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ITERATIVE TECHNIQUES TO ESTIMATE SIGNATURE VECTORS FOR MIXTURE PROCESSING OF MULTISPECTRAL DATA*

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ABSTRACT

Mixture processing of remotely sensed multispectral scanner data involves estimating the percent coverage of individual crops or species contained within the instantaneous field of view of the scanner. In recent years, various mixture processing algorithms have been proposed to solve the so-called "mixture problem". All of the proposed algorithms require, as inputs, the spectral signatures of the various species observed. Often it is extremely difficult to obtain the required spectral signatures of individual species.

In this paper, two methods for obtaining the required spectral signatures for a particular mixture model are considered. For the model considered, the spectral signatures become signature vectors. The first method is based upon determination of the signature vectors such that a measure of the inconsistency between the mixture model and the observed data is minimized. The second method is based upon determination of the signature vectors such that the estimated mean percent coverage of individual species match apriori or ground truth estimates. The two methods proposed are applied to actual multispectral data in order to verify the concepts presented.

1. INTRODUCTION

When a multispectral sensor observes a ground resolution element (GRE) containing several separate objects or species, the reflected radiation sensed will be composed of the sum of the reflected radiation of each separate species. In order to estimate the proportions of the GRE occupied by each spearate species, it is first necessary to construct a model which accurately relates the observed data vector to the proportions of the species contained within the GRE. Two such "mixture models" have recently appeared in the literature (Horwitz, et al, 1971, Detchmendy and Pace, 1972) and each has been applied, with varying degrees of success, in the estimation of proportions of species from multispectral data.

Although these mixture models were arrived at through different lines of reasoning, they are quite similar in several respects, and under a certain set of assumptions they are identical. Both models require the "spectral signatures" of pure species as inputs in order to process data from fields containing various mixtures of the pure species. The spectral signature of a pure species is generally characterized by the mean vector and covariance matrix of multispectral data from a sample field containing only the pure species.

There are some very serious problems associated with obtaining the spectral signatures of pure species from sensors mounted in either aircraft or spacecraft. These problems are due, in part, to the fact that the sensor would rarely, if ever, observe a pure species due to the size of the GRE's.

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This is especially true at spacecraft altitudes. In addition, pure species are often mistakenly identified. For example, a sensor observing a corn field may not sense only a corn species but, depending upon the spatial characteristics of the field, would observe a corn species and a soil species in various proportions. If mixture processing of multispectral data is to become a realistic part of operational ground data processing systems, it is necessary that practical, efficient methods be devised to obtain the spectral signatures required.

The purpose of this paper is to present two iterative techniques for obtaining the spectral signatures required by the mixture model proposed by Detchmendy and Pace (1972) and to present the results obtained by applying the iterative techniques and mixture model to actual agricultural data of selected test sites from the C1 flight line of test area C in Tippecanoe County, Indiana (Fu, et al, 1969).

2. MIXTURE MODELS

The mixture models proposed by Horwitz, et al (1971) and Detchmendy and Pace (1972), although resulting from different formulations of the mixture problem, have certain common features. It is felt that a brief examination and comparison of the equations constituting the two mixture models is appropriate in order to establish a common point of reference for persons familiar with one or the other model. The model proposed by Horwitz, et al, (1971) will be referred to herein as the "Maximum Likelihood Mixture Model" (MLMM), and the model proposed by Detchmendy and Pace (1972) will be referred to as the "Least Squares Mixture Model" (LSMM).

THE LEAST SQUARES MIXTURE MODEL

Under certain simplifying assumptions (Detchmendy and Pace, 1972) the model for the reflected radiation from a GRE as observed by a m-channel multispectral sensor may be represented as

$$y = \sum_{i=1}^{n} a_i s_i + \varepsilon$$

where

y = a m x 1 column vector of observed data

s_i = a m x 1 column vector defined as the basis vector of the ith pure species

a_i = the fractional part of the GRE occupied by the ith pure species

ε = a m x 1 column vector of noise including observation noise

n = the number of separate species existing in the GRE's to be processed

Since the areas, a_i, are defined as fractions of the GRE, they are constrained as follows

$$\sum_{i=1}^{n} a_i = 1$$

and

$$a_i \geq 0 \qquad \text{for } 1 \leq i \leq n$$

The basis vectors, s_i, are considered to be the spectral signatures of the pure species making up the mixture. The key assumption in this model is that the major variability in the observed data is due to variations in the values of a_i from GRE to GRE. The model explicitly assumes that the basis (or signature) vectors corresponding to the pure species are constant. It is noted (Pace and Detchmendy, 1973) that while this assumption may seem drastic, it is sufficient to explain the variations in the data.

Given the observation vector, y, and the basis vectors, $s_i (i=1,\ldots,n)$, the problem becomes one of determining the corresponding fractional area vector, a (where $a = (a_1, a_2, \ldots, a_n)$) subject to the model and constraints. Assuming that the number of channels, m, is greater than the number of species, n, the expression for y is overdetermined in that there are more equations than there are unknowns. In this case, the "best" solution is defined as the "least squares minimum noise solution". This solution is obtained by determining the a-vector such that the square of the noise, ε, is minimized.

