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A GENERAL NON-PARAMETRIC CLASSIFIER APPLIED TO DISCRIMINATING SURFACE WATER FROM TERRAIN SHADOWS*

W. G. Eppler

Lockheed Electronics Company
Aerospace Systems Division
Houston, Texas

I. ABSTRACT

A general non-parametric classifier is described in the context of discriminating surface water from terrain shadows. In addition to using non-parametric statistics, this classifier permits the use of a cost matrix to assign different penalties to various types of misclassifications. The approach also differs from conventional classifiers in that it applies the maximum-likelihood criterion to overall class probabilities as opposed to the standard practice of choosing the most likely individual subclass. The classifier performance is evaluated using two different effectiveness measures for a specific set of ERTS data.

II. INTRODUCTION

An application which has attracted widespread interest (Cartmill, 1974 and Moore, 1973) is that of using ERTS multispectral scanner data to detect surface water. One such system (Anderson, 1973) which has been used with considerable success is based on the fact that values observed in Multispectral Scanner Channels 1 and 4 cluster in different regions of measurement space as shown in Fig. 1. It was found that water could be separated from other confusion classes (e.g., wet fields) by using the Spectral Discriminant Line shown in Fig. 1. In order for a pixel to be classified as water the value for Channel 4 has to be in the range 0 to 12 inclusive and the value in Channel 1 must equal or exceed the value shown. Results of extensive study (Moore, 1973) indicated that this approach can achieve high detection rate (i.e., greater than 90%) with low false-alarm rate (i.e., less than 10%) in cases where there is no significant terrain relief and/or where the sun elevation angle is high.

More recent experience indicates that the false-alarm rate exceeds all reasonable bounds when the nominal spectral discriminant line (shown in Fig. 1) is used with ERTS data acquired at low sun elevation angles in areas with significant ground relief. The particularly troublesome scene used for this paper is ERTS 1191-15381 acquired with a sun elevation angle of 29 degrees on January 30, 1973, in an area of the Great Smoky Mountains near Asheville, North Carolina. Figure 1, based on data to be described in Section III, shows that measurements from terrain shadows cluster near the origin; i.e., there is very little return in either Channel 1 or 4. Unfortunately, the nominal spectral discriminant line passes through the cluster with the result that many of the terrain shadow pixels (specifically, those on and above the line) are misclassified as water. This paper describes a procedure for modifying (specifically, raising) the nominal Spectral Discriminant Line in order to reduce the number of terrain shadow pixels misclassified as water without significantly reducing the detection rate for water pixels. No consideration is given to discrimination between water and any other confusion class.

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From an applications point-of-view the result of this research is a number of Spectral Discriminant Lines which optimize system performance for this particular data set under a variety of different conditions. More importantly, however, this paper describes the straightforward use of statistical decision theory in an easily-understood (i.e., two-dimensional measurement space) application. Features and/or results presented in this paper which are not available in most computer-based classification systems include:

1. The use of a non-parametric representation of the class statistics.

2. The use of a cost matrix to assign different penalties to various kinds of misclassifications.

3. The maximum-likelihood criterion is applied to overall class probabilities (computed as the weighted sum of subclass probabilities) as opposed to the standard practice of choosing the most likely individual subclass.

4. The probability density functions are used to derive two different measures of classification accuracy:

   a) The probability of misclassifying water as "other" and vice versa.

   b) The classification efficiency defined in Section V.

III. DEVELOPING THE TRAINING STATISTICS

The probability density functions for water and terrain shadows were developed from a total of ten locations in ERTS 1191-15381 (near Asheville, North Carolina), ERTS 1092-16305 (in Washington County, Texas), and ERTS 1073-16244 (in Harris County, Texas). Ground truth for the Texas scenes were primarily in the form of aerial photographs but for the North Carolina scene only topographic maps were used. The training samples were those pixels classified as water by the original Spectral Discriminant Line shown in Fig. 1. This somewhat unconventional approach is necessary because terrain shadow pixels are distributed quite randomly throughout an area and therefore are not easily defined by location (i.e., the conventional training field method). The approach is valid because:

1) Previous studies (Moore, 1973) showed it admits almost all of the water samples.

2) By definition, it admits all of the terrain shadows which can be mistaken for water.*

The resulting training samples for each of the ten subclasses were processed to produce the following statistical representations:

1) Two-dimensional and one-dimensional nonparametric probability density functions.

