO TRAN COV MATRICES
C       NTM   - CORE SPACE ALLOCATED TO MEAN VEC USING THE SUBCHAN
C       NTS   - CORE SPACE ALLOCATED TO COV MAT USING THE SUBCHAN
C       SDIM  - TOTAL NO OF CHANNELS AVAILABLE
C
C******************************************************************
C
C******************************************************************
C
70    FORMAT(1H1)
71    FORMAT(/)
72    FORMAT(//)
74    FORMAT(////)
C
536   FORMAT(35X,'ANALYTIC CLASSIFICATION ACCURACY PREDICTION - A C A P
     1)
537   FORMAT(49X,'CLASS ',2A4)
538   FORMAT(1X,'SAMPLING GRID CHARACTERISTICS0')
539   FORMAT(1X,'GRID SIZE=',I3,' CELLS PER DIMENSION')
540   FORMAT(1X,'TOTAL NO OF CELLS IN THE GRID= ',I8)
541   FORMAT(1X,'TRANSFORMED FEATURE SPACE CHARACTERISTICS0')
542   FORMAT(1X,'EIGENVALUES0')
543   FORMAT(1X,3(1PE11.4,3X))
544   FORMAT(1X,'EIGENVECTORS0')
545   FORMAT(3(1PE12.4,3X))
546   FORMAT(1X,'TRANSFORMED MEAN VECTORS0')
547   FORMAT(25X,'P R O B A B I L I T Y  O F   C O R R E C T  C L A S S
     1 F I C A T I O N = ',F7.3,' (')
548   FORMAT(25X,'*****TOTAL PROB OF CORRECT CLASSIFICATION***** ',
     1           F7.3,' PERCENT')
549   FORMAT(80A1)
550   FORMAT(1X,'ALL CLASS STATISTICS HAVE BEEN READ')
      ISS=NS**NDIM
C
      N1=N1+NDIM
      N2=N2+SDIM
      N3=N3+5
      IF(NSIZE.LT.5) N3=NSIZE
C
      FMT1(2)=N1
      FMT2(2)=N2
      FMT3(2)=N3
C
C
C
C******************************************************************
C
C        READ IN THE MEAN VECTORS AND COVARIANCE MATRICES
C
C******************************************************************
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
C
C
      READ (5,FMT2) EMA
      READ (5,FMT3) EA
C
      WRITE(16,550)
      WRITE( 6,550)
C
C
C*****************************************************************************
C
C       SELECT THE REQUIRED SUBSET OF CHANNELS
C
C*****************************************************************************
C
      DO 803 JCLS=1,NCLS
C
          MDS=(JCLS-1)*SDIM
C
      DO 802 KDIM=1,SDIM
      DO 801 J=1,NDIM
C
          IF(KDIM.EQ.NCC(J)) K=K+1
          IF(KDIM.EQ.NCC(J)) MA(K)=EMA(MDS+NCC(J))
C
801   CONTINUE
802   CONTINUE
803   CONTINUE
C
C
      DO 530 JCLS=1,NCLS
      DO 530 KDIM=1,NDIM
C
          MDS=(JCLS-1)*NDIM+KDIM
          MAA(MDS)=MA(MDS)
C
530   CONTINUE
C
C
C*****************************************************************************
C
C       OBTAIN THE ADDRESS OF THE SELECTED ENTRIES INTO COVARIANCE MATRIX
C
C*****************************************************************************
C
C
      CALL ADRES (NADR,NC,SDIM,NDIM)
C
C
      K=0
      DO 603 JCLS=1,NCLS
C
          MDS=(JCLS-1)*NSIZE
C
      DO 602 I=1,NSIZE
      DO 601 J=1,NSIZE
C
          IF(I.EQ.NADR(J)) GO TO 602
C
601   CONTINUE
C
          K=K+1
          A(K)=EA(MDS+I)
C
602   CONTINUE
603   CONTINUE
C
      DO 20 KDIM=1,NTS
C
          AA(KDIM)=A(KDIM)
20    CONTINUE
C
381   CONTINUE
      DO 628 I=1,NCLS
      PC(I)=0.
      PR(I)=0.
 620  CONTINUE
      ICNT=ICNT+1
```

FILE. . .   ACAP   FORTRAN B1

```
      TPCC=0.
C
      DO 888 KNTR =1,NCLS
C
      WRITE(6,70)
      WRITE(6,72)
C
C*****************************************************************************
C
C       RESET THE MEAN VECTORS
C
C*****************************************************************************
C
C
      DO 321 JCLS=1,NCLS
      DO 321 KDIM=1,NDIM
C
          MDS=(JCLS-1)*NDIM
          MA(MDS+KDIM)=MAA(MDS+KDIM)
C
321   CONTINUE
C
C*****************************************************************************
C
C       PERFORM A DECOUPLING TRANSFORMATION ON THE CURRENT CLASS
C
C
C*****************************************************************************
C
C
C
C
C       ***************************************************************
C
C       TRANSLATE THE ORIGIN TO THE MEAN OF THE CURRENT CLASS
C
C       ***************************************************************
C
C
      DO 35 JCLS=1,NCLS
C
          IF(JCLS.EQ.KNTR) GO TO 35
C
      DO 38 KDIM=1,NDIM
C
          MDS=(JCLS-1)*NDIM
          NDS=(KNTR-1)*NDIM
          MA(MDS+KDIM)=MA(MDS+KDIM)-MA(NDS+KDIM)
C
38    CONTINUE
35    CONTINUE
C
      DO 36 KDIM=1,NDIM
C
36        MA(NDS+KDIM)=0
C
C
C
C
C       ***************************************************************
C
C       FIND THE REQUIRED TRANSFORMATION MATRIX
C
C       ***************************************************************
C
51    FORMAT(1X,4(1PE11.4,1X))
C
      NDS=(KNTR-1)*ISIZE+1
      CALL EIGEN (A(NDS),R,NDIM,MV)
C
      L=0
      DO 40 KDIM=1,NDIM
C
          L=L+KDIM
          EV(KDIM)=A(NDS+L-1)
C
40    CONTINUE
C
C
```

```
FILE. . .     ACAP     FORTRAN  B1
C        ****************************************************************
C
C        RESET THE CURRENT CLASS COVARIANCE MATRIX
C
C        ****************************************************************
C
      DO 901 KDIM=1,ISIZE
C
            A(NDS+KDIM-1)=AA(NDS+KDIM-1)
C
901   CONTINUE
C
C
C
C**********************************************************************
C
C
C       PERFORM THE TRANSFORMATION ON COVARIANCE MATRICES
C
C**********************************************************************
C
      NR1=NDIM
      NC1=NDIM
      NC2=NDIM
      MSA=0
      MSB=1
      ICD=0
C
      DO 701 JCLS=1,NCLS
C
            KDS=(JCLS-1)*ISIZE+1
            KSS=(JCLS-1)*ND+1
            CALL DIAG (R,A(KCS),SIGMA(KSS),XPRIM,MSA,MSB,NR1,NC1,NC2,ICD,
     1                NDIM)
C
701   CONTINUE
C
C
C**********************************************************************
C
C
C        PERFORM THE TRANSFORMATION ON MEAN VECTORS
C
C**********************************************************************
C
      CALL GMTRA (R,Q,NDIM,NDIM)
      NR1=NDIM
      NC1=NDIM
      NC2=1
C
      DO 702 JCLS=1,NCLS
C
            MDS=(JCLS-1)*NDIM+1
            CALL GMPRD (Q,MA(MDS),M(MDS),NR1,NC1,NC2)
C
702   CONTINUE
C
C
C**********************************************************************
C
C
C        PREPEARE THE PRINTED OUTPUT
C
C**********************************************************************
C
      KS=2*KNTR
      WRITE(6,536)
      WRITE(6,74)
      WRITE(6,537) (HEAD(KS+I),I=1,2)
C
      WRITE(6,74)
      WRITE(6,538)
C
      WRITE(6,72)
      WRITE(6,539) NS
C
      WRITE(6,72)
      WRITE(6,540) IGS
C
      WRITE(6,74)
      WRITE(6,541)
```

```
FILE. . .     ACAP     FORTRAN  B1
      WRITE(6,72)
      WRITE(6,542)
      WRITE(6,72)
      WRITE(6,FMT1) EV
C
      WRITE(6,74)
      WRITE(6,544)
      WRITE(6,72)
      WRITE(6,FMT1) R
C
      WRITE(6,74)
      WRITE(6,546)
C
      WRITE(6,72)
      WRITE(6,FMT1) M
C
      WRITE(6,74)
C
C**********************************************************************
C
C
C         START ESTIMATION OF THE PROBABILITY OF MISCLASSIFICATION
C
C**********************************************************************
C
99    CONTINUE
      CALL PMC (M,SIGMA,P,PC,EV,Z,SD,DELTA,WV1,WV2,V,XPRIM,W,R,
     1                 Q,DET,INDX,IQ,LL,UL,PR,HEAD)
C
C**********************************************************************
C
C
C         CONVERT THE RESULTING PCC TO PERCENTAGE
C
C**********************************************************************
C
      PC(KNTR)=100.*PC(KNTR)
C
      WRITE(16,72)
      WRITE(16,49) (HEAD(KS+I),I=1,2)
49    FORMAT(25X,'CLASS ',2A4)
      WRITE(16,71)
      WRITE(16,43) PC(KNTR)
43    FORMAT(10X,'--- PROBABILITY OF CORRECT CLASSIFICATION= ',F7.3,' -
     1')
      WRITE(16,72)
C
      WRITE(6,547) PC(KNTR)
C
888   CONTINUE
C
C**********************************************************************
C
C
C       FIND THE TOTAL PROBABILITY OF CORRECT CLASSIFICATION
C
C**********************************************************************
C
      DO 568 I=1,NCLS
C
            TPCC=TPCC+PC(I)
C
568   CONTINUE
      TPCC=TPCC/FLOAT(NCLS)
      WRITE(6,74)
      WRITE(6,548) TPCC
      WRITE(6,72)
C
C
999   CONTINUE
C
C
C
      RETURN
      END
C
C
C......................................................................
C
C
C         SUBROUTINE ADRES
C
```

```
C       PURPOSE
C
C            FIND THE DESIRED SUBSET OF A COVARIANCE MATRIX
C
C       DESCRIPTION OF PARAMETERS
C
C          NADR - ADRESS ARRAY OF THE ENTRIES TO BE DELETED
C          NC   - ARRAY OF THE CHANNELS TO BE DELETED
C          SDIM - TOTAL NO OF BANDS SUPPLIED
C          NDIM - DESIRED SUBSET OF SDIM
C
C       REMARKS
C
C          NONE
C
C       SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C          NONE
C
C       METHOD
C
C          USING A ONE DIMENSIONAL STORAGE MODE FOR EACH COVARIANCE
C          MATRIX IN AN UPPER TRIANGULAR FORM,THE PARAMETRIC ADDRESSES
C          FOR EACH ENTRY IS DERIVED AND USING NC ARRAY THE LOACTION OF
C          ALL THE ENTRIES THAT LIE IN THE UNWANTED LINES AND COLUMNS
C          IS COMPUTED AND STORED IN NADR
C
C.......................................................................
C
C
      SUBROUTINE ADRES (NADR,NC,SDIM,NDIM)
C
      INTEGER*4 NADR(1),PDIM,SDIM
      INTEGER*2 NC(1)
C
      PDIM=SDIM-NDIM
C
      IF(PDIM.LE.0) GO TO 100
      DO 20 J=1,PDIM
      N1=NC(J)
      DO 10 I=1,N1
      K=K+1
      NADR(K)=(NC(J)*(NC(J)-1)/2)+I
10    CONTINUE
20    CONTINUE
C
C
      DO 45 J=1,PDIM
      NFIN=SDIM-NC(J)+1
      DO 35 I=1,NFIN
C
      K=K+1
      NADR(K)=((NC(J)+I-1)*(NC(J)+I-2)/2)+NC(J)
C     WRITE(16,233) K,NADR(K)
233   FORMAT(2(I5,3X))
35    CONTINUE
45    CONTINUE
C
100   RETURN
      END
C
C
C.......................................................................
C
C        SUBROUTINE PMC
C
C        PURPOSE
C
C             TO COMPUTE THE PROBABILITY OF MISCLASSIFICATION OF THE
C             CLASSES
C
C        DESCRIPTION OF PARAMETERS
C
C             DET  - DETERMINANT ARRAY FOR EACH COV MATRIX
C             DELTA- ARRAY OF SAMPLING CELL DIMENSIONS
C             EV   - EIGENVALUES
C             HEAD - HEADER ARRAY
C             INDX - POINTER ARRAY
```



```
C             LL   - LOWER COORD. OF A CELL
C             M    - TRANSFORMED MEAN VECTORS
C             P    - GRID POINT VECTOR
C             PC   - CLASSIFICATION ACCURACY RESULTS
C             PR   - PRCBABILITY ASSOCIATED WITH EACH GRID CELL
C             Q    - WORK VECTOR
C             R    - EIGENVECTORS
C             SD   - CLASS STANDARD DEVIATION VECTOR
C             V    - PART OF THE DISCRIMINANT FUNCTIONS
C             WV1  - WORK VECTOR
C             WV2  - WORK VECTOR
C             W    - DISCRIMINAT FUNCTION
C             XPRIM- WORK VECTOR
C             Z    - COORD OF CELL CENTERS
C
C        REMARKS
C
C             NONE
C
C        SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C             DIAG, GRPROB, MINV
C
C        METHOD
C
C             THE PRIMARY INPUT TO THIS SUBROUTINE IS THE TRANSFORMED
C             MEAN AND COVARIANCE MATRICES. THE STRUCTURE OF THE SAMPLING
C             GRID IS DETERMINED BY COMPUTING THE CELL WICTH ALONG EACH
C             DIMENSION AND THE COORD OF THE CENTER OF EVERY CELL THROUGHOU
C             THE GRID. THE ENTIRE GRID IS SCANNED AND NCLS DISCRIMINAT
C             FUNCTION IS CALCULATED FOR EACH CELL. USING MAXIMUM LIKELIH
C             OOD RULE A CELL IS ASSIGNED TO EITHER THE CURRENT CLASS OR
C             OUTSIDE(NEED NOT KNOW EXACTLY WHICH CLASS). SUBROUTINE
C             GRPROB CALCULATES THE HYPERVOLUME UNDER THE PDF AND OVER THE
C             GRID CELL. AFTER ALL THE POINTS ARE EXHAUSTIVELY TESTED THE
C             ELEMENTARY UNITS OF PROBABILITY ARE SUMMED AND STORED IN THE
C             PC ARRAY AND RETURNED TO THE CALLING PROGRAM.
C
C.......................................................................
C
C
       SUBROUTINE PMC (M,SIGMA,P,PC,EV,Z,SD,DELTA,WV1,WV2,V,XPRIM,W,R,
     1                 Q,DET,INDX,IC,LL,UL,PR,HEAD)
C
C
C***********************************************************************
C
C        LOCAL VARIABLES DEFINITION
C
C***********************************************************************
C
       REAL M(1),LL(1),UL(1),PR(1),
     1      SIGMA(1),Z(NDIM,NS),SD(1),DELTA(1),
     2      P(1),WC(1),WV1(1),WV2(1),PC(1),V(1),
     3      XPRIM(NDIM),W(1),R(1),Q(1),DET(1),EV(1)
C
       INTEGER*4 GS,HEAD(1),IQ(1),INDX(1)
C
       COMMON /CAPCOM/ ND,NDIM,NS,NCLS,NP,ISIZE,KNTR
C
       GS=NS**NDIM
       ICD=1
C
C
C
C***********************************************************************
C
C        FIND THE VARIANCES ALONG EACH FEATURE AXIS
C
C***********************************************************************
C
       DO 706 KDIM=1,NDIM
C
            SD(KDIM)=SQRT(EV(KDIM))
C
706    CONTINUE
C
C
```