Thus, it is required to determine the a-vector such that $\phi(a)$ is minimized, where

$$\phi(a) = \left(y - \sum_{i=1}^{n} a_i s_i\right)^T W \left(y - \sum_{i=1}^{n} a_i s_i\right)$$

and W is an arbitrary m x m weight matrix to account for observation noise. However, this minimization is subject to the required constraints.

Various techniques may be used to solve this constrained minimization problem. One of the fastest methods computationally is proposed by Pace and Detchmendy (1973) and was used in the present analysis.

THE MAXIMUM LIKELIHOOD MIXTURE MODEL

The formulation leading to this model is described fully by Horwitz, et al (1971) where it is assumed that the signature of an ith species ($1 \leq i \leq n$) is represented by an m-dimensional Gaussian distribution with mean μ_i and covariance Λ_i. If the proportion of a species in the GRE is a_i and $a = (a_1, \ldots, a_n)$, then the signature of this combination of species will have a mean μ_a and a covariance matrix Λ_a. The expressions for μ_a and Λ_a are assumed to be

$$\mu_a = \sum_{i=1}^{n} a_i \mu_i$$

and

$$\Lambda_a = \sum_{i=1}^{n} a_i \Lambda_i$$

If the observed data for a GRE is represented by the m-vector y, then the maximum likelihood procedure leads to choosing a fractional area vector, a, such that a likelihood function $\theta(a)$ is minimized, where

$$\theta(a) = \ln |\Lambda_a| + (y-\mu_a)^T \Lambda_a^{-1} (y-\mu_a)$$

subject to the constraints that the a-vector be a proportion vector, i.e.,

$$\sum_{i=1}^{n} a_i = 1$$

and

$$a_i \geq 0 \qquad 1 \leq i \leq n$$

Here $|\Lambda_a|$ denotes the determinant of the matrix Λ_a, and Λ_a^{-1} denotes the inverse of Λ_a. It is noted that, except for a constant term, $\theta(a)$ is the natural log of the Gaussian density function with mean μ_a and covariance Λ_a, evaluated at point y.

It is pointed out by Horwitz, et al (1971) that this constrained minimization problem is extremely difficult to solve due to the fact that $\theta(a)$ is not convex and its derivatives are impractical to compute. Two special cases which involve certain simplifying assumptions are considered in order to reduce the expression for $\theta(a)$ to a convex function.

In one of the cases considered, it is assumed that the covariance matrices of each of the n species in the GRE are equal (i.e., $\Lambda_i = \Lambda$). Under this assumption the expression for Λ_a becomes

$$\Lambda_a = \Lambda \sum_{i=1}^{n} a_i$$

or, in view of the constraint on a_i,

$$\Lambda_a = \Lambda$$

In this case, Λ_a is no longer a function of the a-vector. Thus, the first term in the expression for $\theta(a)$, being a constant, contributes nothing to the minimization of $\theta(a)$ and may be dropped out. The problem then becomes one of determining the a-vector such that $\psi(a)$ is a minimum where

$$\psi(a) = (y - \sum_{i=1}^{n} a_i \mu_i)^T \Lambda^{-1} (y - \sum_{i=1}^{n} a_i \mu_i)$$

subject to the required constraints. It is suggested by Horwitz, et al (1971) that Λ be computed as the average of the covariance matrices for the n species, i.e.,

$$\Lambda = \frac{1}{n} \sum_{i=1}^{n} \Lambda_i$$

since for real data the Λ_i matrices will not be equal in general.

A comparison of the expressions for $\phi(a)$ in the LSMM and $\psi(a)$ for the MLMM shows that if the basis vectors for the species are taken to be the mean vectors for the species (i.e., $s_i = \mu_i$), and if the weight matrix is taken to be the inverse of the average covariance matrix (i.e., $W = \Lambda^{-1}$), the two models become mathematically identical.

The LSMM was used as the mixture model in the present analysis.

3. MIXTURE GEOMETRY

The mixture model has associated with it, certain geometric properties which may be represented pictorially in simple cases. In order to illustrate the relationships between the basis vectors, the fractional area vector, and the observed data vector, consider the case of three channels and three species. In this case, the equations defining the model become

$$y = a_1 s_1 + a_2 s_2 + a_3 s_3 + \varepsilon$$

$$a_1 + a_2 + a_3 = 1$$

$$a_1 \geq 0,\ a_2 \geq 0,\ a_3 \geq 0$$

where y is a 3 x 1 column vector.

Figure 1 shows the basis vectors in the three-channel space. From Figure 1, it is noted that the tips of the basis vectors form a triangle which defines a plane. Observation vectors, y, whose tips lie above or below the s-plane will violate the constraint $a_1 + a_2 + a_3 = 1$. On the other hand, y-vectors whose tips lie in the s-plane, but do not lie inside the triangle will violate $a_i \geq 0$ for some i (where $1 \leq i \leq 3$). These types of geometric properties are extendable to higher dimensions.