2) Mean vectors, covariance matrices, standard deviations, and correlation coefficients.

3) Two-dimensional and one-dimensional probability density functions computed assuming the data is Normal with parameters obtained in step 2.

The means, standard deviations, and correlation coefficients for each of the ten subclasses are given in Table 1. This table supports the following observations:

1) The mean value in Channel 1 is always less for terrain shadows than for water. This suggests that the Spectral Discriminant Line in Fig. 1 should be raised in order to reduce confusion between these two classes.

2) The mean value in Channel 4 is sometimes lower and sometimes higher for terrain shadows than for water. This indicates that classification based on Channel 4 alone will confuse water and terrain shadows.

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*Because of the way the training samples were selected, the *a priori* probability of terrain shadows is interpreted to mean "the *a priori* probability of that subset of terrain shadows misclassified as water by the original Spectral Discriminant Line."

Figure 2 shows the Nonparametric and Normal representations for a particular area of terrain shadows. From this figure it can be seen that:

1) Both distributions occupy very nearly the same portion of measurement space.

2) The Nonparametric and the Normal density functions have approximately the same shapes.

3) The magnitudes are approximately comparable; the Normal representation is generally smoother, having smaller peak values with no holes or multiple modes.

IV. PARTITIONING THE OBSERVATION SPACE

In a general sense the object of the computer-based classification system is "to separate water pixels from other kinds of pixels." To accomplish this it is reasonable to classify as water those pixels where the likelihood of water is greater than any other confusion class (e.g., terrain shadows). Statistical Decision Theory (Anderson, 1958) takes this into account through the use of conditional probability density functions estimated on the basis of training samples; Fig. 2 is a typical example.

In actual practice it is usually impossible to completely separate water from the confusion classes and it is necessary to make certain compromises. For example, it may not be appropriate to give the same consideration to classes which occur only very rarely as to classes which comprise the majority of pixels. In this case the objective might be "to minimize the number of pixels which are misclassified." Statistical Decision Theory incorporates this consideration through the use of *a priori* probabilities.

Another factor which the user should take into account is that all errors do not have the same consequence. For example, one might prefer to allow a number of false alarms rather than to miss small bodies of surface water. Statistical Decision Theory permits the user to specify* the relative importance of various types of misclassifications by use of a payoff matrix. In this case the objective is "to maximize the expected benefit (i.e., payoff) over the entire data set."

All computer-based classification systems operate by partitioning the observation space into non-overlapping regions associated with each known class. This partitioning is accomplished by an algorithm which specifies the desired classification for each point in observation space (i.e., every combination of measurements). Four different algorithms were considered in this investigation:

1) The Two-Dimensional Table (Eppler, 1974) which maximizes the expected benefit.

2) The Spectral Discriminant Line (Anderson, 1973) which maximizes the expected benefit.

3) The Threshold Value in Channel 4 which maximizes the expected benefit.

4) The conventional LARS classification (Fu, 1969) which selects the Most-Likely Subclass.

The first three classifiers operate to maximize the expected benefit defined by Eq. (1).†

$$\overline{B} = \sum_{n=1}^{N} P_n (\tilde{P}_{n1} B_{n1} + \tilde{P}_{n2} B_{n2}) \tag{1}$$

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*An inability by the user to specify the relative consequences of the two types of misclassification implies that he does not care which type of error is made.

†Symbols used in this paper are defined in the Glossary of Symbols in Section VII.

Equation (2) follows from the fact that, for the purpose of this paper, a pixel is considered to be either water or terrain shadow.

$$\tilde{P}_{n2} = (1 - \tilde{P}_{n1}) \quad (2)$$

It is helpful to define a differential benefit according to Eq. (3); the value of D_n is positive for water subclasses and negative for terrain shadows.

$$D_n = (B_{n1} - B_{n2}) \quad (3)$$

Equation (4) follows from defining $R_1(X)$ as that region in observation space which the classification algorithm associates with Class 1.

$$\tilde{P}_{n1} = \int_{R_1(X)} p_n(X)dX \quad (4)$$

Equation (5) is a definition of the local differential benefit.

$$B(X) = \sum_{n=1}^{N} D_n P_n p_n(X) \quad (5)$$

By combining Eqs. (1)-(5) it is possible to express the expected benefit according to Eq. (6).

$$\bar{B} = \sum_{n=1}^{N} P_n B_{n2} + \int_{R_1(X)} B(X)dX \quad (6)$$

From Eq. (6) it is apparent that the expected benefit is maximized simply by partitioning the observation space in such a way that X is included in region R_1 wherever $B(X) > 0$. Points in observation space where $B(X) < 0$ are assigned to region R_2 and points where $B(X) = 0$ are regarded as a threshold class.