```
FILE. . .     ACAP     FORTRAN  B1
C*********************************************************************
C
C       INVERT THE TRANSFORMED COVARIANCE MATRICES
C
C*********************************************************************
C
      DO 707 JCLS=1,NCLS
         KDS=(JCLS-1)*ND+1
         CALL MINV (SIGMA(KDS),NDIM,DET(JCLS),WV1,WV2)
707   CONTINUE
C
33    FORMAT(1X,3(1PE12.4,2X))
32    FORMAT(1X,2(1PE12.4))
C
C
C*********************************************************************
C
C       SAMPLE THE FEATURE SPACE BY A BINOMIAL APPROX. TO NORMAL D.F.
C
C*******************************************************************
C
C
C
C       *************************************************************
C
C       FIND THE WIDTH OF A SAMPLING CELL ALONG EACH DIMENSION
C
C       *************************************************************
C
      DO 44 KDIM=1,NDIM
C
         DELTA(KDIM)=2.*SD(KDIM)/SQRT(FLOAT(NP))
C
44    CONTINUE
C
C
C       *************************************************************
C
C       FIND THE COORDINATES OF EACH AND EVERY SAMPLING CELL CENTER
C
C       *************************************************************
C
C
      DO 30 KDIM=1,NDIM
      DO 30 I=1,NS
C
         Z(KDIM,I)=2.*SD(KDIM)*(FLOAT(I-1)-NP/2)/SQRT(FLOAT(NP))
C
30    CONTINUE
C
C
C*********************************************************************
C
C+      CALCULATE THE QUADRATIC DISCRIMINANT FUNCTIONS               +
C
C*********************************************************************
C
      ICNT=0
C
      DO 404 KDIM=1,NDIM
C
404      INDX(KDIM)=1
C
C
C       *************************************************************
C
C       PERFORM A ONE-TO-N DIMENSIONAL MAPPING OF POSITIVE INTEGERS
C
C       *************************************************************
      DO 520 I=1,NDIM
C
         IQ(I)=NS**(NDIM-I)
C
520   CONTINUE
C
      DO 120 J=1,GS
C
         IF(J.EQ.1) GO TO 38
```

```
FILE. . .     ACAP     FORTRAN  B1
C
      DO 122 KDIM=1,NDIM
C
         IF(MOD(J-1,IQ(KDIM)).EQ.0) INDX(KDIM)=INDX(KDIM)+1
         IF(INDX(KDIM).GT.NS) INDX(KDIM)=1
C
122   CONTINUE
38    CONTINUE
C
C
C       *************************************************************
C
C       COMPLETE THE DISCRIMINANT FUNCTION CALCULATION
C
C       *************************************************************
C
      DO 124 JCLS=1,NCLS
      DO 126 KDIM=1,NDIM
C
         MDS=(JCLS-1)*NDIM
         P(MDS+KDIM)=Z(KDIM,INDX(KDIM))-M(MDS+KDIM)
C
126   CONTINUE
124   CONTINUE
C
      DO 128 JCLS=1,NCLS
C
         MDS=(JCLS-1)*NDIM+1
         KDS=(JCLS-1)*ND+1
         MSA=0
         MSB=0
         CALL DIAG (P(MDS),SIGMA(KDS),V(JCLS),XPRIM,MSA,MSB,NR1,
     1              NC1,NC2,ICD,NDIM)
C
         W(JCLS)=V(JCLS)+ALOG(DET(JCLS))
C
128   CONTINUE
C*********************************************************************
C
C+      CHECK THE CONDITION FOR CORRECT CLASSIFICATION
C
C*********************************************************************
C
      TERM=1.0E 10
      DO 134 JCLS=1,NCLS
C
         IF(W(JCLS).LE.TERM) TERM=W(JCLS)
C
134   CONTINUE
C
C
C*********************************************************************
C
C       FIND THE ELEMENTARY UNIT OF PROBABILITY
C
C*********************************************************************
C
         IF(TERM.NE.W(KNTR)) GO TO 120
      ICNT=ICNT+1
         CALL GRPROB (INDX,DELTA,SD,NS,NDIM,Z,PR,LL,UL)
C
855   CONTINUE
         PRP=1.
      DO 710 KDIM=1,NDIM
C
         PRP=PRP*PR(INDX(KDIM))
C
710   CONTINUE
C
         PC(KNTR)=PC(KNTR)+PRP
C
C
120   CONTINUE
C
      RETURN
      END
C     ***************************************************************
```

FILE. . .   ACAP   FORTRAN B1

```
C
C       SUBROUTIN GRPROB
C
C       PURPOSE
C          COMPUTE THE GRID PROBABILITY
C
C       DESCRIPTION OF PARAMETERS
C
C            DELTA - GRID CELL DIMENSIONS
C            INDX  - POINTER ARRAY
C            LL    - LOWER COORD OF A CELL
C            NDIM  - NO OF SELECTED SUBSET CHANNELS
C            NS    - NO OF CELL PER DIMENSION
C            PR    - PROBABILITY ARRAY AROUND EACH CELL
C            SD    - STANDARD DEVIATICN VECTOR
C            UL    - UPPER COORD OF A CELL
C            Z     - CELL CENTER COORD ARRAY
C
C       REMARKS
C
C          NONE
C
C       SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C          ERF
C
C       METHOD
C          THE NORMALIZED CLASS DENSITY FUNCTION IS INTEGRATED OVER
C          A HYPERVOLUME WHOSE SIZE IS DETERMINED BY 'DELTA'.
C
C       ..................................................................
C
        SUBROUTINE GRPROB (INDX,DELTA,SD,NS,NDIM,Z,PR,LL,UL)
C
        REAL*4 Z(NDIM,NS),DELTA(1),PR(1),SD(1),LL(1),UL(1)
        INTEGER*4 INDX(1)
C
C
        DO 100 KDIM=1,NDIM
C
C
            UL(KDIM)=Z(KDIM,INDX(KDIM))+DELTA(KDIM)/2.
            LL(KDIM)=Z(KDIM,INDX(KDIM))-DELTA(KDIM)/2.
C
            UL(KDIM)=UL(KDIM)/SD(KDIM)
            LL(KDIM)=LL(KDIM)/SD(KDIM)
            PR(INDX(KDIM))=0.5*(ERF(UL(KDIM)/SQRT(2.))-ERF(LL(KDIM)/SQRT(
     1      2.)))
C
100     CONTINUE
C
        RETURN
        END
C
C       ..................................................................
C
C       SUBROUTINE DIAG
C
C       PURPOSE
C          TO PERFORM THE FOLLOWING OPERATION. (QTRANSPOSE)*X*(Q)
C
C       USAGE
C          CALL DIAG (Q,X,SIGMA,XPRIM,MSA,MSB,NR1,NC1,NC2,ICO,N)
C
C       DESCRIPTION OF PARAMETERS
C          Q      -THE FIRST MATRIX
C          X      -THE SECOND MATRIX
C          SIGMA  -THE OUTPUT MATRIX
C          MSA    -FLAG FOR THE STORAGE MODE OF FIRST MATRIX
C          MSB    -FLAG FOR THE STORAGE MODE OF SECOND MATRIX
C          NR1    -NO OF ROWS IN THE FIRST MATRIX
C          NC1    -NO OF COLUMNS IN THE FIRST MATRIX
C          NC2    -NO OF COLUMNS IN THE SECOND MATRIX
C          ICO    -FLAG TO RESET NR AND NC PARAMETERS
C          N      -NO OF DIMENSIONS
```

FILE. . .   ACAP   FORTRAN B1

```
C
C       REMARKS
C          NONE
C
C       SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C          TPRD,GMPRD
C
C       METHOD
C          TWO SUBROUTINES TPRD AND GMPRD ARE USED IN SECUENCE TO
C          PERFORM THE TRANSPCSITON AND MULTIPLICATION.
C
C
C       ..................................................................
C
C
C
        SUBROUTINE DIAG (Q,X,SIGMA,XPRIM,MSA,MSB,NR1,NC1,NC2,ICO,N)
        REAL Q(1),X(1),SIGMA(1),XPRIM(1)
        WRITE(16,76) X
C
76      FORMAT(10(F5.2,1X))
        IF(ICO.EQ.0) GO TO 55
C
        NR1=N
        NC1=1
        NC2=N
C
55      CONTINUE
        CALL TPRD (Q,X,XPRIM,NR1,NC1,MSA,MSB,NC2)
C
        IF(ICO.EQ.0) GO TO 121
C
        NR1=1
        NC1=N
        NC2=1
C
121     CONTINUE
        CALL GMPRD (XPRIM,Q,SIGMA,NR1,NC1,NC2)
        RETURN
        END
C
C       ..................................................................
C
C          SUBROUTINE GMTRA
C
C          PURPOSE
C             TRANSPOSE A GENERAL MATRIX
C
C          USAGE
C             CALL GMTRA(A,R,N,M)
C
C          DESCRIPTION CF PARAMETERS
C             A - NAME OF MATRIX TO BE TRANSPOSED
C             R - NAME OF RESULTANT MATRIX
C             N - NUMBER OF ROWS OF A AND COLUMNS OF R
C             M - NUMBER OF COLUMNS OF A AND ROWS OF R
C
C          REMARKS
C             MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A
C             MATRICES A AND R MUST BE STORED AS GENERAL MATRICES
C
C          SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C             NONE
C
C          METHOD
C             TRANSPOSE N BY M MATRIX A TO FORM M BY N MATRIX R
C
C          ..................................................................
C
           SUBROUTINE GMTRA(A,R,N,M)
           DIMENSION A(1),R(1)
C
           IR=0
           DO 10 I=1,N
           IJ=I-N
           DO 10 J=1,M
           IJ=IJ+N
           IR=IR+1
   10      R(IR)=A(IJ)
```

```
      RETURN
      END
C
C     ..................................................................
C
C        SUBROUTINE TPRD
C
C        PURPOSE
C           TRANSPOSE A MATRIX AND POSTMULTIPLY BY ANOTHER TO FORM
C           A RESULTANT MATRIX
C
C        USAGE
C           CALL TPRD(A,B,R,N,M,MSA,MSB,L)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF FIRST INPUT MATRIX
C           B - NAME OF SECOND INPUT MATRIX
C           R - NAME OF OUTPUT MATRIX
C           N - NUMBER OF ROWS IN A AND B
C           M - NUMBER OF COLUMNS IN A AND ROWS IN R
C           MSA - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX A
C                  0 - GENERAL
C                  1 - SYMMETRIC
C                  2 - DIAGONAL
C           MSB - SAME AS MSA EXCEPT FOR MATRIX B
C           L - NUMBER OF COLUMNS IN B AND R
C
C        REMARKS
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRICES A OR B
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           LOC
C
C        METHOD
C           MATRIX TRANSPOSE OF A IS NOT ACTUALLY CALCULATED. INSTEAD,
C           ELEMENTS IN MATRIX A ARE TAKEN COLUMNWISE RATHER THAN
C           ROWWISE FOR MULTIPLICATION BY MATRIX B.
C           THE FOLLOWING TABLE SHOWS THE STORAGE MODE OF THE OUTPUT
C           MATRIX FOR ALL COMBINATIONS OF INPUT MATRICES
C                         A                B                R
C                      GENERAL          GENERAL          GENERAL
C                      GENERAL          SYMMETRIC        GENERAL
C                      GENERAL          DIAGONAL         GENERAL
C                      SYMMETRIC        GENERAL          GENERAL
C                      SYMMETRIC        SYMMETRIC        GENERAL
C                      SYMMETRIC        DIAGONAL         GENERAL
C                      DIAGONAL         GENERAL          GENERAL
C                      DIAGONAL         SYMMETRIC        GENERAL
C                      DIAGONAL         DIAGONAL         DIAGONAL
C
C     ..................................................................
C
      SUBROUTINE TPRD(A,B,R,N,M,MSA,MSB,L)
      DIMENSION A(1),B(1),R(1)
C
C        SPECIAL CASE FOR DIAGONAL BY DIAGONAL
C
      MS=MSA*10+MSB
      IF(MS-22) 30,10,30
   10 DO 20 I=1,N
   20 R(I)=A(I)*B(I)
      RETURN
C
C        MULTIPLY TRANSPOSE OF A BY B
C
   30 IR=1
      DO 90 K=1,L
      DO 90 J=1,M
      R(IR)=0.0
      DO 80 I=1,N
      IF(MS) 40,60,40
   40 CALL LOC(I,J,IA,N,M,MSA)
      CALL LOC(I,K,IB,N,L,MSB)
      IF(IA) 50,80,50
   50 IF(IB) 70,80,70
   60 IA=N*(J-1)+I
      IB=N*(K-1)+I
   70 R(IR)=R(IR)+A(IA)*B(IB)
```

FILE. . . ACAP FORTRAN B1