For the three-channel, three-species example, the solution to the constrained minimization problem is illustrated conceptually in Figure 2. First, the observed vector y, is projected into the s-plane to form the vector, y_p. The a-vector associated with y_p will satisfy the constraint $a_1 + a_2 + a_3 = 1$. Next, the y_p vector is projected to the nearest exterior side of the triangle in the s-plane to form y_c. The a-vector associated with y_c will satisfy both $a_1 + a_2 + a_3 = 1$ and $a_i \geq 0$, $1 \leq i \leq 3$.

4. METHODS OF DETERMINING BASIS VECTORS

In considering the methods of determining basis vectors proposed herein, it is convenient to think of the three-channel, three-species case previously discussed. In the ideal situation, where there is no observation noise, the tips of all of the observation vectors, y, would lie in the plane formed by the tips of the three basis vectors, s_i. In addition, the tips of the y-vectors would lie inside the triangular region formed by the tips of the three basis vectors.

The distribution of points in this region would depend upon the distribution of a_i's in the data set. It is important to note that the "thickness" of the region occupied by the data in the direction normal to the plane formed by the s_i-vectors would be zero in this ideal case.

With real data, these ideal conditions do not exist. The region occupied by the data is not a plane. However, with the assumption that the major variability of the data is due to variations in the a_i's, it would be expected that, for the case under consideration, the covariance matrix of the data would have two "significant" eigenvalues. The eigenvectors corresponding to these eigenvalues define the plane in which the major variability in the data occurs. Experiments with actual data from the C1 flight line (Pace and Detchmendy, 1973) support this conjecture.

This same line of reasoning can be extended to higher dimensions. The important point is that the structure of the covariance matrix of the data can be used to define the subspace in which the tips of the basis vectors should lie. This subspace is defined by the mean vector of the data and the eigenvectors corresponding to the significant eigenvalues. An arbitrary decision must be made about the number of significant eigenvalues to be explained by the mixture theory.

When the data vectors are projected into the subspace defined by the basis vectors, they may not, in general, lie inside the figure formed by the tips of the basis vectors. A necessary requirement, in order that the mixture model is consistent with the observed data, is that the figure formed by the tips of the basis vectors should enclose the data vectors when they are projected into the subspace defined by the basis vectors. While this requirement is necessary, it is, of course, not sufficient to assure that the arbitrary basis vectors are indeed the basis vectors corresponding to the pure species making up the mixture. However, if this basic requirement is not met, the model is certainly inconsistent with the observed data.

A measure of the inconsistency between the model and the data is given by Φ, where

$$\Phi = \sum_{j=1}^{q} \phi_j(a)$$

The function $\phi_j(a)$ is $\phi(a)$ (defined earlier) for the jth observed data vector, y, and q is the total number of observed data vectors.

Since the solution to the constrained minimization problem minimizes each $\phi(a)$, and since each observation vector, y_j, is independent of all other observations, it follows that Φ is also a minimum with respect to the a-vectors.

Because it is known that a set of arbitrary s_i-vectors forming the required subspace are not the true basis vectors of the mixture observed, it is natural to pose the following question: "If the function Φ can be minimized with respect to all of the a-vectors for a given set of s_i-vectors, could a smaller value of Φ be obtained with a different set of s_i-vectors?" The answer to this question is, of course, yes. This is obvious from Figure 2, where it is noted that the vector Δy (where $\Delta y = y - y_c$) may be written as

$$\Delta y = \Delta y_1 + \Delta y_2$$

The vector Δy_1 is the component orthogonal to the s-plane (i.e., the required subspace) with respect to the metric W, and the vector Δy_2 is parallel to the s-plane with respect to W. For any observation vector, y, the function $\phi(a)$ thus becomes

$$\phi(a) = \phi_1(a) + \phi_2(a)$$

where

$$\phi_1(a) = \Delta y_1^T \, W \, \Delta y_1$$

and

$$\phi_2(a) = \Delta y_2^T \, W \, \Delta y_2$$

Since the s_i-vectors must be constrained to remain in the required subspace, $\phi_1(a)$, the part of $\phi(a)$ due to the out-of-plane component, Δy_1, cannot be reduced by varying s_i. However, the value of $\phi_2(a)$ can be driven to zero. In the simple example cited previously (see Figure 2), $\phi_2(a)$ can be driven to zero by simply moving s_1 and/or s_3 such that the line connecting them passes through the tip of the vector y_p.