It is apparent from Eq. (5) that the local differential benefit depends on the a) conditional probability density functions, b) *a priori* probabilities, and c) the payoff matrix for each subclass; these are the three key elements in Statistical Decision Theory. Figure 3a shows B(X) for a typical case; the subclasses are defined in Table 1. In this case the *a priori* probabilities are 0.04, 0.08, and 0.16 that a given pixel is from an area of small ponds, large lakes, and terrain shadows, respectively. The benefit of classifying a pixel as water when it is actually a small pond, a large lake, and a terrain shadow is +100, +30, and -10, respectively.

Using the decision rule given by Eq. (7) results in the observation space

$$B(X) > 0 \qquad X \rightarrow R_1 = \text{Water} \quad (7a)$$

$$B(X) = 0 \qquad X \rightarrow R_0 = \text{Threshold} \quad (7b)$$

$$B(X) < 0 \qquad X \rightarrow R_2 = \text{Terrain Shadow} \quad (7c)$$

partition shown in Fig. 3b. The first type of classification algorithm investigated stores the partition in core memory as a <u>Two-Dimensional Table</u> where the desired classification can be looked up for any (x_1, x_4) combination. The second classification algorithm investigated stores only the <u>Spectral Discriminant Line</u>. A pixel is classified as water if $0 \leq x_1 \leq 12$ and if x_4 is above a line such as the one shown in Figs. 3a and 3b. This rule can result in suboptimal performance; for example, note the two circled entries in Figs. 3a and 3b.

The third algorithm investigated classifies a pixel as water if x_4 is less than a prescribed <u>Channel 4 Threshold</u>. This simple classifier has been used with considerable success in scenes where the ground relief is not significant and where the sun elevation angle is high. For a partition based on x_4 alone, the region R_1 in Eq. (6) is a vertical line in the (x_1, x_4)-plane. In this case Eq. (6) can be reduced to Eq. (8) where $B(x_4)$ is given by Eq. (9).

$$\bar{B} = \sum_{n=1}^{N} P_n B_{n2} + \int_{R_1(x_4)} B(x_4)dx_4 \tag{8}$$

$$B(x_4) = \sum_{x_1=9}^{43} B(x_1,x_4) \tag{9}$$

From Eq. (8) it is apparent that the expected benefit is maximized simply by including in region R_1 only those values x_4 for which $B(x_4)$ is positive. The upper limit on x_4 is dictated by other confusion classes (e.g., wet fields) not considered in this study.

The first three algorithms all make use of $p_n(X)$, the conditional probability density functions. In all cases considered, both the Normal representation and the Nonparametric representation were investigated. In one case $B(X)$ was smoothed using a two-dimensional filter according to Eq. (10) prior to applying Eq. (7).

$$\hat{B}(x_1,x_4) = \sum_{i=-1}^{+1} \sum_{j=-1}^{+1} H(i,j)\cdot B[(x_1 - i),(x_4 - j)] \tag{10}$$

This smoothing reduces the magnitude of peak values and extends the radius of non-zero values of B. This has the effect of generalizing and extrapolating the training data. These functions are particularly important when only a few training samples are available or when this technique is extended to more than two dimensions.

The fourth classifier investigated was the well-known LARS algorithm which assigns to a pixel the designation of the Most-Likely Subclass. This approach assumes the probability density functions are Normal with parameters computed from training samples. One variation from the conventional LARS approach was to use the product $D_n P_n$ in place of the usual *a priori* probabilities P_n. The resulting partition and Spectral Discriminant Line in a typical case are shown in Fig. 4.

V. EVALUATING CLASSIFIER PERFORMANCE

This section describes two different measures used for evaluating the performance of the various classifiers discussed in Section IV. These performance measures are a) Classification Efficiency, and b) Probability of Misclassification.