```
   80 CONTINUE
   90 IR=IR+1
      RETURN
      END
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
FILE. . .      SPEST    FORTRAN  B1
C***********************************************************************
C
C     SUPERVISOR FOR THE STRATIFIED POSTERIOR CLASSIFICATION ACCURACY
C                              ESTIMATOR
C
C              WRITTEN 01/24/78  D. J. WIERSMA
C
C***********************************************************************
C
C     AMENDED 10/31/78   B. G. MOBASSERI
C
C     DESCRIPTION AND PURPOSE
C
C        THE STRATIFIED POSTERIOR ESTIMATOR - SPEST- IS SIMILAR  TO TH
C        ACAP PROCESSOR IN PURPOSE. IT PROVIDES A SET OF CLASSIFICATIO
C        ACCURACY ESTIMATES FOR A BAYES CLASSIFIER WHEN THE POPULATION
C        STATISTICS ARE MULTIVARIATE NORMAL. ALTHOUGH THE ALGORITHM
C        EMPLOYS THE LIKELIHOOD PRINCIPAL, IT DIFFERS FROM ACAP APPROC
C        BY USING A RANDOMLY GENERATED DATA BASE. FURTHER DETAILS ON
C        THE THEORY CAN BE FOUND IN THE LARS FINAL REPORT,NOV 30 ,1978
C
C
C     DESCRIPTION OF CONTROL CARDS
C
C        *SPEST
C
C              THIS CARDS SPECIFIES THE PARTICULAR PROCESSOR REQUESTED
C
C        CHANNELS
C
C              THE DESIRED SUBSET OF THE AVAILABLE CHANNELS IS GIVEN HE
C              IT IS IMPORTANT TO REMEMBER THAT THE NUMBERS APPEARING
C              ON THIS CARD IS THE ORDER OF THE SELECTED CHANNELS NOT
C              THEIR ACTUAL NUMBER. FOREXAMPLE, IF THE AVAILABLE
C              CHANNELS ARE 8,9,12,14 AND CHANNELS 8,9 AND 14 ARE REQUE
C              THIS CARD SHOULD READ 1,2,4.
C
C        CLASSES
C
C              THIS CARD SPECIFIES THE NAME OF EACH CLASS. EACH NAME
C              MUST BE PLACED IN A FIELD 7 CHARACTERS LONG FOLLOWED
C              BY A BLANK. THE CONTINUATION CARD , IF REQUESTED, MUST H
C              VE THE NAME 'CLASSES' IN THE BEGINNIG FOLLOWED BY THE
C              REST OF THE NAMES.
C
C        END
C
C              SIGNALS THE END OF THE CONTROL CARDS
C
C     *** STAT DECK FOLLOWS IMMEDIATELY. MUST BE IN CHARACTER FORMAT
C
C     REMARKS
C
C              THIS PROGRAM IS CURRENTLY CAPABLE OF PROCESSING UP TO 20
C              CLASSES AND 8 SPECTRAL BANDS.
C
C     HOW TO RUN THE PROGRAM
C
C              TYPE 'GETDISK OHSYS' TO ESTABLISH THE PROPER LINKS.THE RE
C              ER FILE CONTAINS ONE DECK CONSISTING OF THE CONTROL CARD
C              AND LARSYS STATISTICS DECK. TYPE 'SPEST' FOR THE PROGRAM
C              EXECUTION.
C
C     EXAMPLE OF A CONTROL CARD SET UP
C
C              *SPEST
C              CHANNELS 1,2,4
C              CLASSES BARESOI CORN    PASTURE WHEAT
C              END
C
C***********************************************************************
C
C
C
      REAL*4 PR(20),PHI(8,8,20),P(20),AM(8,20),COV(36,20)
      REAL*4 QP(20),COVT(36),GAM(8,20),DET(20),COVIN(36,20)
      REAL*4 Y(8),DEL(8),COVU(8,8),PX(20),SDET(20)
      REAL*4 EXAM(8,20),EXCOV(36,20)
```

```
FILE. . .      SPEST    FORTRAN  B1
      REAL*4 ICSET(90)
      INTEGER*4 NADR(720),HEAD(20),SDIM
      INTEGER*4 LIST(4)/'*SPE','CHAN','CLAS','END'/
     1          ,IVEC(1),ICARD(20)
     2          ,BLANK/'    '/,FSTCRD/'LARS'/
      INTEGER*2 ICSEL(30),NC(30),NCC(30)
      LOGICAL*1 FLAG(4)/4*.FALSE./
      COMMON /SPCOM/ NTM,NTS,ISIZE,NSIZE,NSA,SDIM,NDIM
C
11    FORMAT(//)
1010  FORMAT(1X,'ERROR IN CTLWRD. EXECUTION TERMINATED.')
1012  FORMAT(1X,'ERROR IN CHANEL. EXECUTION TERMINATED')
1013  FORMAT(1X,'MISSING CONTROL WORD. EXECUTION TERMINATED')
1014  FORMAT(1X,'SPEST PROCESSING STARTED')
1015  FORMAT(1X,'ALL CONTROL CARDS HAVE BEEN READ')
C***********************************************************************
C
C     DECODE CONTROL CARDS
C
C***********************************************************************
C
      DO 777 I=1,30
      ICSEL(I)=0
      NC(I)=0
777   CONTINUE
C
      DO 778 I=1,90
778   ICSET(I)=-50000.0
C
100   CONTINUE
      LSZ=4
      IER=0
      INRD=5
C
      CALL CTLWRD (ICARD,ICOL,LIST,LSZ,ICODE,INRD,IER)
      IF(IER.NE.0) GO TO 1001
      GO TO (99,101,102,103), ICODE
99    FLAG(1)=.TRUE.
      GO TO 100
C
C     CHANNELS CARD
C
101    CALL CHANEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,900)
      FLAG(2)=.TRUE.
      NDIM=NCR
      GO TO 100
C
C     CLASS NAMES CARD
C
102   DO 10 I=1,20
10    HEAD(I)=ICARD(I)
      FLAG(3)=.TRUE.
      GO TO 100
C
C     END CARD
C
103   FLAG(4)=.TRUE.
      GO TO 201
C
C
201   CONTINUE
C
C     CHECK IF ALL CONTROL CARDS HAVE BEEN READ
C
      DO 250 I=1,4
      IF(.NOT.FLAG(I)) GO TO 321
      GO TO 250
321   WRITE(16,1013)
      WRITE( 6,1013)
      GO TO 999
250   CONTINUE
C
      WRITE(16,11)
      WRITE( 6,11)
      WRITE(16,1014)
      WRITE( 6,1014)
C
      WRITE(16,1015)
```

```
      WRITE( 6,1015)
C
      GO TO 680
1001  WRITE(16,1010)
      WRITE( 6,1010)
      GO TO 999
680   CONTINUE
C
      GO TO 720
900   WRITE(6,1012)
      WRITE(16,1012)
      GO TO 999
720   CONTINUE
C*************************************************************************
C
C     READ THE TOTAL NO OF CHANNELS AND CLASSES FROM THE STAT DECK
C
C*************************************************************************
C
502   READ (5,501) ICRD
501   FORMAT(A4)
      ICRDSQ=ICRDSQ+1
C
      IF(ICRD.EQ.BLANK) GO TO 503
      GO TO 502
503   CONTINUE
C
      REWIND 5
602   READ(5,501) ICRD
      IF(ICRD.EQ.FSTCRD) GO TO 601
      GO TO 602
601   CONTINUE
C
      NUM=ICRDSQ-2
      DO 506 I=1,NUM
      READ(5,507)
507   FORMAT(18A4,I8)
506   CONTINUE
C
C
C
      READ(5,508) NCLS,NFLD,SDIM
508   FORMAT(I5,6X,I5,6X,I5)
C
      M=NCLS
      N=NDIM
      NUM2=SDIM+1
      DO 509 I=1,NUM2
      READ(5,507)
509   CONTINUE
C
C
C     FIND THE CHANNEL SET THAT IS NOT REQUESTED
C
      DO 611 I=1,SDIM
      DO 612 J=1,NDIM
C
          IF(I.EQ.NCC(J)) GO TO 611
C
612   CONTINUE
C
          K=K+1
          NC(K)=I
C
611   CONTINUE
C
      ISIZE=NDIM*(NDIM+1)/2
      NSIZE=SDIM*(SDIM+1)/2
C
      NTS=NCLS*ISIZE
      NTM=NCLS*NDIM
      NSS=NCLS*ND
      NSA=NCLS*NSIZE
      NSM=NCLS*SDIM
C
C
      NCT = N*(N+1)/2
      DO 12 I=1,M
12    P(I)=1./FLOAT(M)
```

FILE. . .   SPEST   FORTRAN 81

```
C
C     CALL MCOVP(N,AM(1,I),COV(1,I))
30    CONTINUE
      CALL SPESTM(M,N,PHI,P,AM,COV,PR,PC,QP,COVT,GAM,DET,COVIN,
     1            Y,TEI,DEL,COVU,PX,SDET,NADR,NC,NCC,EXAM,EXCOV,
     2            HEAD)
C
C
C
C
      PC=100.*PC
      WRITE(6,60)PC
60    FORMAT(/////30X,'OVERALL PROBABILITY OF CORRECT RECOGNITION = ',F7
     1.3)
999   STOP
      END
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C  SPEST IS AN ESTIMATOR OF THE CLASSIFICATION PERFORMANCE FROM A
C  GIVEN SET OF STATISTICS FROM M CLASSES.  THE ESTIMATOR IS A
C  STRATIFIED POSTERIOR ESTIMATOR (REF. WHITSITT AND LANDGREBE).
C  THE PROBABILITY DISTRIBUTIONS ARE ASSUMED TO BE MULTIVARIATE
C  GAUSSIAN
C
C  19 JANUARY, 1978
C
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      SUBROUTINE SPESTM(M,N,PHI,P,AM,COV,PR,PC,QP,COVT,GAM,DET,COVIN,
     1                  Y,TEI,DEL,COVU,PX,SDET,NADR,NC,NCC,EXAM,EXCOV,
     2                  HEAD)
      INTEGER*4 NADR(NSA),SDIM
      INTEGER*4 FMT1(4) /'(2X,','   0','E14.','7)'/
      INTEGER*4 FMT2(4) /'(2X,','   0','E14.','7)'/
      INTEGER*4 N1/'   0'/,N2/'   0'/
      REAL*4 QP(M),P(M),PR(M),EXAM(SDIM,M),EXCOV(NSIZE,M),COVT(ISIZE)
      REAL*4 GAM(N,M),PHI(N,N,M),DET(M),COVIN(ISIZE,M)
      REAL*4 Y(N),TEI(N),DEL(N),COVU(N,N)
      REAL*4 AM(N,M),HEAD(20),COV(ISIZE,M)
      REAL*8 PX(M),BIG,DEN,SDET(M),BETA,Z0,Z1,Z2,Z3
      INTEGER*2 NC(30),NCC(30)
      COMMON /SPCOM/ NTM,NTS,ISIZE,NSIZE,NSA,SDIM,NDIM
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C  LIST OF VARIABLES
C     M = NUMBER OF CLASSES
C     N = NUMBER OF DIMENSIONS
C     P(I) = APRIORI PROBABILITIES OF CLASS I
C     PR(I) = CLASS CONDITIONAL PERFORMANCE
C     PC = OVERALL PERFORMANCE
C     AM(J,I)  = MEAN VECTOR OF CLASS I
C     COV(J,I) = COVARIANCE MATRIX OF CLASS I(STORED IN UPPER TRIANGULAR
C                FORM)
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C
C*************************************************************************
C
C
C     READ IN THE MEAN VECTORS AND COVARIANCE MATRICES
C
C
C*************************************************************************
C
      N1=N1+SDIM
      N2=N2+5
      IF(NSIZE.LT.5) N2=N2+NSIZE
C
      FMT1(2)=N1
      FMT2(2)=N2
C
      DO 30 I=1,M
      READ (5,FMT1) (EXAM(J,I),J=1,SDIM)
30    CONTINUE
C
      DO 31 I=1,M
      READ (5,FMT2) (EXCOV(K,I),K=1,NSIZE)
31    CONTINUE
C
```

```
C
C***********************************************************************
C
C     SELECT THE REQUIRED SUBSET OF THE CHANNELS
C
C***********************************************************************
C
      DO 803 JCLS=1,M
C
         K=0
C
      DO 802 KDIM=1,SDIM
      DO 801 J=1,N
C
         IF(KDIM.EQ.NCC(J)) K=K+1
         IF(KDIM.EQ.NCC(J)) AM(K,JCLS)=EXAM(NCC(J),JCLS)
C
801   CONTINUE
802   CONTINUE
803   CONTINUE
C
C***********************************************************************
C
C     OBTAIN THE ADDRESS OF THE SELECTED ENTRIES INTO COVARIANCE MATRIX
C
C***********************************************************************
C
      CALL ADRES (NADR,NC,SDIM,NDIM)
C
      K=0
      DO 603 JCLS=1,M
C
         K=0
C
      DO 602 I    =1,NSIZE
      DO 601 J   =1,NSIZE
C
         IF(I.EQ.NADR(J)) GO TO 602
C
601   CONTINUE
C
      K=K+1
      COV(K,JCLS)=EXCOV(I,JCLS)
C
602   CONTINUE
603   CONTINUE
C
C
      IX = 947913
      NCT = N*(N+1)/2
C
C  COPPUTE EIGENVALUES AND EIGENVECTORS FOR EACH MATRIX
C
      MV = 0
      EPS = 1.0E-6
      DO 100 IJ=1,M
      DO 55 I=1,NCT
55    COVT(I) = COV(I,IJ)
      CALL EIGEN(COVT,PHI(1,1,IJ),N,MV)
      L = 0
      DO 60 I =1,N
      L = L + I
60    GAM(I,IJ) = COVT(L)
C
C  COMPUTE DETERMINANT AND INVERSE OF EACH MATRIX
C
      DO 65 I=1,NCT
65    COVT(I) = COV(I,IJ)
      CALL SMINV(COVT,N,DET(IJ),MV,EPS,IER)
      IF(IER)1000,70,1000
70    CONTINUE
      SDET(IJ) = SQRT(DET(IJ))
      DO 75 I=1,NCT
75    COVIN(I,IJ) = COVT(I)
100   CONTINUE
      MV = 0
      DO 105 I=1,M
105   QP(I) = 0.0
```



```
C
C  LOOP ON CLASS ICL
C
      PC = 0.0
      DO 500 ICL=1,M
      AVEQ = 0.0
C
C  LOOP ON THE NUMBER OF SAMPLES
C
      NS = 1000
      DO 300 IJ=1,NS
C
C  GENERATE Y VECTOR FROM CLASS ICL
C
      DO 110 I=1,N
      CALL RANDU(IX,IY,XP)
      IX = IY
      CALL NDTRI(XP,Y(I),XD,IER)
110   CONTINUE
C
C  COMPUTE CONDITIONAL PROBABILITIES FOR EACH CLASS
C
      DO 200 JCL=1,M
      IF(JCL .EQ. ICL) GO TO 180
      DO 130 I=1,N
      TE1(I) = 0.0
      DEL(I) = AM(I,ICL) - AM(I,JCL)
      DO 130 J=1,N
      TE1(I) = TE1(I) + SQRT(GAM(J,ICL))*Y(J)*PHI(I,J,ICL)
130   CONTINUE
      JJ = 0
      DO 140 I=1,N
      DO 140 J=1,I
      JJ = JJ + 1
      COVU(I,J) = COVIN(JJ,JCL)
      COVU(J,I) = COVIN(JJ,JCL)
140   CONTINUE
      Z1 = 0.0
      Z2 = 0.0
      Z3 = 0.0
      DO 150 I=1,N
      DO 150 J=1,N
      Z1 = Z1 - 0.5*TE1(I)*COVU(I,J)*TE1(J)
      Z2 = Z2 - TE1(I)*COVU(I,J)*DEL(J)
      Z3 = Z3 - 0.5*DEL(I)*COVU(I,J)*DEL(J)
150   CONTINUE
      ZSUM = Z1 + Z2 + Z3
      IF(ZSUM .LT. -100) GO TO 190
      BETA = P(JCL)*1.0
      PX(JCL) = BETA*DEXP(Z1+Z2+Z3)/SDET(JCL)
      IF(PX(JCL) .EQ. 0.0) WRITE(16,919) ICL,JCL,ZSUM,SDET(JCL),PX(JCL
170   CONTINUE
      GO TO 200
180   CONTINUE
      Z0 = 0.0
      DO 185 I=1,N
      Z0 = Z0- 0.5*Y(I)*Y(I)
185   CONTINUE
      IF(Z0 .LT. -100) GO TO 190
      BETA = P(JCL)*1.0
      PX(JCL) = BETA*DEXP(Z0)/SDET(JCL)
      IF(PX(JCL) .EQ. 0.0) WRITE(16,919) ICL,JCL,Z0,SDET(JCL),PX(JCL)
      GO TO 200
190   PX(JCL) = 0.0
200   CONTINUE
919   FORMAT(5X,2I5,3E12.4)
C
C
C  CHOOSE THE LARGEST
C
      BIG = -1000
      DO 220 I=1,M
      IF(PX(I) .GT. BIG) LOC = I
      IF(PX(I) .GT. BIG) BIG = PX(I)
220   CONTINUE
      DEN = 0.0
      DO 230 I=1,M
      DEN = DEN + PX(I)
```

FILE. . .   SPEST   FORTRAN B1