ESTIMATION OF BASIS VECTORS TO ACHIEVE CONSISTENCY BETWEEN MIXTURE MODEL AND OBSERVED DATA

The technique used to minimize the function Φ with respect to the s_i vectors is iterative in nature and may be summarized by the following equations. The equation relating an observed data vector to the basis vectors and fractional areas may be written as

$$y = [S] \, a + \varepsilon$$

where

S - a m x n matrix whose columns are the basis vectors s_i

a - the n x 1 column vector of fractional areas a_i

Consider an observed data vector, y, which when projected such that its tip lies in the basis vector subspace, becomes y_p (see Figure 2). The mixture equation for y_p becomes

$$y_p = [S] \, a_p + \varepsilon_p$$

where ε_p is the corresponding projected noise vector, and a_p is the vector of fractional areas associated with y_p. The vector a_p will satisfy the constraint

$$\sum_{i=1}^{n} (a_p)_i = 1$$

However, suppose that a_p does not satisfy $(a_p)_i \geq 0$, $1 \leq i \leq n$. A change in the S-matrix is sought such that

$$y_p = [S + \Delta S] \, a_c + \varepsilon_p$$

where a_c is the vector of fractional areas associated with y_c (i.e., the data vector enclosed by the basis vectors). The vector a_c will satisfy both

$$\sum_{i=1}^{n} (a_c)_i = 1$$

and

$$(a_c)_i \geq 0 \qquad 1 \leq i \leq n.$$

The two expressions for y_p may be equated to form

$$[\Delta S] \, a_c = [S] \, \Delta a$$

where $\Delta a = a_p - a_c$. The expression involving $[\Delta S]$ corresponds to a single observation vector.

Out of a total of q observed data vectors, suppose that some number, r (where $r \leq q$) fail to meet the constraint $a_i \geq 0$, $1 \leq i \leq n$ after they have been projected into the required subspace. Then r of the q projected data vectors fall outside the region bounded by the tips of the s_i-vectors. For these r data vectors, the expression involving $[\Delta S]$ becomes

$$[\Delta S] \, [A_c] = [S] \, [\Delta A]$$

where

$[\Delta A]$ = a n x r matrix whose columns contain r vectors Δa (where $\Delta a = a_p - a_c$)

$[A_c]$ = a n x r matrix whole columns contain r vectors a_c

r = the number of data vectors, y, that fail to meet the constraint $a_i \geq 0$, $1 \leq i \leq n$ after being projected into the required subspace

The solution for $[\Delta S]$ may be written as

$$[\Delta S] = [S]\ [\Delta A]\ [A_c]^{\dagger}$$

where $[A_c]^{\dagger}$ denotes the "generalized inverse" (Deutsch, 1965) of the matrix $[A_c]$. For the case under consideration, $[A_c]^{\dagger}$ may be written as follows

- For $r \geq n$

$$A_c^{\dagger} = A_c^T\ (A_c A_c^T)^{-1}$$

- For $r \leq n$

$$A_c^{\dagger} = (A_c^T A_c)^{-1}\ A_c^T$$

Prior to being applied, the computed changes to the basis vectors, ΔS, are constrained to lie in certain acceptable regions of the required subspace (see Figure 3). This is required in order to be sure that no change in s_i causes data already within the figure formed by the s_i-vectors to be placed outside the figure on the next iteration. Figure 3 shows an example of a change in s_1 being outside an acceptable region. In this case, a value $(\Delta s_1)_c$ would be computed as the projection of Δs_1 on to the vector $(s_1 - s_3)$.

The new S-matrix may be formed as

$$[S] = [S + \Delta S_c]$$

where $[\Delta S_c]$ is the matrix of constrained changes in the basis vectors. Repeated entries into the mixture processing algorithm using the new S-matrix will lower the value of Φ to its minimum value.

The iterative technique may be terminated when

$$R \leq TOL$$

where R is the relative change in Φ, i.e.,

$$R = 1.0 - \frac{(\Phi)\ \text{current}}{(\Phi)\ \text{previous}}$$

and TOL is a preselected tolerance.

While Φ does have a minimum with respect to the s_i vectors, the minimum is not unique since many different sets of basis vectors can be used to surround the data. However, the method for varying the s_i-vectors proposed will yield a minimum movement in the initial s_i-vectors required to just surround the data.

ESTIMATION OF BASIS VECTORS TO ACHIEVE GROUND-TRUTH ESTIMATES OF MEAN FRACTIONAL AREAS

Ground-truth estimates for areas of particular test sites covered by individual species are often known apriori. This is true for certain test sites in the C1 flight line. The average, or mean, value of the fractional areas for the test site, as computed by the mixture processing algorithm, should match the ground-truth estimates of the areas covered by individual species in the test site.

The mean vector of the fractional areas from the mixture processing algorithm, $\bar{a}$, and the ground-truth mean vector of the fractional areas for the particular field processed, $\hat{a}$, may be written as

$$\bar{a} = \begin{bmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_n \end{bmatrix} \qquad \text{and} \qquad \hat{a} = \begin{bmatrix} \hat{a}_1 \\ \vdots \\ \hat{a}_n \end{bmatrix}$$

If the assumed matrix of basis vectors, [S], does not contain the true spectral signatures of the species observed, then it is expected that $\bar{a} \neq \hat{a}$. The desired agreement between $\bar{a}$ and $\hat{a}$ can be obtained by an iterative technique for moving the basis vectors from a first guess.

The expression relating the mean observed data vector, $\bar{y}$, to the assumed matrix of basis vectors [S] and the mean fractional area vector, $\bar{a}$, may be written as

$$\bar{y} = [S]\,\bar{a} + \bar{\varepsilon}$$

where $\bar{\varepsilon}$ is the mean of the noise vectors. However, in order to fit the ground-truth estimates, $\hat{a}$, a change in the S-matrix is sought such that

$$\bar{y} = [S + \Delta S]\,\hat{a} + \bar{\varepsilon}$$

where [ΔS] represents the required change in the S-matrix.

Equating the expressions for $\bar{y}$ yields

$$[\Delta S]\,\hat{a} = \bar{q}$$

where the m x 1 column vector $\bar{q}$ is given as

$$\bar{q} = [S]\,(\bar{a} - \hat{a})$$

Considering the elements of the ΔS-matrix as unknowns, it is noted that there are m equations and m x n unknowns. Thus, the system is "under determined" and has an infinite number of solutions.

Because the system is under determined, the natural tendency is to seek a "minimum norm" solution such that the norms of the row vectors of the ΔS-matrix are minimized. This solution is given by

$$[\Delta S] = \bar{q}\,\hat{a}^{\dagger}$$

where

$$\hat{a}^{\dagger} = \frac{\hat{a}^T}{\hat{a}^T \hat{a}}$$

The changes in the individual basis vectors for this solution may be written as

$$\Delta s_i = \frac{\hat{a}_i}{\hat{a}^T \hat{a}}\,\bar{q} \qquad 1 \leq i \leq n$$

It is noted that this solution yields values of Δs_i which are all in the same direction, $\bar{q}$. This solution is unacceptable because changing the s_i-vectors in the same direction may cause data already within the figure formed by the s_i-vectors to be moved outside the figure on the next iteration. This causes a degradation in the consistency between the mixture model and the observed data as noted previously.

In seeking other solutions to the system for [ΔS], it was found that it was necessary to change only one basis vector on any iteration to achieve the desired agreement between $\bar{a}$ and $\hat{a}$. The one vector solutions to the system are given by

Solution 1: $\Delta s_1 = \frac{1}{\hat{a}_1} \bar{q}, \quad \Delta s_2 = 0, \ldots, \Delta s_n = 0$

Solution 2: $\Delta s_1 = 0, \quad \Delta s_2 = \frac{1}{\hat{a}_2} \bar{q}, \quad \Delta s_3 = 0, \ldots, \Delta s_n = 0$

$\vdots$

Solution n: $\Delta s_1 = 0, \ldots, \Delta s_{n-1} = 0, \quad \Delta s_n = \frac{1}{\hat{a}_n} \bar{q}$

If any one of the one vector solutions lies within an acceptable region in the required subspace (see Figure 3) then it is chosen. If no solution lies in an acceptable region, then the solution lying nearest to an acceptable region is chosen and is projected (as in the first iterative scheme) into an acceptable region before it is applied.

Once an acceptable value of $[\Delta S_c]$ has been determined, the value of the S-matrix for the next iteration is computed as

$$[S] = [S + \Delta S_c]$$

Iteration on the S-matrix is terminated when $\bar{a}$ is sufficiently close to $\hat{a}$. This is determined by testing two factors, R_1 and R_2 where

$$R_1 = \frac{\bar{a}^T \bar{a}}{\hat{a}^T \hat{a}}$$

and

$$R_2 = \frac{\bar{a}^T \hat{a}}{|\bar{a}||\hat{a}|}$$

Iteration is terminated when $.8 \leq R_1 \leq 1.1$ and $.8 \leq R_2 \leq 1.0$.

The iteration scheme to yield $\bar{a} = \hat{a}$ is started with first guesses for the basis vectors resulting from the iterative scheme used to minimize the inconsistency between the model and the data. The consistency achieved by the first scheme is maintained by the second scheme. Again it must be noted that the basis vectors yielding $\bar{a} = \hat{a}$ are not unique. This is due to the fact that for one test site, the system solved is under determined. Unique basis vectors may be determined if ground-truth estimates of area coverage are available for n test sites, each containing different proportions of n species. In this case, the required change is the S-matrix may be determined by a straightforward non-iterative method.

It was shown previously that for one test site

$$[\Delta S] \hat{a} = \bar{q}$$

where $\bar{q}$ is an m x 1 column vector given as

$$\bar{q} = [S] (\bar{a} - \hat{a})$$

For n test sites, the expression becomes

$$[\Delta S] [\hat{A}] = [S] [\bar{A} - \hat{A}]$$

where

$\hat{A}$ - an n x n matrix whose columns contain the n ground-truth mean vectors of fractional areas , $\hat{a}$, for each field

$\bar{A}$ - an n x n matrix whose columns contain the n computed mean fractional areas, $\bar{a}$, for each field.