```
230    CONTINUE
       Q = BIG/DEN
C
C  AVERAGE
C
       QP(LOC) = QP(LOC) + P(ICL)*Q/P(LOC)
300    CONTINUE
500    CONTINUE
       WRITE(6,73)
       WRITE(6,536)
536    FORMAT(35X,'STRATIFIED POSTERIOR ERROR ESTIMATOR')
73     FORMAT(1H1)
       DO 510 ICL=1,M
       PR(ICL) = QP(ICL)/FLOAT(NS)
       PC = PC + P(ICL)*PR(ICL)
       WRITE(6,72)
72     FORMAT(///)
       DSPL=2*ICL
       WRITE(6,537) (HEAD(DSPL+I),I=1,2)
537    FORMAT(49X,'CLASS ',2A4)
C
       PCPR=100.*PR(ICL)
       WRITE(6,82)
82     FORMAT(//)
       WRITE(6,482) PCPR
482    FORMAT(25X,'P R O B A B I L I T Y   O F   C O R R E C T   C L A S
      1 S I F I C A T I O N = ',F7.3,'(')
510    CONTINUE
       RETURN
1000   WRITE(6,1100)IER
1100   FORMAT(10X,'***INVERSION ERROR(',I2,')***')
       RETURN
       END
C.....................................................................
C
C      SUBROUTINE ADRES
C
C      PURPOSE
C
C           FIND THE DESIRED SUBSET OF A COVARIANCE MATRIX
C
C      DESCRIPTION OF PARAMETERS
C
C           NADR - ADDRESS ARRAY OF THE ENTRIES TO BE DELETED
C           NC   - ARRAY OF THE CHANNELS TO BE DELETED
C           SDIM - TOTAL NO OF BANDS SUPLIED
C           NDIM - DESIRED SUBSET OF SDIM
C
C      REMARKS
C
C           NONE
C
C      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C            NONE
C
C      METHOD
C
C         USING A ONE DIMENSIONAL STORAGE MODE FOR EACH COVARIANCE
C          MATRIX IN AN UPPER TRIANGULAR FORM, THE PARAMETRIC ADDRESSES
C          FOR EACH ENTRY IS DERIVED AND USING THE NC ARRAY THE LOCATION
C          OF ALL THE ENTRIES THAT LIE IN THE UNWANTED LINES AND COLUMNS
C          IS COMPUTED AND STORED IN NADR.
C
C.....................................................................
C
       SUBROUTINE ADRES(NADR,NC,SDIM,NDIM)
C
       INTEGER*4 NADR(1),PDIM,SDIM
       INTEGER*2 NC(1)
C
       PDIM=SDIM-NDIM
C
       IF(PDIM.LE.0) GO TO 100
       DO 20 J=1,PDIM
       N1=NC(J)
       DO 10 I=1,N1
       K=K+1
```

FILE. . .   SPEST   FORTRAN B1

```
       NADR(K)=(NC(J)*(NC(J)-1)/2)+I
10     CONTINUE
20     CONTINUE
C
C
       DO 45 J=1,PDIM
       NFIN=SDIM-NC(J)+1
       DO 35 I=1,NFIN
C
       K=K+1
       NADR(K)=((NC(J)+I-1)*(NC(J)+I-2)/2)+NC(J)
35     CONTINUE
45     CONTINUE
C
100    RETURN
       END
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
FILE. . .     CORELAT  FORTRAN  B1
C*****************************************************************************
C
C     SUPERVISOR FOR A TWO DIMENSIONAL SPATIAL CORRELATION
C
C          WRITTEN 10/04/78   BIJAN G. MOBASSERI
C
C*****************************************************************************
C
C
C     DESCRIPTION AND PURPOSE
C
C        THIS PROGRAM IS A TWO DIMENSIONAL SPATIAL CORRELATOR THE
C        PRIMARY OUTPUT OF WHICH IS A NORMALIZED SPATIAL CORRELATION
C        MATRIX FOR ANY SPECIFIED AREA.
C
C        THE USER SPECIFIES THE COORDINATES OF HIS DESIRED AREA IN THE
C        FORM OF INITIAL AND FINAL LINES AND COLUMNS ALONG WITH THE
C        RESPECTIVE SPECTRAL BAND(S). IF THE AUTOCORRELATION FUNCTION
C        IS DESIRED ONLY ONE CHANNEL NUMBER NEED BE SPECIFIED.
C
C        FOLLOWING THE ESTIMATION OF THE CORRELATION MATRIX, THE
C        EXPONENTIAL FIT OPTION ,IF INVOKED, WILL FIT AN EXPONENTIALLY
C        DROPPING FUNCTION TO THE EXPERIMENTAL DATA USING A WEIGHTED
C        LINEAR LEAST SQUARES FIT TECHNIQUE. IN CASES WHERE THIS
C        ASSUMPTION IS NOT VALID, THIS OPERATION IS BYPASSED.
C
C        THE PURPOSE HERE IS TO DEVELOP A MARKOV MODEL FOR THE
C        SPATIAL CORRELATION FUNCTION OF THE MSS DATA.
C
C
C     DESCRIPTION OF CONTROL CARDS
C
C       *CORRELATE
C
C          THIS CARD SPECIFIES THE PARTICULAR PROCESSOR REQUESTED
C
C       INPUT RUN(.),TAPE(.),FILE(.)
C
C          THE INPUT RUNTABLE FROM WHICH DATA IS READ
C
C       BLOCK LINE(.,.),COLUMN(.,.)
C
C          SEGMENT TO BE CORRELATED
C
C       FUNCTION
C
C          EITHER AUTO OR CROSS FUNCTION CAN BE SPECIFIED
C
C       CHANNELS
C
C          SPECTRAL BANDS USED IN CORRELATING
C
C       SAMPLELAG
C
C          CROSS TRACK LAG USED IN ESTIMATING THE CORRELATION FUNCTION
C          EXPRESSED AS A PERCENTAGE OF TOTAL NO OF SAMPLES IN
C          THE AREA. THIS CARD IS OPTIONAL.
C
C
C       LINELAG
C
C          SAME AS SAMPLELAG EXCEPT FOR LINES (ALONG TRACK)
C
C       EXPOFIT
C
C          THIS CARD STARTS THE EXPONENTIAL FITTING PROCEDURE. OPTIONAL
C
C       END
C
C          END OF CONTROL CARDS
C
C     REMARKS
C
C        THIS PROGRAM IS CURRENTLY CAPABLE OF PROCESSING AN AREA
C        2401 PIXELS LARGE .SINCE ALL THE SUBROUTINES ARE DYNAMICALLY
C        DIMENSIONED, ANY ENLARGEMENTS CAN BE ACCOMPLISHED BY ALTERIN(
C        THE DIMENSIONS OF THE ARRAYS IN THE MAIN PROGRAM.
C
```

```
FILE. . .     CORELAT  FORTRAN  B1
C     HOW TO RUN THE PROGRAM
C
C        USE THE COMMAND 'GETDISK DHSYS' TO ESTABLISH THE PROPER LINK
C        THEN TYPE *CORRELATE.
C
C     EXAMPLE OF CONTROL CARD SETUP
C
C        *CORRELATE
C        INPUT RUN(74028500),TAPE(2689),FILE(3)
C        BLOCK LINE(1,25),COLUMN(1,25)
C        FUNCTION AUTO
C        CHANNELS 2
C        SAMPLELAG 25
C        LINELAG 25
C        EXPOFIT
C        END
C
C     OR FOR CROSSCORRELATION,
C
C        *CORRELATE
C        INPUT RUN(74028500),TAPE(2689),FILE(3)
C        BLOCK LINE(1,25),COLUMN(1,25)
C        FUNCTION CROSS
C        CHANNELS 1,3
C        SAMPLELAG 25
C        LINELAG 25
C        EXPOFIT
C        END
C
C
C     IF SAMPLELAG AND LINELAG ARE LEFT OUT, THE DEFAULT IS 20
C     PERCENT OF THE TOTAL NO OF LINES AND COLUMNS.
C
C*****************************************************************************
C
C
      IMPLICIT INTEGER (A-Z)
      REAL*4 F(2500),G(2500),R(2500),ICSET(90),RNORM,MEAN1,MEAN2
      INTEGER*4 RUNTAB(10,3),ID(200),LDATA1(2500),LDATA2(2500)
      INTEGER*4 LIST1(9)/'*COR','INPU','BLOC','FUNC','CHAN','SAMP',
     1                   'LINE','EXPO','END '/,
     2          LIST2(3)/'RUN ','TAPE','FILE'/,
     3          LIST3(2)/'LINE','COLU'/
      INTEGER*4 ICARD(20),FCT(20),IVEC(2)
      INTEGER*2 CSEL(8),ICSEL(30),NCC(30)
      LOGICAL*1 IDATA(25000),FLAG(12)/12*.FALSE./
      COMMON /CORCOM/ FSTLN,LLINE,FSTCL,LCOL,NSIZEX,NSIZEY,LAGX,LAGY,
     1                PCTX,PCTY
C
 1011 FORMAT(1X,'ERROR IN CTLWRD. ERROR CODE=',I3,' EXECUTION TERMINAT
     1')
 1012 FORMAT(1X,'ERROR IN CTLPRM. EXECUTION TERMINATED')
 1013 FORMAT(1X,'ERROR IN IVAL. EXECUTION TERMINATED')
 1014 FORMAT(1X,'ERROR IN CHANEL. EXECUTION TERMINATED')
 1015 FORMAT(1X,'MISSING CONTROL CARD OR PARAMETER. EXECUTION TERMINATI
     1')
C
C
      PCTX=20
      PCTY=20
      DO 777 I=1,30
      NCC(I)=0
      ICSEL(I)=0
  777 CONTINUE
C
      DO 778 I=1,90
  778 ICSET(I)=-50000.0
C
  100 CONTINUE
      LSZ=9
      IER=0
      INRD=5
      CALL CTLWRD (ICARD,ICOL,LIST1,LSZ,ICODE,INRD,IER)
      IF(IER.NE.0) GO TO 1001
      GO TO (99,101,102,103,104,105,106,107,108),ICODE
   99 FLAG(1)=.TRUE.
      GO TO 100
C
C     INPUT RUNTABLE
```

```
C
101     LSZ=3
        CALL CTLPRM(ICARD,ICOL,LIST2,LSZ,ICODE,1002)
        LSZ=1
        CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
        GO TO (501,502,503), ICODE
C
C       RUN NUMBER
C
501     RUNTAB(1,1)=IVEC(1)
        FLAG(2)=.TRUE.
        IF(ICOL.LT.72) GO TO 101
        GO TO 100
C
C       TAPE NUMBER
C
502     RUNTAB(1,2)=IVEC(1)
        FLAG(3)=.TRUE.
        IF(ICOL.LT.72) GO TO 101
        GO TO 100
C
C       FILE NUMBER
C
503     RUNTAB(1,3)=IVEC(1)
        FLAG(4)=.TRUE.
        IF(ICOL.LT.72) GO TO 101
        GO TO 100
C
102     LSZ=2
        CALL CTLPRM(ICARD,ICOL,LIST3,LSZ,ICODE,1002)
        LSZ=2
        CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
        GO TO (601,602), ICODE
C
C       NO OF LINES
C
601     FSTLN=IVEC(1)
        LLINE=IVEC(2)
        FLAG(5)=.TRUE.
        IF(ICOL.LT.72) GO TO 102
        GO TO 100
C
C       NO OF COLUMNS
C
602     FSTCL=IVEC(1)
        LCOL  =IVEC(2)
        FLAG(6)=.TRUE.
        IF(ICOL.LT.72) GO TO 102
        GO TO 100
C
C       AUTO OR CROSS
C
103     DO 151 I=1,20
        FCT(I)=ICARD(I)
151     CONTINUE
        FLAG(7)=.TRUE.
        GO TO 100
C
C       CHANNELS CARD
C
104     CALL CHANEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,1004)
        NDIM=NCR
        FLAG(8)=.TRUE.
        GO TO 100
C
C       CROSS TRACK LAG
C
105     LSZ=1
        CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
        PCTX=IVEC(1)
        FLAG(9)=.TRUE.
        IF(ICOL.LT.72) GO TO 101
        GO TO 100
C
C       ALONG TRACK LAG
C
106     LSZ=1
        CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
```