If each field contains different proportions of the n species, the $\hat{A}$-matrix is non-singular and the expression for [ΔS] becomes

$$[\Delta S] = [S]\ [\overline{A} - \hat{A}]\ [\hat{A}]^{-1}$$

The required S-matrix is then

$$[S] = [S + \Delta S]$$

Because this solution is unique, the consistency function minimized by the first iterative scheme cannot be maintained by the solution.

5. RESULTS

The iterative schemes discussed herein and the mixture processing algorithm (Pace and Detchmendy, 1973) were applied to multispectral data from selected test sites in the C1 flight line in order to verify the concepts presented. The test sites selected consist of three soybean fields with the estimated ground coverage given (Fu, et al, 1969).

The first test field chosen is denoted herein as S1. The upper left hand corner of the field is located at the 79th pixel of scan line 171. This field (see Figure 4) consists of soybeans and soil. One-half of the field is bare soil and is denoted in Figure 4 as S1B. The remaining half of the field, denoted as S1A, is made up of an estimated 20% coverage of soybeans and 80% soil. The soybeans are approximately 12 inches tall. Field S1 is square and has 20 pixels to the side for a total of 400 pixels.

The initial values of the basis vectors, s_1 and s_2, were chosen as

$$s_1 = \overline{y}_1 + \sqrt{\lambda}_1\ \mu_1$$

and

$$s_2 = \overline{y}_1 - \sqrt{\lambda}_1\ \mu_1$$

where

$\overline{y}_1$ = the mean vector for field S1

λ_1 = the largest eigenvalue of the covariance matrix of field S1

μ_1 = the eigenvector corresponding to λ_1

These expressions place s_1 and s_2 in the required subspace. The tips of s_1 and s_2 lie 1σ away from $\overline{y}_1$ along μ_1. The vector s_1 corresponds to soybeans and s_2 corresponds to bare soil. The initial values of s_1 and s_2 are presented in Table 1.

RESULTS OF ITERATIVE SCHEME NO. 1

The iterative progress of iterative scheme no. 1 is presented in Table 2. The value Φ is the total sum-of-squares of the differences between the observed data vectors and the computed data vectors which meet the constraints of the mixture model. The parameter NPO in Table 2 represents the number of pixels in field S1 having a negative fractional area.

It is seen in Table 2 that the iterative scheme cuts NPO approximately in half on each iteration. This is as expected by the least squares formulation of the scheme. The parameters $\overline{a}_1$ and $\overline{a}_2$ are the mean fractional areas of soybeans and soil, respectively, for field S1.

The final values of the basis vectors s_1 and s_2 from iterative scheme no. 1 are listed in Table 3. These vectors correspond to the values obtained on the final, seventh iteration.

RESULTS OF ITERATIVE SCHEME NO. 2

The basis vectors obtained from iterative scheme no. 1 yield a minimum value of Φ. While these basis vectors are not unique, they do make the mixture model and data consistent.

It is noted that from Table 2 the mean fractional areas for field S1 resulting from the final values of the basis vectors are

$$\bar{a}_1 = .374$$

and

$$\bar{a}_2 = .626$$

From Figure 4 it is seen that the apriori mean fractional areas for soybeans and soil for field S1 may be computed from the estimated ground coverage as

$$\hat{a}_1 = 1/2(.20) = .1$$

and

$$\hat{a}_2 = 1/2(.80) + 1/2(1.00) = .9$$

Iterative scheme no. 2 seeks to change the basis vectors from a first guess such that $\bar{a} = \hat{a}$. The initial values of the basis vectors to start iterative scheme no. 2 were chosen to be the final values from iterative scheme no. 1 (see Table 3).

Iterative scheme no. 2 achieved convergence in one step. The final values of the basis vectors s_1 and s_2 from iterative scheme no. 2 are presented in Table 4.

FURTHER TEST RESULTS

In order to test the validity of considering the converged basis vectors for field S1 (from iterative scheme no. 2) to be the true spectral signatures of soybeans and soil, these basis vectors were used to compute the mean fractional areas of two other soybean fields in the C1 flight line. The two fields are denoted here as S2 and S3.

The estimated ground coverage for these fields is given by Fu, et al (1969) as follows:

Field	Soybean Coverage	Soil Coverage
S2	20%	80%
S3	40%	60%

The mean fractional areas computed for fields S2 and S3 using the converged basis vectors for field S1 are given as follows:

Field	$\bar{a}_1$	$\bar{a}_2$
S2	.265	.735
S3	.327	.673

A comparison of the mean fractional areas with the estimated ground coverage for fields S2 and S3 shows that the use of the converged basis vectors from field S1 as spectral signatures to process fields S2 and S3 results in an error of less than 7%.

This error could be due to several factors. First, the apriori ground coverage estimates given may possibly not be accurate to within 7% for any of the three soybean fields. Secondly, it is highly probable that more than two spectral signatures are necessary to describe the fields to a high degree of accuracy. For example, the fields could consist of four species requiring the following signatures:

1) The signature of soybeans in sunlight.
2) The signature of soybeans in shade.
3) The signature of soil in sunlight.
4) The signature of soil in shade.