```
        PCTY=IVEC(1)
        FLAG(10)=.TRUE.
        IF(ICOL.LT.72) GO TO 101
        GO TO 100
C
C       EXPONENTIAL FIT
C
107     FLAG(11)=.TRUE.
        GO TO 100
C
C       END CARD
108     FLAG(12)=.TRUE.
C
C
C       CHECK IF ALL CONTROL CARDS HAVE BEEN READ
C
        DO 651 I=1,8
        IF(.NOT.FLAG(I)) GO TO 652
651     CONTINUE
        IF(.NOT.FLAG(12)) GO TO 652
C
        GO TO 654
652     WRITE(16,1015)
        GO TO 999
C
654     CONTINUE
        GO TO 125
1001    WRITE(16,1011) IER
        GO TO 999
1002    WRITE(16,1012)
        GO TO 999
1003    WRITE(16,1013)
        GO TO 999
1004    WRITE(16,1014)
        GO TO 999
C
125     NSIZEX=LCOL-FSTCL+1
        NSIZEY=LLINE-FSTLN+1
C
C
        LAGX=PCTX*NSIZEX/100
        LAGY=PCTY*NSIZEY/100
C
        CALL CRLT (F,G,R,LDATA1,LDATA2,RUNTAB,NCC,FCT,FLAG)
C
999     STOP
        END
C
C...................................................................
C
C       SUBROUTINE CRLT
C
C     PURPOSE
C          TO PERFORM A TWO DIMENSIONAL CORRELATION OF AN IMAGE IN TWO
C          ARBITRARY BANDS AND DETERMINE A SPATIAL CORRELATION MATRIX.
C       DESCRIPTION OF PARAMETERS
C
C        F            - ARRAY TO STORE THE INPUT IMAGE (FIRST CHANNEL)
C        G            - ARRAY TO STORE THE INPUT IMAGE (SECOND CHANNEL)
C        R            - RESULTING SPATIAL CORRELATION MATRIX
C        LDATA1       - STORAGE ARRAY
C        LDATA2       - STORAGE ARRAY
C        RUNTAB(1,1)  - RUN NUMBER OF THE DESIRED AREA
C        RUNTAB(1,2)  - TAPE NUMBER OF THE DESIRED AREA
C        RUNTAB(1,3)  - FILE NUMBER OF THE DESIRED AREA
C        NCC          - CHANNEL ARRAY
C        FCT          - DESIGNATING AUTO OR CROSS FUNCTION
C        FLAG         - OPTION ARRAY
C
C       REMARKS
C
C        THIS SUBROUTINE HAS VARIABLE DIMENSION PROPERTY. ALL THE
C        ARRAYS MUST BE DIMENSIONED IN THE MAIN PROGRAM ACCORDING
C        TO THE SIZE OF THE PICTURE.
C        THE LAG IN ESTIMATING THE CORRELATION FUNCTION IS USER SUPPLIE
C        IT IS ADVISABLE HOWEVER TO KEEP THIS QUANTITY AT
C        OR BELOW 20 PERCENT OF THE AREA IN ORDER TO PROVIDE AN
C        ESTIMATE WITH SMALL VARIANCE.
```

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```
FILE. . .     CORELAT  FORTRAN  B1
C       ARRAYS F,G,AND R ARE EMPTY UPON ENTERING THE SUBROUTINE.
C
C      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C       ACF, TAPOP, GETRUN
C
C      METHOD
C
C       THE STANDARD LAGGED-PRODUCT SUM METHOD IS IMPLEMENTED IN TWO
C       DIMENSIONS AND CARRIED OUT BY SUBROUTINE ACF.
C
C
C**********************************************************************
C
C
C
C
      SUBROUTINE CRLT (F,G,R,LDATA1,LDATA2,RUNTAB,NCC,FCT,FLAG)
C
C**********************************************************************
C
C              LOCAL VARIABLES DEFINITION
C
C**********************************************************************
C
      IMPLICIT INTEGER (A-Z)
      REAL*4 F(NSIZEX,NSIZEY),G(NSIZEX,NSIZEY),R(LAGX,LAGY),
     1       H(5),P(6),S(6),WGT(3),WR1(2),WR2(2),ERROR(3),
     2       IER(3),RMSE(1),FN(3),FHAT(3),RNORM,MEAN1,MEAN2,
     3       ER1,ER2,RHOX,RHOY,C(2)
      INTEGER*4 RUNTAB(10,3),ID(200),LDATA1(NSIZEX),LDATA2(NSIZEX)
      INTEGER*4 ACOR/' AUT'/,CCOR/' CRO'/
      INTEGER*4 FMT1(6)/'((//','30X,',' ','  0',',(F5.',',2,1X',','))'/
      INTEGER*4 FMT2(5)/'(',' ','  0','(F5.','2,1X',')))'/
      INTEGER*4 FCT(20), N1/'   0'/
      INTEGER*2 CSEL(8),NCC(30)
      LOGICAL*1 IDATA(25000),FLAG(12)
      COMMON /CORCOM/ FSTLN,LLINE,FSTCL,LCOL,NSIZEX,NSIZEY,LAGX,LAGY,
     1                PCTX,PCTY
C
C**********************************************************************
C
C           COMMON BLOCK VARIABLES DESCRIPTION
C
C
C
C      FSTLN  - FIRST LINE
C      FSTCL  - FIRST COLUMN
C      LLINE  - LAST LINE
C      LCOL   - LAST COLUMN
C      NSIZEX- NO OF COLUMNS
C      NSIZEY - NO OF LINES
C      PCTX   - LAG AS A PERCENTAGE OF NSIZEX
C      PCTY   - LAG AS A PERCENTAGE OF NSIZEY
C**********************************************************************
60    FORMAT(1X,'GETRUN ERROR=',I2)
61    FORMAT(1X,'TOPFS ERROR=',I2)
62    FORMAT(1X,'TOPRV ERROR=',I2)
C
      INUNIT=12
      LAG=3
C
C**********************************************************************
C
C*    SETUP THE RUNTABLE AND FIND THE PROPER FLIGHTLINE
C
C**********************************************************************
C
C
C
C
      FSTCL=FSTCL-1
C
      RUNSEL=RUNTAB(1,1)
C
      CALL GETRUN (RUNSEL,INUNIT,ID,ERR,RUNTAB,1)
      IF(ERR.NE.0) WRITE(16,60) ERR
      IF(ERR.NE.0) GO TO 881
```

```
FILE. . .     CORELAT  FORTRAN  B1
1     NRTRY=2
C
C**********************************************************************
C
C*    POSITION THE TAPE AT THE START OF DESIRED RECORD
C
C**********************************************************************
C
      ICOUNT=FSTLN-1
      IF(ICOUNT.EQ.0) GO TO 211
      CALL TOPFS (INUNIT,ICOUNT,ERR)
      IF(ERR.NE.0) WRITE(16,61) ERR
      IF(ERR.NE.0) GO TO 881
211   CONTINUE
C
      NSC=ID(6)
      NC =ID(5)
C
      IF(FCT(3).EQ.ACOR) DSPL=FSTCL
      IF(FCT(3).EQ.CCOR) DSPL=NSC+FSTCL
      DO 11 I=1,8
11    CSEL(I)=0
C
      IF(FCT(3).EQ.ACOR) GO TO 14
      CSEL(NCC(1))=1
      CSEL(NCC(2))=1
14    CSEL(NCC(1))=1
C
C**********************************************************************
C*    READ IN THE DESIRED AREA
C**********************************************************************
C
      DO 30 J=1,NSIZEY
      CALL TOPRV (INUNIT,NSC,ERR,IDATA,NRTRY,NC,CSEL,LNID)
      IF(ERR.NE.0) WRITE(16,62) ERR
      IF(ERR.NE.0) GO TO 881
C
      CALL MOVBYT (IDATA,FSTCL,1,LDATA1,3,4,NSIZEX)
      CALL MOVBYT (IDATA,DSPL,1,LDATA2,3,4,NSIZEX)
      DO 20 I=1,NSIZEX
      F(I,J)=LDATA1(I)
      G(I,J)=LDATA2(I)
20    CONTINUE
30    CONTINUE
C
C
C
C**********************************************************************
C*    FIND THE MEAN OF THE PICTURE
C**********************************************************************
C
      DO 200 J=1,NSIZEY
      DO 200 I=1,NSIZEX
      MEAN1=MEAN1+F(I,J)
      MEAN2=MEAN2+G(I,J)
200   CONTINUE
      MEAN1=MEAN1/FLOAT(NSIZEX*NSIZEY)
      MEAN2=MEAN2/FLOAT(NSIZEX*NSIZEY)
C
C
C**********************************************************************
C*    SUBTRACT THE MEAN OUT
C**********************************************************************
C
      DO 255 J=1,NSIZEY
      DO 255 I=1,NSIZEX
      F(I,J)=F(I,J)-MEAN1
      G(I,J)=G(I,J)-MEAN2
652   FORMAT(1X,I2,1X,I2,1X,2(1PE11.4,1X))
255   CONTINUE
220   CONTINUE
C
C**********************************************************************
C
C     COMPUTE THE AUTO(CROSS) CORRELATION FUNCTION
C
C**********************************************************************
```

```
FILE. . .     CORELAT  FORTRAN  B1
C
      CALL ACF (F,G,R,NSIZEX,NSIZEY,LAGX,LAGY)
C
C
C
C
C
C
C***********************************************************************
C      NORMALIZE THE RESULTING CORRELATION FUNCTION                    *
C***********************************************************************
C
      RNORM=R(1,1)
      DO 68 J=1,LAGY
      DO 68 I=1,LAGX
      R(I,J)=R(I,J)/(RNORM)
68    CONTINUE
C
      IF(.NOT.FLAG(11)) GO TO 902
C
C***********************************************************************
C
C      EXPONENTIAL CORRELATION MODEL DEVELOPMENT
C
C***********************************************************************
C
C     USE A LINEAR LEAST SQUARE FIT TO THE LOGARITHM OF THE FUNCTION
C
      IF(R(1,1).LT.R(2,1)) GO TO 901
      IF(R(1,1).LT.R(1,2)) GO TO 901
C
      DO 831 I=1,LAG
      IF(R(I,1).LE.0) GO TO 901
      FN(I)=-ALOG(R(LAG-(I-1),1))
831   CONTINUE
C
      CALL LSF (H,WGT,P,S,FN,FHAT,C,IER,ERROR,WR1,WR2,LAG,RMSE,RMU)
      ER1=RMSE(1)
      RHOX=EXP(-C(2))
C
      DO 832 I=1,LAG
      IF(R(1,I).LE.0) GO TO 901
      FN(I)=-ALOG(R(1,LAG-(I-1)))
832   CONTINUE
C
      CALL LSF (H,WGT,P,S,FN,FHAT,C,IER,ERROR,WR1,WR2,LAG,RMSE,RMU)
      ER2=RMSE(1)
      RHOY=EXP(-C(2))
C
      GO TO 902
901   WRITE(16,1013)
      WRITE(6,1013)
1013  FORMAT(1X,'EXPONENTIAL FUNCTION DOES NOT DESCRIBE THE ACF OR CCF O
     1F THIS AREA. REQUEST  IGNORED')
902   CONTINUE
C
C***********************************************************************
C
C                         GENERATE THE OUTPUT
C
C***********************************************************************
C
      IF(FCT(3).EQ.ACOR) NCC(2)=NCC(1)
      FMT1(3)=LAGX+N1
      IF(LAGX.GE.10) FMT1(3)=N1+9
      FMT2(2)=FMT1(3)
      WRITE(6,1000)
1000  FORMAT(1H1)
      WRITE(6,1001)
1001  FORMAT(40X,'TWO DIMENSIONAL SPATIAL CORRELATION ANALYSIS')
      WRITE(6,1002)
1002  FORMAT(//)
      WRITE(6,1003) NCC(1),NCC(2)
1003  FORMAT(50X,'CHANNELS0 ',2(I2,1X))
      WRITE(6,1002)
C     WRITE(6,1004) (CLASS(I+2),I=1,2)
1004  FORMAT(40X,'CLASS',2A4)
```

```
FILE. . .     CORELAT  FORTRAN  B1
      WRITE(6,1005)
1005  FORMAT(////)
      WRITE(6,1006)
1006  FORMAT(30X,'2-D SPATIAL CORRELATION MATRIX')
      WRITE(6,1002)
      DO 320 J=1,LAGY
      WRITE(6,FMT1) (R(I,J),I=1,LAGX)
320   CONTINUE
C
      IF(R(1,1).LT.R(2,1)) GO TO 671
      IF(R(1,1).LT.R(1,2)) GO TO 671
      IF(.NOT.FLAG(11)) GO TO 671
C
      WRITE(6,1005)
      WRITE(6,1007)
1007  FORMAT(40X,'WEIGHTED LEAST SQUARES FIT INFORMATION')
      WRITE(6,1002)
      WRITE(6,1008) RMU
1008  FORMAT(30X,'WEIGHTING MATRIX DIAGONAL BASE=',F4.2)
      WRITE(6,1002)
      WRITE(6,1009) ER1
1009  FORMAT(30X,'WEIGHTED LSF ERROR (CROSS TRACK)=',E14.7)
      WRITE(6,1002)
      WRITE(6,1010) RHOX
1010  FORMAT(30X,'ADJACENT SAMPLE CORRELATION=',E14.7)
      WRITE(6,1002)
      WRITE(6,1011) ER2
1011  FORMAT(30X,'WEIGHTED LSF ERROR (ALONG TRACK)=',E14.7)
      WRITE(6,1002)
      WRITE(6,1012) RHOY
1012  FORMAT(30X,'ADJACENT LINE CORRELATION    =',E14.7)
C
671   CONTINUE
      WRITE(16,1002)
C
2001  FORMAT(/)
      WRITE(16,2002)
2002  FORMAT(15X,'TWO DIMENSIONAL SPATIAL CORRELATION ANALYSIS')
      WRITE(16,2001)
      WRITE(16,2003) NCC(1),NCC(2)
2003  FORMAT(20X,'CHANNELS0',2(I2,1X))
      WRITE(16,2001)
      WRITE(16,2004)
2004  FORMAT(17X,'2-D SPATIAL CORRELATION MATRIX')
      WRITE(16,2001)
      DO 330 J=1,LAGY
      WRITE(16,FMT2) (R(I,J),I=1,LAGX)
330   CONTINUE
      WRITE(16,1005)
881   RETURN
      END
C
C
C.......................................................................
C
C         USAGE
C          CALL ACF (F,G,R,NSIZEX,NSIZEY,LAGX,LAGY)
C
C         DESCRIPTION OF PARAMETERS
C          F      - ARRAY CONTAINING THE AREA TO BE CORRELATED(1ST CHANNEL
C          G      - ARRAY CONTAINING THE AREA TO BE CORRELATED(2ND CHANNEL
C          R      - RESULTING SPATIAL CORRELATION MATRIX
C          NSIZEX - NUMBER OF COLUMNS IN THE PICTURE
C          NSIZEY - NUMBER OF LINES IN THE PICTURE
C          LAGX   - LAG IN PIXELS ALONG THE COLUMNS
C          LAGY   - LAG IN PIXELS ALONG THE LINES
C
C         REMARKS
C
C          NONE
C
C         SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C          NONE
C
C         METHOD
C
C          THE LAGGED PRODUCT SUM METHOD IS CARRIED OUT
```

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```
FILE. . .    CORELAT  FORTRAN  B1
C
C
C*************************************************************************
C
      SUBROUTINE ACF (F,G,R,NSIZEX,NSIZEY,LAGX,LAGY)
C
C*************************************************************************
C
C                  LOCAL VARIABLES DEFINITION
C
C*************************************************************************
C
      IMPLICIT INTEGER (A-Z)
      REAL*4 F(NSIZEX,NSIZEY),G(NSIZEX,NSIZEY),R(LAGX,LAGY),
     1        MEAN1,MEAN2,RDIM
      RDIM=NSIZEX*NSIZEY
C
C*************************************************************************
C
C                  SUM THE LAGGED PRODUCTS
C
C*************************************************************************
C
      DO 10 ETA=1,LAGY
      DO 10 TAU=1,LAGX
      DO 30 J   =1,NSIZEY
      DO 30 I   =1,NSIZEX
C
      IF(((I+TAU-1).GT.NSIZEX).OR.((J+ETA-1).GT.NSIZEY)) GO TO 25
C
C
      R(TAU,ETA)=R(TAU,ETA)+G(I+TAU-1,J+ETA-1)*F(I,J)
25    CONTINUE
30    CONTINUE
      R(TAU,ETA)=R(TAU,ETA)/RDIM
10    CONTINUE
C
      RETURN
      END
C
C       *****************************************************************
C
C     SUBROUTINE LSF
C
C     PURPOSE
C
C         TO FIT A LINEAR FUNCTION THROUGH A DATA SET,USING LEAST SQUARES
C
C      DESCRIPTION OF PARAMETERS
C
C         H  -MATRIX OF VALUES OF LINEAR FUNCTIONS
C         WGT-WEIGHTING MATRIX
C         P  -HTRANSPOSE*WGT
C         R  -P*H
C         S  -RINVERSE*P
C         C  -LEAST SQUARES COEFF.
C         F  -MATRIX OF DATA VALUES
C         FHT-ESTIMATE OF F
C         ERR-WEIGHTED F-FHAT
C         LAG-NO OF DATA VALUES
C         WR1-WORK VECTOR
C         WR2-WORK VECTOR
C
C
C     REMARKS
C
C         THE EQUATION OF THE LINEAR FIT IS OF THE FORM,C1+C2*X
C          THE FOLLOWING MATRICES ARE DIMENSIONED 2*LAG
C
C          H,P,S
C
C          THE FOLLOWING MATRICES ARE DIMENSIONED LAC IN THE MAIN PROG
C
C         WGT,WR1,WR2,ERROR,ERR,FN,FHAT
```