It is felt that the differences between the ground truth estimates and the computed ground coverages for fields S2 and S3 are not too great when considering the assumed simple two species model.

Although more testing of the iterative techniques is required, it is felt that they show considerable promise as methods by which spectral signatures may be accurately estimated.

6. REFERENCES

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Horwitz, H. M., et al, "Estimating the Proportions of Objects Within a Single Resolution Element of a Multispectral Scanner," Proceedings of the 7th International Symposium on Remote Sensing of the Environment, University of Michigan, May 1971, pp 1307-1320.

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TABLE 1. INITIAL BASIS VECTORS FOR FIELD S1

Channel	s_1	s_2
1	87.778	90.303
2	82.632	85.387
3	63.838	66.113
4	64.719	67.431
5	91.254	93.877
6	93.346	91.436
7	68.519	68.377
8	91.254	96.966
9	76.897	84.814
10	89.870	92.603
11	97.032	75.176
12	76.092	63.402

TABLE 2. ITERATIVE PROGRESS OF SCHEME NO. 1

Iteration Number	Φ	NPO	$\bar{a}_1$	$\bar{a}_2$
0	.26294962+5	206	.474	.526
1	.23258357+5	89	.453	.547
2	.22904458+5	32	.437	.563
3	.22371341+5	16	.418	.582
4	.22274832+5	6	.389	.611
5	.22253118+5	3	.375	.625
6	.22252115+5	1	.373	.627
7	.22252107+5	1	.374	.626

TABLE 3. FINAL BASIS VECTORS FOR FIELD S1 FROM ITERATIVE SCHEME NO. 1

Channel	s_1	s_2
1	86.004	90.914
2	80.697	86.052
3	62.240	66.662
4	62.814	68.086
5	89.412	94.511
6	94.688	90.975
7	68.619	68.343
8	87.242	98.346
9	71.336	86.726
10	87.950	93.263
11	112.383	69.897
12	85.005	60.337

TABLE 4. FINAL BASIS VECTORS FOR FIELD S1 FROM ITERATIVE SCHEME NO. 2

Channel	s_1	s_2
1	72.551	90.914
2	66.024	86.052
3	50.124	66.662
4	48.370	68.086
5	75.442	94.511
6	104.860	90.975
7	69.375	68.343
8	56.821	98.346
9	29.171	86.726
10	73.395	93.263
11	228.786	69.897
12	152.591	60.337