```
FILE. . .    CORELAT  FORTRAN  B1
C
C         THE FOLLOWING MATRICES HAVE CONSTANT DIMENSIONS
C
C         R(4),RMSE(1),C(2) ,
C
C      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C        TPRO,GMPRD,MINV
C
C     METHOD
C         GENERALIZED LEAST SQUARES TECHNIQUE IS IMPLEMENTED.FORMULA USE
C         (HTR*WGT*H)**(-1)*HTR*WGT*F
C         FOR FURTHER DETAILS SEE 'DICRETE PARAMETER ESTIMATION ',BY
C         J.M.MENDEL,1973
C
C
C      ******************************************************************
C
      SUBROUTINE LSF (H,WGT,P,S,F,FHAT,C,ERR,ERROR,WR1,WR2,LAG,RMSE,RMU)
C
C*************************************************************************
C
C                  LOCAL VARIABLES DEFINITION
C
C*************************************************************************
C
      IMPLICIT INTEGER (A-Z)
      REAL*4 H(1),WGT(1),P(1),S(1),F(1),FHAT(1),C(1),ERR(1),ERROR(1)
     1       ,WR1(1),WR2(1),RMSE(1),R(4),RMU,DET
C
80    FORMAT(//)
81    FORMAT(///)
      NPNTS=LAG
C
      DO 30 I=1,NPNTS
      H(I)=1
30    CONTINUE
C
      K1=NPNTS+1
      K2=2*NPNTS
      DO 40 I=K1,K2
      H(I)=K2-I
40    CONTINUE
C
C*************************************************************************
C*     INITIALIZE THE WEIGHTING MATRIX
C*************************************************************************
C
C
      RMU=0.4
C
      DO 28 I=1,NPNTS
      K=NPNTS-I
      WGT(I)=RMU**K
28    CONTINUE
C
C*************************************************************************
C
C     DETERMINE THE LEAST SQUARES FIT COEFFICIENTS
C
C*************************************************************************
C
C
      N=NPNTS
      M=2
      MSA=0
      MSB=2
      L=NPNTS
C
      CALL TPRD (H,WGT,P,N,M,MSA,MSB,L)
C
      N=2
      M=NPNTS
      L=2
C
```