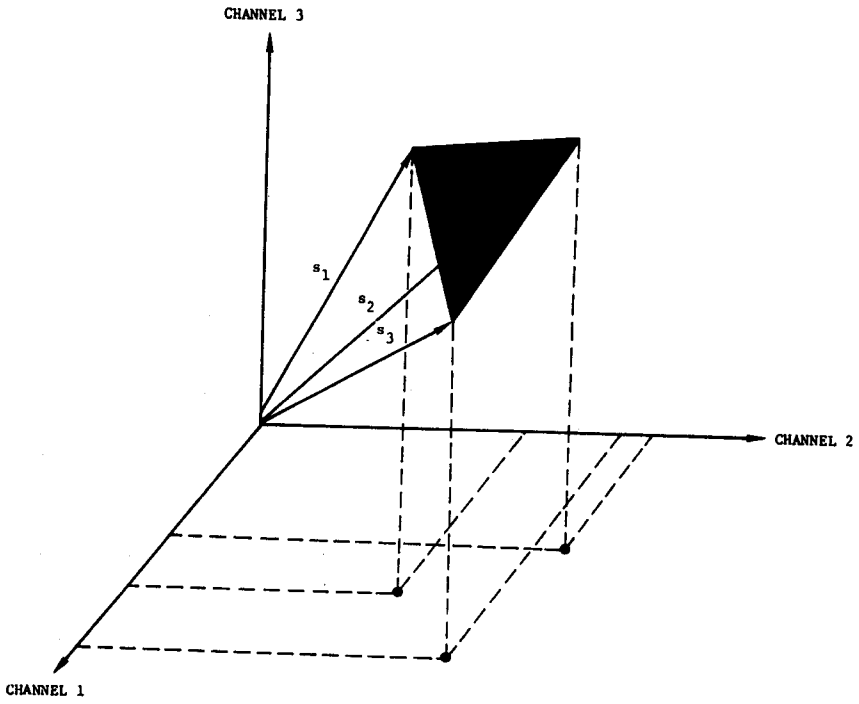

FIGURE 1. THE BASIS VECTORS IN THREE CHANNEL SPACE

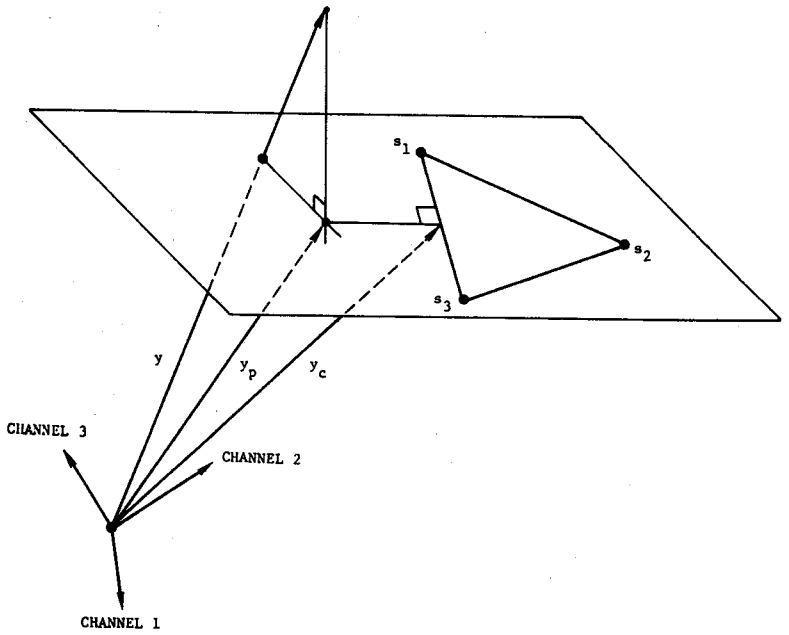

FIGURE 2. CONCEPTUAL SOLUTION TO THE CONSTRAINED OPTIMIZATION PROBLEM

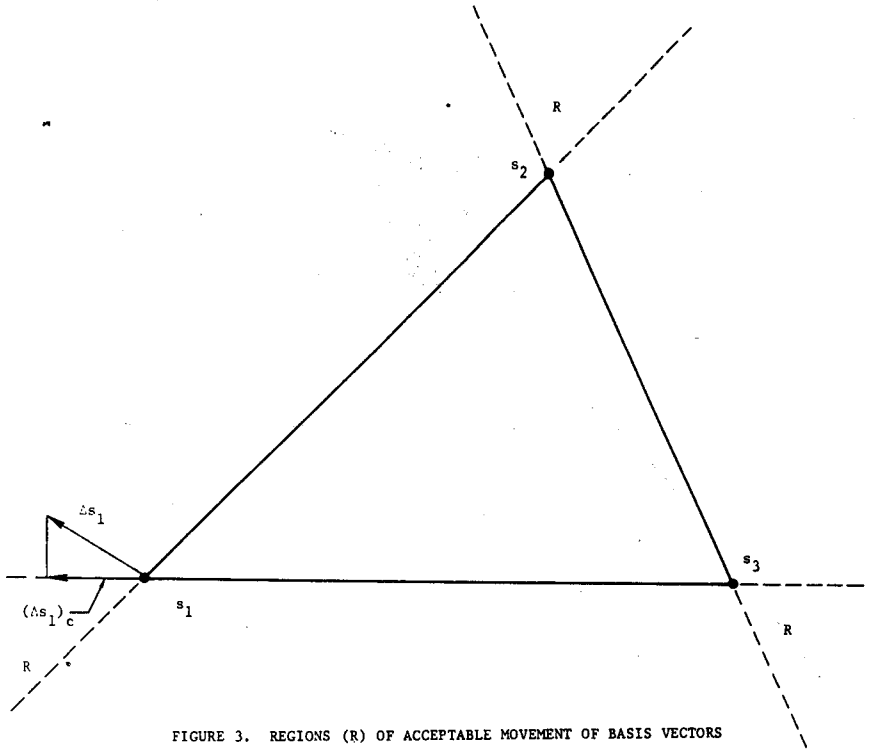

FIGURE 3. REGIONS (R) OF ACCEPTABLE MOVEMENT OF BASIS VECTORS

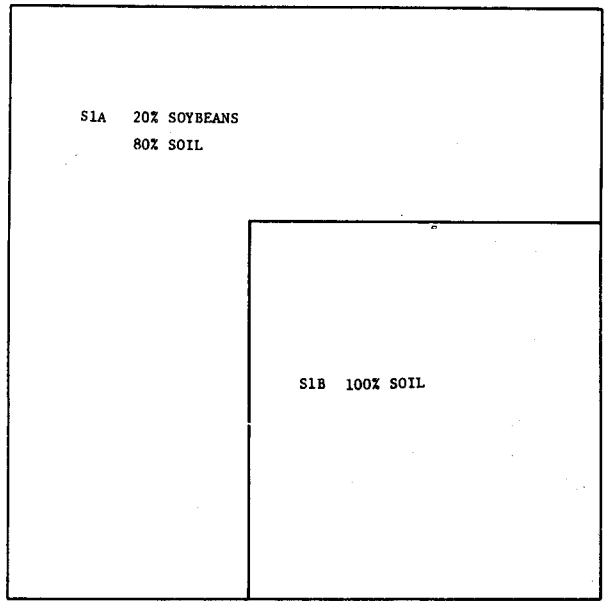

FIGURE 4. FIELD S1