```
FILE. . .     CORELAT  FORTRAN  B1
      CALL GMPRD (P,H,R,N,M,L)
C
      N=2
      CALL MINV (R,N,DET,WR1,WR2)
C
C
      N=2
      M=2
      L=NPNTS
      CALL GMPRD (R,P,S,N,M,L)
C
      N=2
      M=NPNTS
      L=1
      CALL GMPRD (S,F,C,N,M,L)
C
C**********************************************************************
C*    FIND THE LEAST SQUARE ESTIMATE                                  *
C**********************************************************************
C
      N=NPNTS
      M=2
      L=1
      CALL GMPRD (H,C,FHAT,N,M,L)
C
C
C**********************************************************************
C*    FIND THE WEIGHTED LEAST SQUARES ERROR
C**********************************************************************
C
      DO 100 I=1,NPNTS
      ERROR(I)=F(I)-FHAT(I)
100   CONTINUE
C
      N=NPNTS
      M=1
      MSA=0
      MSB=2
      L=NPNTS
      CALL TPRO (ERROR,WGT,ERR,N,M,MSA,MSB,L)
C
      N=1
      M=NPNTS
      L=1
      CALL GMPRD (ERR,ERROR,RMSE,N,M,L)
C
C
C
      RETURN
      END
```

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```
C*********************************************************************
C
C      SUPERVISOR FOR THE COMPUTATION OF THE SCANNER OUTPUT STATISTICS
C
C           WRITTEN 09/25/78   BIJAN G. MOBASSERI
C
C*********************************************************************
C
C
C     DESCRIPTION AND PURPOSE
C
C        THE SCANNER OUTPUT STATISTICS  PROGRAM IMPLEMENTS A LINEAR
C        TRANSFORMATION TO OBTAIN A SET OF STATISTICS AT THE OUTPUT
C        OF A MULTIBAND SCANNING RADIOMETER IN TERMS OF THE CORRESPONDIN
C        INPUT QUANTITIES. THIS TRANSFORMATION IS ACCOMPLISHED BY
C        THE COMPUTATION OF THE ' SCANNER CHARACTERISTIC FUNCTION '. AN
C        ANALYTICAL EXPRESSION, THE MAIN PARAMETERS OF WHICH ARE THE
C        IFOV SIZE AND INFORMATION ON DATA SPATIAL CORRELATION.
C
C     DESCRIPTION OF CONTROL CARDS
C
C     *SCANSTAT
C
C        THIS CARD SPECIFIES THE PARTICULAR PROCESSOR REQUESTED
C
C     CHANNELS
C        THE DESIRED SUBSET OF THE AVILABLE CHANNELS IS GIVEN HERE.
C        IT IS IMPORTANT TO REMEMBER THAT THE NUMBERS APPEARING
C        ON THIS CARD ARE THE ORDER OF THE SELECTED CHANNELS NOT
C        THIER ACTUAL NUMBER. FOR EXAMPLE IF THE AVAILABLE CHANNELS
C        ARE 8,9,12,14 AND CHANNELS 8,9 AND 14 ARE SELECTED, CHANNELS
C        CARD SHOULD READ 1,2,4
C
C     CLASSES
C
C        THIS CARD SPECIFIES THE NAME OF EACH CLASS. EACH NAME MUST BE
C        PLACED IN A FIELD 7 CHARACTER LONG FOLLOWED BY A BLANK. THE
C        CONTINUATION CARD, IF REQUIRED, MUST HAVE THE WORD 'CLASSES'
C        IN THE BEGINNING FOLLOWED BY THE REST OF THE NAMES.
C
C     IFOV
C
C        THIS CARD SPECIFIES THE SPATIAL RESOLUTION OF THE OUTPUT DATA
C        IN TERMS OF THE INPUT. BASICALLY IT IS THE NUMBER OF HIGH
C        RESOLUTION PIXELS WITHIN ONE IFOV OF THE SCANNER. E. G. FOR
C        MSS OPERATING ON A 6 METER DATA AND IFOV=5 ACCORDING TO THE
C        ABOVE CONVENTION THE ACTUAL SPATIAL RESOLUTION IS 30 METERS
C
C     APERTURE
C
C        THE CHOICES HERE ARE 'GAUSSIAN' OR 'RECTANGULAR'
C
C      SNR (OPTIONAL)
C
C        THIS CARD SIMULATES THE EFFECT OF RANDOM ADDICTIVE NOISE ON THE
C        POPULATIONS STATISTICS AT THE SCANNER OUTPUT.THE NOISE COVARIAN
C        MATRIX IS DIAGONAL WITH OFF DIAGONAL ELEMENTS EQUAL TO ZERO.
C        THE SNR IS DEFINED AS THE RATIO OF SIGNAL ENERGY (DIAGONAL ELEM
C        ENTS) TO NOISE ENERGY.SNR MUST BE GIVEN IN DECIBELS DEFINED
C        SNR = 10.*ALOG10(SIGNAL ENERGY/NOISE ENERGY). THIS SNR WILL
C        BE THE SAME IN ALL CHANNELS.
C
C     PUNCH (OPTIONAL)
C
C        IF PRESENT THE OUTPUT STATISTCS IS PUNCHED OUT. NOTE THAT SOME
C        BLANK CARDS ARE INCLUDED IN THE DECK FOR COMPATIBILITY REASONS
C
C     END
C
C        THIS ARD SIGNALS THE END OF CONTROL CARDS. DATA FOLLOWS IMMEDIA
C        TELY
C
C     INPUT DATA STRUCTURE
C
C        INPUT TO *SUS CONSISTS OF 3 SEPERATE DECKSO
C        1- LARSYS STATISTICS DECK WITH NO CHANGES   IT SHOULD
C           HOWEVER BE IN  ** CHARACTER FORMAT **
C        2- SPATIAL CORRELATION PARAMETERS ARE ENTERED VIA A NDIM X NDIM
```



```
C           MATRIX. THE (I,J) ELEMENT OF IT IS THE PIXEL-TO PIXEL
C           CORRELATION IN CHANNELS I AND J. THIS DECK CORRESPONDS TO
C           THE CROSS TRACK CORRELATION.
C
C        3- SAME AS 2 EXCEPT FOR ALONG TRACK DIRECTION.
C
C     REMARKS
C        THIS PROGRAM IS CURRENTLY CAPABLE OF PROCESSING UP TO 20 CLASSE
C        AND 8 SPECTRAL BANDS.THE EXECUTION TIME IS QUITE SHORT AND
C        EXTENSION TO A HIGHER NO OF CLASSES AND DIMENSIONS PRESENTS
C        NO PARTICULAR PROBLEM. THE STATISTICS DECK PRODUCED HERE DOES
C        NOT CARRY A SEQUENCE NUMBER IN THE 72-80 COLUMNS.
C
C      HOW TO RUN THE PROGRAM
C
C        THE SOURCE AND TEXT FILES ARE LOCATED ON THE DHDSK AND DHSYS
C        DISKS RESPECTIVELY. AUTHORIZED ID'S ARE AUTOMATICALLY LINKED
C        TO BOTH DISKS AT LOGIN TIME. OTHERWISE ANYONE CAN ACCESS THE
C        DISKS THRU THE COMMANDS 'GETDISK DHSYS' FOR TEXT AND
C        'GETDISK DHDSK' FOR THE SOURCE.
C        AFTER THE PROPER LINKS ARE ESTABLISHED, TYPE IN *SOS.
C
C     EXAMPLE OF THE CONTROL CARD SET UP
C
C        *SCANSTAT
C        CHANNELS 1,2,4
C        CLASSES BARESOI CORN    SOYBEAN WHEAT
C        IFOV 2
C        APERTURE GAUSSIAN
C        PUNCH
C        END
C
C*********************************************************************
C
      INTEGER*4 LIST(8)/'*SCA','CHAN','CLAS','IFOV','APER','SNR','PUNC',
     1'END '/,          IVEC(1),ICARD(20),NADR(720)
      INTEGER*4 BLANK/'    '/,FSTCRD/'LARS'/
      INTEGER*4 SDIM,SIGMAX,SIGMAY,HEAD(20),APERT(20)
      INTEGER*4 FMT1(5)/'(7X,',' 0','(F8.','2,1X',')'/,
     1          FMT2(6)/'(75X',',',' 0','(F8.','2,1X',')'/,
     2          FMT3(6)/'(1 ,',' 0','(F7.','1,1X',')'/,
     3          FMT4(6)/'(4CX',',',' 0','(F7.','1,1X',')'/,
     4          NL/'  0'/
      INTEGER*2 SPREAD(30),ICSEL(30),NC(30),NCC(30)
      REAL*4 COVIN(720),COVOUT(720),SUBCOV(720),SLPX(720),SLPY(720),
     1       SUBSLX(720),SUBSLY(720),PX(720),PY(720),SPX(720),
     2       SPY(720),C1(720),C2(720),C(720),RHO(720),
     3       MU(160),SUBMU(160)
C
      REAL*4 ICSET(90)
      LOGICAL*1 FLAG(8)/8*.FALSE./
      COMMON /SUSCOM/ NCLS,SDIM,NDIM,NSIZE,ISIZE,NTS,NSA,SIGMAX,SIGMAY,
     1                NTM,NSM,SNR
C
C
 1010 FORMAT(1X,'ERROR IN CTLWRD, EXECUTION TERMINATED.')
 1011 FORMAT(1X,'ERROR IN IVAL. EXECUTION TERMINATED.')
 1012 FORMAT(1X,'ERROR IN CHANEL. EXECUTION TERMINATED.')
 1013 FORMAT(1X,'MISSING CONTROL WORD. EXECUTION TERMINATED.')
 1015 FORMAT(1H1)
 1016 FORMAT(/)
 1017 FORMAT(35X,'S C A N N E R   O U T P U T   S T A T I S T I C S')
 1018 FORMAT(20X,'SCANNER OUTPUT SPECTRAL STATISTICS')
 1019 FORMAT(35X,'APERTUREO ',3A4)
 1020 FORMAT(20X,'APERTUREO ',3A4)
 1021 FORMAT(35X,'IFOV SIZEO',I2,' HIGH RESOLUTION PIXELS')
 1022 FORMAT(20X,'IFOV SIZEO',I2,' HIGH RESOLUTION PIXELS')
 1023 FORMAT(49X,'CLASS ',2A4)
 1024 FORMAT(25X,'CLASS ',2A4)
 1025 FORMAT(10X,'INPUT COVARIANCE MATRIX',45X,'OUTPUT COVARIANCE MATRIX
     1')
 1026 FORMAT(1X,'INPUT COVARIANCE MATRIX',10X,'OUTPUT COVARIANCE MATRIX'
     1)
 1027 FORMAT(///)
C
C*********************************************************************
C
C
C                       DECODE CONTROL CARDS
```

```
C
C*****************************************************************************
C
C
      DO 777 I=1,30
      ICSEL(I)=0
777   CONTINUE
C
      DO 778 I=1,90
778   ICSET(I)=-50000.0
C
100   CONTINUE
      LSZ=8
      IER=0
      INRD=5
C
      CALL CTLWRD (ICARD,ICOL,LIST,LSZ,ICODE,INRD,IER)
      IF(IER.NE.0) GO TO 1001
      GO TO (99,101,102,103,104,105,106,107),ICODE
99    FLAG(1)=.TRUE.
      GO TO 100
C
C     CHANNELS CARD
C
101   CALL CHANEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,900)
      FLAG(2)=.TRUE.
      NDIM=NCR
      GO TO 100
C
C     CLASS NAMES CARD
C
102   DO 10 I=1,20
10    HEAD(I)=ICARD(I)
      FLAG(3)=.TRUE.
      GO TO 100
C
C
C     IFOV SIZE SPECIFICATION
C
103   LSZ=1
      CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
      FLAG(4)=.TRUE.
      SIGMAX=IVEC(1)
      SIGMAY=SIGMAX
      GO TO 100
C
C     IFOV SHAPE SPECIFICATION
C
104   DO 30 I=1,20
30    APERT(I)=ICARD(I)
      FLAG(5)=.TRUE.
      GO TO 100
C
C     SIGNAL TO NOISE RATIO
C
105   LSZ=1
      CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
      SNR=IVEC(1)
      FLAG(6)=.TRUE.
      GO TO 100
C
C     TO PUNCH OR NOT TO PUNCH
C
106   FLAG(7)=.TRUE.
      GO TO 100
C
C     END CARD
C
107   LSZ=1
      FLAG(8)=.TRUE.
      GO TO 201
C
1002  WRITE(6,1011)
      WRITE(16,1011)
      GO TO 999
C
900   WRITE(6,1012)
      WRITE(16,1012)
```



```
      GO TO 999
201   CONTINUE
C
C     CHECK IF ALL CONTROL CARDS HAVE BEEN READ
C
      DO 250 I=1,5
      IF(.NOT.FLAG(I)) GO TC 321
      GO TO 250
321   WRITE(16,1013)
      WRITE( 6,1013)
      GO TO 999
250   CONTINUE
C
      IF(.NOT.FLAG(8)) GO TO 321
C
      GO TO 680
1001  WRITE(16,1010)
      WRITE( 6,1010)
      GO TO 999
680   CONTINUE
C
C
C
C
C*****************************************************************************
C
C     READ THE TOTAL NO OF CHANNELS AND CLASSES FROM THE STAT DECK
C
C*****************************************************************************
C
502   READ(5,501) ICRD
501   FORMAT(A4)
      ICRDSQ=ICRDSQ+1
C
      IF(ICRD.EQ.BLANK) GO TO 503
      GO TO 502
503   CONTINUE
C
      REWIND 5
602   READ(5,501) ICRD
      IF(ICRD.EQ.FSTCRD) GO TO 601
      GO TO 602
601   CONTINUE
C
      NUM=ICRDSQ-2
      DO 506 I=1,NUM
      READ(5,507)
507   FORMAT(18A4,I8)
506   CONTINUE
C
C
      READ(5,1014) NCLS,NFLD,SDIM
1014   FORMAT(I5,6X,I5,6X,I5)
C
C
      NUM2=SDIM+1
      DO 509 I=1,NUM2
      READ(5,507)
509   CONTINUE
C
      NSIZE=SDIM*(SDIM+1)/2
      ISIZE=NDIM*(NDIM+1)/2
C
      NTS=NCLS*ISIZE
      NTM=NCLS*NDIM
      NSM=NCLS*SDIM
      NSA=NCLS*NSIZE
C
C     FIND THE CHANNEL SET THAT IS NOT REQUESTED
C
      K=0
      DO 611 I=1,SDIM
      DO 612 J=1,NDIM
C
        IF(I.EQ.NCC(J)) GO TO 611
C
612   CONTINUE
C
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
      K=K+1
      NC(K)=I
C
611   CONTINUE
C
C
C*********************************************************************
C
C.    START COMPUTATION OF THE STATISTICS AT THE MSS OUTPUT
C*********************************************************************
C
      CALL SCANER (COVIN,COVOUT,SUBCOV,SLPX,SLPY,SUBSLX,SUBSLY,PX,PY,
     1             C1,C2,C,NC,NCC,RHO,NADR,FLAG,MU,SUBMU,APERT)
C
C
C*********************************************************************
C
C                       GENERATE THE OUTPUT
C
C*********************************************************************
C
      DO 842 JCLS=1,NCLS
C
        FMT1(2)=N1
        KS=2*JCLS
        MS=(JCLS-1)*ISIZE
C
        WRITE(6,1015)
        WRITE(6,1016)
        WRITE(16,1016)
        WRITE(6,1017)
        WRITE(16,1018)
        WRITE(6,1016)
        WRITE(16,1016)
        WRITE(6,1019) (APERT(I+2),I=1,3)
        WRITE(16,1020) (APERT(I+2),I=1,3)
        WRITE(6,1016)
        WRITE(16,1016)
        WRITE(6,1021) SIGMAX
        WRITE(16,1022) SIGMAX
        WRITE(6,1027)
        WRITE(15,1016)
        WRITE(6,1023) (HEAD(KS+I),I=1,2)
        WRITE(16,1024) (HEAD(KS+I),I=1,2)
        WRITE(16,1016)
C
        WRITE(6,1027)
        WRITE(6,1025)
        WRITE(16,1026)
C
        WRITE(6,1016)
        WRITE(16,1016)
C
      DO 841 KDIM=1,NDIM
C
        ISTRT=KDIM*(KDIM-1)/2+1
        ISTOP=KDIM*(KDIM+1)/2
C.
        FMT1(2)=FMT1(2)+1
        FMT2(3)=FMT1(2)
        FMT3(2)=FMT1(2)
        FMT4(3)=FMT2(3)
C
        WRITE(6,FMT1) (SUBCOV(I+MS),I=ISTRT,ISTOP)
        WRITE(6,FMT2) (COVOUT(J+MS),J=ISTRT,ISTOP)
        WRITE(16,FMT3) (SUBCOV(I+MS),I=ISTRT,ISTOP)
        WRITE(16,FMT4) (COVOUT(J+MS),J=ISTRT,ISTOP)
C
        WRITE(6,1016)
C
841   CONTINUE
C
842   CONTINUE
C
C
999   STOP
      END
```



```
C
C
C.....................................................................
C
C     SUBROUTINE SCANER
C
C     PURPOSE
C
C          TO COMPUTE THE SCANNER CHARACTERISTICS FUNCTION AND GENERATE
C          THE TRANSFORMED STATISTICS
C
C     DESCRIPTION OF PARAMETERS
C
C          APERT  - ARRAY CONTAINING THE IFOV SHAPE
C          C1     - ONE DIMENSIONAL SCANNER CHARACTERISTIC FUNCTION
C          C2     - ONE DIMENSIONAL SCANNER CHARACTERISTIC FUNCTION
C          C      - C1*C2
C          COVIN  - INPUT COVARIANCE MATRICES
C          COVOUT - OUTPUT COVARIANCE MATRICES
C          FLAG   - FLAG IN CONTROL CARDS
C          MU     - INPUT MEAN VECTORS
C          NADR   - ADDRESS OF THE MATRIX ELEMENTS TO BE DELETED
C          NC     - THE CHANNEL SET THAT IS NOT REQUESTED
C          NCC    - COMPLEMENT OF NC
C          RHO    - SPECTRAL CORRELATION MATRIX
C          PX     - AUX ARRAY
C          PY     - AUX ARRAY
C          SLPX   - SPATIAL CORRELATION PARAMETERS. CROSS TRACK
C          SLPY   - SPATIAL CORRELATION PARAMETERS. ALONG TRACK
C          SUBSLX - SUBSET OF SLPX
C          SUBSLY - SUBSET OF SLPY
C          SUBCOV - SUBSET OF THE INPUT SPECTRAL COV MATRICES
C          SUBMU  - SUBSET OF THE INPUT MEAN VECTORS
C
C
C      REMARKS
C
C          NONE
C
C      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C          ADRES
C      METHOD
C
C         THE REQUIRED SUBSET OF THE INPUT SPECTRAL AND SPATIAL CORRELA
C         MATRICES IS COMPUTED. THE SCANNER CHARACTERISTIC FUNCTION IS
C         CALCULATED BASED ON EITHER A GAUSSIAN OR RECTANGULAOR IFOV
C         AND SPECIFIED SIZE. THIS WEIGHTING FACTOR IS THEN APPLIED
C         TO THE INPUT STATISTICS AND THE RESULTING OUPUT IS PRINTED
C         AND PUNCHED (IF REQUESTED). THE PUNCHED STAT DECK IS COMPATIB
C         WITH *ACAP AND VARIOUS LARSYS PROCESSORS. UNITY FILTER
C         GAIN MAINTAINS THE EQUALITY OF INPUT AND OUPUT MEAN VECTORS
C
C
C.....................................................................
C
      SUBROUTINE SCANER (COVIN,COVCUT,SUBCOV,SLPX,SLPY,SUBSLX,SUBSLY,PX,
     1              PY,C1,C2,C,NC,NCC,RHO,NADR,FLAG,MU,SUBMU,APERT)
C
C
C*********************************************************************
C
C                LOCAL VARIABLES DEFINITION
C
C
C
      REAL*4 COVIN(NSA),MU(NSM),COVOUT(NTS),SUBCOV(NTS),SUBMU(NTM),SLPX(
     1       NSA),SLPY(NSA),SUBSLX(NTS),SUBSLY(NTS),PX(NTS),PY(NTS),
     2       C1(NTS),C2(NTS),C(NTS),RHO(NTS)
      INTEGER*4 FMT1(4)/'(2X,',' 0','E14.','7)'/
      INTEGER*4 FMT2(4)/'(2X,',' 0','E14.','7)'/
      INTEGER*4 N1/'  0'/,N3/'  0'/,N2/'  0'/
      INTEGER*4 SIGMAX,SIGMAY,SDIM,NADR(NSA),APERT(20)
      INTEGER*4 SHAPE/' REC'/
      INTEGER*2 NC(30),NCC(30)
      LOGICAL*1 FLAG(8)
```

```
C
C*************************************************************************
C
      COMMON /SOSCOM/ NCLS,SDIM,NDIM,NSIZE,ISIZE,NTS,NSA,SIGMAX,SIGMAY,
     1               NTM,NSM,SNR
C
C*************************************************************************
C
C                COMMON BLOCK VARIABLES DESCRIPTION
C
C     ISIZE - CORE SPACE FOR ONE SUBSET OF COVRIANCE MATRIX
C     NCLS  - NO OF CLASSES
C     NDIM  - NO OF REQUESTED CHANNELS
C     NTM   - CORE SPACE FOR A SUBSET OF MEAN VECTORS
C     NSA   - NCLS*NSIZE
C     NSIZE - CORE SPACE FOR ONE COV MATRIX
C     NSM   - CORE SPACE FOR THE ENTIRE INPUT MEAN VECTORS
C     NTS   - NCLS*ISIZE
C     SDIM  - NO OF AVAILABLE CHANNELS
C     SIGMAX- IFOV SPREAD. CROSS TRACK
C     SIGMAY- IFOV SPREAD. ALONG TRACK
C
C*************************************************************************
C
507   FORMAT(20A4)
508   FORMAT('LARSYS VERSION 3 STATISTICS FILE')
509   FORMAT('CLASS NAME')
510   FORMAT(I5,6X,I5,6X,I5)
511   FORMAT(10(F4.2,1X))
C
C*************************************************************************
C
C     READ IN THE INPUT SPECTRAL AND SPATIAL STATISTICS
C
C*************************************************************************
C
C
      N1=N1+SDIM
      N2=N2+5
      IF(NSIZE.LT.5) N2=N3+NSIZE
C
      FMT1(2)=N1
      FMT2(2)=N2
C
      READ(5,FMT1) MU
      READ(5,FMT2) COVIN
C
      READ(5,507)
C
      READ(5,511) SLPX
      READ(5,511) SLPY
C
C
      DO 521 I=1,NSA
      SLPX(I)=-ALOG(SLPX(I))
      SLPY(I)=-ALOG(SLPY(I))
521   CONTINUE
C
C*************************************************************************
C
C     SELECT THE REQUESTED SUBSET OF THE INPUT MATRICES
C
C*************************************************************************
C
C
      CALL ADRES (NADR,NC,SDIM,NDIM)
C
C
      K=0
      DO 603 JCLS=1,NCLS
C
         MDS=(JCLS-1)*NSIZE
C
      DO 602 I=1,NSIZE
      DO 601 J=1,NSIZE
C
         IF (I.EQ.NADR(J)) GO TO 602
C
```



```
601   CONTINUE
         K=K+1
         SUBCOV(K)=COVIN(MDS+I)
         SUBSLX(K)=SLPX(MDS+I)
         SUBSLY(K)=SLPY(MDS+I)
C
C
602   CONTINUE
603   CONTINUE
C
      K=0
      DO 803 JCLS=1,NCLS
C
         MDS=(JCLS-1)*SDIM
C
      DO 802 KDIM=1,SDIM
      DO 801 J=1,NDIM
C
         IF(KDIM.EQ.NCC(J)) K=K+1
         IF(KDIM.EQ.NCC(J)) SUBMU(K)=MU(MDS+NCC(J))
C
801   CONTINUE
802   CONTINUE
803   CONTINUE
C
C*************************************************************************
C
C     DETERMINE THE SCANNER CHARACTERISTIC FUNCTION
C
C*************************************************************************
C
C
C     GAUSSIAN SCANNER POINT SPREAD FUNCTION
C
      DO 100 I=1,NTS
C
         IF(APERT(3).EQ.SHAPE) GO TO 641
         PX(I)=(SIGMAX**2)*(SUBSLX(I)**2)
         PY(I)=(SIGMAY**2)*(SUBSLY(I)**2)
C
         SPX=SQRT(PX(I))
         SPY=SQRT(PY(I))
C
         C1(I)=2.*(0.5*ERFC(SPX/SQRT(2.)))*EXP(PX(I)/2.)
         C2(I)=2.*(0.5*ERFC(SPY/SQRT(2.)))*EXP(PY(I)/2.)
C
         GO TO 646
C
C      RECTANGUALR SCANNER POINT SPREAD FUNCTION
C
641     CONTINUE
C
        PX(I)=SIGMAX*SUBSLX(I)
        PY(I)=SIGMAY*SUBSLY(I)
C
        C1(I)=(2./PX(I))*(1-(1-EXP(-PX(I)))/PX(I))
        C2(I)=(2./PY(I))*(1-(1-EXP(-PY(I)))/PY(I))
C
        C(I)=C1(I)*C2(I)
C
646     CONTINUE
         C(I)=C1(I)*C2(I)
C
C*************************************************************************
C
C      FIND THE OUTPUT COVARIACE MATRICES
C
C*************************************************************************
C
C
         COVOUT(I)=SUBCOV(I)*C(I)
C
100   CONTINUE
C
      IF(.NOT.FLAG(6)) GO TO 456
C*************************************************************************
C
C     ADD  NOISE TO THE SCANNER OUPUT SIGNAL
```

```
FILE. . .      SCANSTAT FORTRAN  B1

C
C*********************************************************************
C
      DO 456 JCLS=1,NCLS
C
           MOS=(JCLS-1)*ISIZE
C
      DO 455 KDIM=1,NDIM
C
           NAD=KDIM*(KDIM+1)/2
           VARNSE=((1C)**(-SNR/10.))*COVOUT(MDS+NAD)
           COVOUT(MOS+NAD)=COVOUT(MDS+NAD)+VARNSE
C
455   CONTINUE
456   CONTINUE
C
      IF(.NOT.FLAG(7)) GO TO 544
C
C*********************************************************************
C
C
C      PUNCH OUT THE SCANNER OUTPUT STATISTICS
C
C
C*********************************************************************
C
      WRITE(7,508)
C
      WRITE(7,509)
      WRITE(7,523)
523   FORMAT(1X,' ')
      WRITE(7,510) NCLS,NFLO,NDIM
C
C
      NUM2=SDIM+1
      DO 533 I=1,NUM2
      WRITE(7,523)
533   CONTINUE
C
      FMT1(2)=N3+NDIM
      FMT2(2)=N2
C
      WRITE(7,FMT1) SUBMU
      WRITE(7,FMT2) CGVOUT
C
544   CONTINUE
      RETURN
      END
C
C
C
C.....................................................................
C
C     SUBROUTINE ADRES
C
C     PURPOSE
C
C           FIND THE DESIRED SUBSET OF A COVARIANCE MATRIX
C
C     DESCRIPTION OF PARAMETERS
C
C           NADR - ADCRESS ARRAY OF THE ENTRIES TO BE DELETED
C           NC   - ARRAY OF THE CHANNELS TO BE DELFTED
C           SDIM - TOTAL NO OF BANDS AVAILABLE
C           NDIM - DESIRED SUBSET OF SDIM
C
C     REMARKS
C
C           NONE
C
C     SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C
C           NONE
C
C     METHOD
C
C           USING A ONE DIMENSIONAL STORAGE MODE FOR EACH COVARIANCE
C           MATRIX IN AN UPPER TRIANGULAR FORM, THE PARAMETRIC ADDRESSE
C           FOR EACH ENTRY IS DERIVED. USING THE NC ARRAY, THE LOCATION
C           OF ALL THE ENTRIES THAT LIE IN THE UNWANTED LINES AND COLU
C           IS COMPUTED AND STORED IN NADR.
```

```
FILE. . .      SCANSTAT FORTRAN  B1

C
C.....................................................................
C
C
      SUBROUTINE ADRES (NADR,NC,SDIM,NDIM)
C
      INTEGER*4 NADR(1),PDIM,SDIM
      INTEGER*2 NC(1)
C
      PDIM=SDIM-NDIM
C
      IF(PDIM.LE.0) GO TO 100
C
      DO 20 J=1,PDIM
C
          N1=NC(J)
C
      DO 10 I=1,N1
C
           K=K+1
           NADR(K)=(NC(J)*(NC(J)-1)/2)+I
C
10    CONTINUE
20    CONTINUE
C
C
      DO 45 J=1,PDIM
C
          NFIN=SDIM-NC(J)+1
C
      DO 35 I=1,NFIN
C
          K=K+1
C
          NADR(K)=((NC(J)+I-1)*(NC(J)+I-2)/2)+NC(J)
C
35    CONTINUE
45    CONTINUE
C
100   RETURN
      END
```

FILE. . .   SPRDCT   FORTRAN P1

```
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C   SPRDCT
C
C   PURPOSE
C      EXOSYS DATA IN PUNCHED FORMAT IS READ AND STORED ON TAPE
C
C   REVISED
C      3 JULY, 1978
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      COMMON ID(100)
      INTEGER*4 IN(5),DAY(3),TIME(3),NOGRPS,GROUPS(5)
      INTEGER*4 ISAM,SINT,DATE(15),INFO(10,7)
      REAL*4 WBCOEF(2,5),DATA(2500),X(100)
      EQUIVALENCE(DATE(1),ID(1)),(INFO(1,1),ID(30)),(INFO(1,2),ID(40)),
     *(INFO(1,3),ID(50)),(INFO(1,4),ID(60)),(INFO(1,5),ID(70)),(INFO(1,
     *),ID(80)),(INFO(1,7),ID(90))
      REWIND 11
      NT = 100
C
C   ID INFORMATION
C
      WRITE(16,10)
10    FORMAT(5X,'TYPEIN DATE',/1X,15('/'))
      READ(15,15)DATE
15    FORMAT(15A1)
C
C   EXP. NO., NUMBER OFCLASSES, AND NUMBER OF DIMENSIONS
C
      WRITE(16,20)
20    FORMAT(5X,'TYPE EXP.NO.,CLASSES,AND DIMENSIONS',/' ///  //   ///'
      READ(15,25)ID(16),ID(17),ID(18)
25    FORMAT(I3,2X,I2,3X,I3)
C
C   EXPERIMENT INFORMATION
C
      NCLS = ID(17)
      DO 35 I=1,NCLS
      WRITE(16,30)I
30    FORMAT(5X,'TYPE CLASS INFO AND NO . SAMPLES FOR CLASS ',I1,/1X,10
     *'/'),4X,'///')
35    READ(15,40)(INFO(L,I),L=1,10),ID(20+I)
40    FORMAT(10A1,4X,I3)
      WRITE(11) ID
      CALL SPLBL
C
C   READ SAMPLE FUNCTIONS FROM EACH CLASS
C
      DO 500 K=1,NCLS
      WRITE(6,80)
80    FORMAT(5(/),10X,'SAMPLE FUNCTIONS')
      NF = ID(20+K)
      DO 200 JJ=1,NF
100   READ(5,1000)DAY,TIME,IN,NOGRPS,NOSAMS
1000  FORMAT(3A4,2X,3(I2,1X),/,45X,5A4,/,20X,I1,8X,I3,/)
      READ(5,1100)(DATA(I),I=1,NOSAMS)
1100  FORMAT(20A4)
      WRITE(6,150)IN
150   FORMAT(10X,20A4)
      DO 160 I=1,NT
160   X(I) = DATA(I+1)
      WRITE(11) X
200   CONTINUE
500   CONTINUE
      END FILE 11
      STOP
      END
```

FILE. . .   SPOPTM   FORTRAN P1

```
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C   SPOPTM
C
C
C   PURPOSE
C      TO DESIGN THE OPTIMUM SENSOR FOR A GIVEN DATA SET.
C
C   USAGE
C      CALLED FROM EXEC ROUTINE
C
C   DESCRIPTION OF PARAMETERS
C      AM  -      MEAN VECTOR OF DATA
C      COV -      COVARIANCE MATRIX OF DATA
C      PHI   -    MATRIX OF EIGENVECTORS.
C      GAM   -    EIGENVALUES
C      N     -    DIMENSIONALITY OF DATA SET
C      NCLS  -    NUMBER OF CLASSES
C
C   SUBROUTINE AND FUNCTION SUBPROGRAMS CALLED
C      EIGENP,EISORT,SPWGTI
C
C   METHOD
C      THE KARHUNEN-LOEVE EXPANSION WITH THE MAXIMUM LIKELIHOOD ESTIMATE
C      OF THE COVARIANCE MATRIX AS THE KERNEL IS USED TO REPRESENT THE
C      RANDOM PROCESS.
C
C   REVISED
C   14 AUG, 1978
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      COMMON ID(100)
      REAL*4 AM(100),Y(100),COV(5050),PHIP(100,100)
      REAL*8 VECI(100,100),EVI(100),INDIC(100),ACOV(100,100),GAM(100)
      REAL*8 PHI(100,100),SUM
      REAL*4 X(100),W(100)
      REWIND 2
      WRITE(16,5)
5     FORMAT(5X,'OPTIMUM SENSOR DESIGN')
C
C   READ ID INFORMATION
C
      READ(2) ID
      WRITE(6,8)
8     FORMAT(1H1,5(/))
      CALL SPLBL
      N = ID(18)
      NCT = N*(N+1)/2
C
C   COMPUTE COVARIANCE
C
      WRITE(16,10)
10    FORMAT(5X,'COVARIANCE BEING ESTIMATED   (SPOPTM)')
      NCLS = ID(17)
      NFT = 0
      DO 20 I=1,NCLS
20    NFT = NFT + ID(20+I)
      CON = DFLOAT(NFT)/DFLOAT(NFT-1)
      DO 30 I=1,N
30    AM(I) = 0.0
      DO 35 I=1,NCT
35    COV(I) = 0.0
      DO 65 IJ=1,NFT
      READ(2) X
      IN = 0
      DO 50 I=1,N
      AM(I) = AM(I) + X(I)/DFLOAT(NFT)
      DO 50 J=1,I
      IN = IN + 1
      COV(IN) = COV(IN) + X(I)*X(J)/DFLOAT(NFT-1)
50    CONTINUE
65    CONTINUE
      IN = 0
      DO 60 I=1,N
      DO 60 J=1,I
      IN = IN + 1
      COV(IN) = COV(IN) - CON*AM(I)*AM(J)
```

```
FILE. . .      SPOPTM   FORTRAN  P1

60     CONTINUE
C
C  WEIGHTING FUNCTION
C
       IN = 0
       DO 210 I=1,N
       DO 210 J=1,I
       IN = IN + 1
       ACOV(I,J) = COV(IN)
       ACOV(J,I) = COV(IN)
210    CONTINUE
C
       CALL SPWGT3(W)
C
       DO 250 I=1,N
       DO 250 J=1,N
250    ACOV(I,J) = ACOV(I,J)*W(J)
C
C  COMPUTE TRACE OF COVARIANCE
C
       SUM = 0.0
       DO 80 I=1,N
80     SUM = SUM + ACOV(I,I)
C
C  COMPUTE EIGENVALUES AND EIGENVECTORS
C
       WRITE(16,75)
75     FORMAT(5X,'EIGENVALUES AND EIGENVECTORS   (EIGENP)')
       NM = N
       T = 56.
       CALL EIGENP(N,NM,ACOV,T,GAM,EVI,PHI,VECI,INDIC,W)
       CALL EISORT(N,GAM,PHI)
C
C  PRINT EIGENVALUES AND MEAN-SQUARE ERROR
C
       CO = FLOAT(NFT)/(FLOAT(NFT-1)*FLOAT(NFT-1))
       CI = FLOAT(4*NFT-1)/(FLOAT(NFT-1)*FLOAT(NFT-1))
       WRITE(6,110)
110    FORMAT(5(/),5X,'N',5X,'EIGENVALUE',5X,'VAR(GAM)',5X,'VAR(PHI)',5X
      *'MEAN-SQUARE ERROR')
       DO 150 I=1,30
       VARP = 0.0
       DO 120 J=1,100
       IF(J .EQ. I) GO TO 115
       VARP = VARP + CO*GAM(I)*GAM(J)/(GAM(I) - GAM(J))**2
115    CONTINUE
120    CONTINUE
       VARG = C1*GAM(I)*GAM(I)
       SUM = SUM - GAM(I)
       WRITE(6,145) I,GAM(I),VARG,VARP,SUM
145    FORMAT(4X,I2,4X,F10.4,4X,F10.4,2X,F10.4,2X,F14.6)
150    CONTINUE
       DO 155 J=1,N
       DO 155 I=1,N
155    PHIP(I,J) = PHI(I,J)
       DO 180 J=1,20
       WRITE(7,160)(PHIP(I,J),I=1,N)
160    FORMAT(20A4)
180    CONTINUE
       STOP
       END
```

```
FILE. . .      SPWGT3   FORTRAN  P1
C
C
C  WEIGHTING FUNCTION NUMBER 3
C
C
       SUBROUTINE SPWGT3(W)
       REAL*4 W(100)
C
       WRITE(6,15)
15     FORMAT(//5X,'WEIGHTING FUNCTION NUMBER 3'//)
C
       DO 20 I=1,100
       W(I) = 1.0
20     CONTINUE
       DO 30 I=48,53
30     W(I) = 0.0
       DO 40 I=72,76
40     W(I) = 0.0
       W(47) = 0.5
       W(54) = 0.5
       W(71) = 0.5
       W(77) = 0.5
       RETURN
       END
```

```
FILE. . .      SPTES      FORTRAN  P1
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     SPTES TRANSFORMS THE DATA USING THE OPTIMUM SET OF BASIS
C     VECTORS, COMPUTES THE MEAN-SQUARE ERROR, AND COMPUTES THE
C     STATISTICS FOR EACH CLASS.
C
C     6 FEBRUARY, 1978
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMMON ID(100)
      REAL*4 P(10),PHI(100,10),X(100),Y(100),Z(100)
      REAL*4 AM(100),AVE(20,10),COV(210,10)
C
C     SELECT NUMBER OF TERMS
C
      WRITE(16,10)
10    FORMAT(5X,'NUMBER OF TERMS ')
      READ(15,15)NTERM
15    FORMAT(I2)
      REWIND 2
      READ(2) ID
      NCLS = ID(17)
      N = ID(18)
      NCT = NTERM*(NTERM + 1)/2
      NFT = 0
      DO 20 I=1,NCLS
20    NFT = NFT + ID(20+I)
      DO 25 I=1,NCLS
      P(I) = 1./FLOAT(NCLS)
25    CONTINUE
      WRITE(7,28)NCLS,NTERM
28    FORMAT(I2,3X,I2)
      WRITE(7,30)(P(I),I=1,NCLS)
30    FORMAT(10F6.4)
C
C     COMPUTE MEAN FUNCTION
C
      DO 300 I=1,N
300   AM(I) = 0.0
      DO 320 K=1,NFT
      READ(2) X
      DO 320 I=1,N
      AM(I) = AM(I) + X(I)/FLOAT(NFT)
320   CONTINUE
      REWIND 2
      READ(2) ID
C
C     READ EIGENVECTORS
C
      DO 40 J=1,NTERM
      READ(5,35)(PHI(I,J),I=1,N)
35    FORMAT(20A4)
40    CONTINUE
C
C
C     LOOP ON THE SAMPLE FUNCTIONS IN THE DATA SET
C
      AVESQ = 0.0
      DO 200 ICLS=1,NCLS
      DO 50 I=1,NTERM
50    AVE(I,ICLS) = 0.0
      DO 55 I=1,NCT
55    COV(I,ICLS) = 0.0
C
C
      NF = ID(20+ICLS)
      CON = FLOAT(NF)/FLOAT(NF-1)
      DO 150 ISAM=1,NF
C
C
C     READ SAMPLE POINTS FROM FUNCTION
C
      READ(2) X
C
C
C     TRANSFORM DATA USING BASIS FUNCTIONS
C
      DO 70 J=1,NTERM
      Y(J) = 0.0
      DO 70 I=1,N
      Y(J) = Y(J) + PHI(I,J)*(X(I) - AM(I))
```

```
FILE. . .      SPTES      FORTRAN  P1
70    CONTINUE
C
C     COMPUTE SQURED ERROR
C
      DO 80 I=1,N
80    Z(I) = 0.0
      DO 85 J=1,NTERM
      DO 85 I=1,N
      Z(I) = Z(I) + PHI(I,J)*Y(J)
85    CONTINUE
      DO 88 I=1,N
88    Z(I) = Z(I) + AM(I)
      XSQ = 0.0
      ZSQ = 0.0
      XZ = 0.0
      TSQ = 0.0
      DO 90 I=1,N
      XSQ = XSQ + X(I)*X(I)
      ZSQ = ZSQ + Z(I)*Z(I)
      XZ = XZ + 2.0*X(I)*Z(I)
90    CONTINUE
      ESQ = (XSQ - XZ + ZSQ)/FLOAT(N)
      AVESQ = AVESQ + ESQ
C
C     COMPUTE STATISTICS
C
      DO 100 I=1,NTERM
      AVE(I,ICLS) = AVE(I,ICLS) + Y(I)/FLOAT(NF)
100   CONTINUE
      IN = 0
      DO 110 J=1,NTERM
      DO 110 I=1,J
      IN = IN + 1
      COV(IN,ICLS) = COV(IN,ICLS) + Y(I)*Y(J)/FLOAT(NF-1)
110   CONTINUE
150   CONTINUE
C
C     PRINT STATISTICS
C
      IN = 0
      DO 160 J=1,NTERM
      DO 160 I=1,J
      IN = IN + 1
      COV(IN,ICLS) = COV(IN,ICLS) - CON*AVE(I,ICLS)*AVE(J,ICLS)
160   CONTINUE
      WRITE(6,165)ICLS
165   FORMAT(5(/),10X,'STATISTICS FOR CLASS',I4)
      CALL MCOVP(NTERM,AVE(1,ICLS),COV(1,ICLS))
      WRITE(7,170)(AVE(I,ICLS),I=1,NTERM)
170   FORMAT(20A4)
      WRITE(7,175)(COV(I,ICLS),I=1,NCT)
175   FORMAT(20A4)
C
C
200   CONTINUE
      AVESQ = AVESQ/FLOAT(NFT)
      WRITE(6,210)AVESQ
210   FORMAT(///10X,'MEAN-SQUARE ERROR = ',E10.4)
      STOP
      END
```

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