From Eq. (1) it is apparent that the expected benefit ranges between $\bar{B}_{min}$ and $\bar{B}_{max}$ given by Eqs. (11) and (12) in which the notation $n\varepsilon 1$ denotes

$$\bar{B}_{min} = \sum_{n\varepsilon 1} P_n B_{n2} + \sum_{n\varepsilon 2} P_n B_{n1} \tag{11}$$

$$\bar{B}_{max} = \sum_{n\varepsilon 1} P_n B_{n1} + \sum_{n\varepsilon 2} P_n B_{n2} \tag{12}$$

water subclasses for which $D_n > 0$ and $n\varepsilon 2$ denotes terrain shadow subclasses for which $D_n < 0$. The value $\bar{B}_{min}$ corresponds to the case in which pixels are assigned to the incorrect class with probability 1.0, and $\bar{B}_{max}$ corresponds to the case in which pixels are assigned to the correct class with probability 1.0.

Note that $\bar{B}_{min}$ and $\bar{B}_{max}$ depend only on *a priori* probabilities and user-defined differential benefits; they do not depend on how effectively the observation space is partitioned by the classification algorithm. As a measure of the classifier performance it is useful to define a measure called Classification Efficiency defined by Eq. (13).

$$E = \frac{\bar{B} - \bar{B}_{min}}{\bar{B}_{max} - \bar{B}_{min}} \quad (13$$

From Eq. (13) it is apparent that $0 \leq E \leq 1.0$. By combining Eqs. (6), (11), (12), and (13) the Classification Efficiency can be expressed by Eq. (14) to show a

$$E = \frac{\int_{R_1(X)} B(X)dX - \sum_{n\varepsilon 2} P_n D_n}{\sum_{n\varepsilon 1} P_n D_n - \sum_{n\varepsilon 2} P_n D_n} \quad (14$$

dependence on the partition R_1 .* It is clear from Eq. (14) that the Classification Efficienty is maximized by including X in R_1 only if $B(X) \geq 0$; this is the criteric used throughout Section IV.

The second measure of performance used in this investigation is the standard Probability of Misclassification. Equation (15) gives the probability that water will be misclassified as terrain shadows and Eq. (16) gives the probability that terrain shadows wil be misclassified as water.

$$P_{12} = \int_{R_2(X)} \frac{1}{\sum_{n\varepsilon 1} P_n} \sum_{n\varepsilon 1} P_n p_n(X) dX \quad (15)$$

$$P_{21} = \int_{R_1(X)} \frac{1}{\sum_{n\varepsilon 2} P_n} \sum_{n\varepsilon 2} P_n p_n(X) dX \quad (16)$$

Figures 5a and 5b show the integrand in Eqs. (15) and (16) for a typical case. In Fig. (5a) the region of integration $R_2(X)$ is below the Spectral Discriminant Line. In Fig. (5b) the region of integration $R_1(X)$ is above the Spectral Discriminant Line.

VI. RESULTS AND CONCLUSIONS

The major quantitative results of this investigation are contained in Tables 2 and 3. The Cases 1-9 consist of three groups, each having different combinations of *a priori* probabilities and user-specified differential benefits. In Group 1, consisting of Cases 1-3, the *a priori* probabilities and the magnitude of differential benefits are the same for all subclasses. In Group 2, consisting of Cases 4-6, the magnitude of differential benefits are the same for all subclasses but the *a priori* probabilities for terrain shadows and water are 0.64 and 0.36, respectively. In Group 3, consisting of Cases 7-9, the *a priori* probabilities are the same as for Group 2 but the magnitude of differential benefits vary with subclass to put the emphasis on detecting small ponds at the cost increased terrain shadow false alarms. Within each group three different classification algorithms were used to partition the observation space. Case 10 is the same as Case 1 except that smoothing is applied to B(X). The original Spectral Discriminant Line, designated Case 0, is included for the purpose of comparison.

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*It is clear from Eq. (14) and the definition of B(X) in Eq. (5) that E also depends on the inherent separability of the classes.

The observation-space partition determined by the classifier is specified in the form of a) an irregularly-shaped (usually) multiply-connected area such as Fig. 3b, b) a spectral discriminant line, and c) a threshold value for Channel 4. Exceptions are Cases 0, 3, 6, and 9 for which no Channel 4 threshold value is given. Not counting the original spectral discriminant line, a total of 27 different classifiers are derived. Three different performance measures given by Eqs. (14)-(16) are computed for each of the classifiers and given in Table 3.

Information presented in Tables 2 and 3 supports the following conclusions:

(1) In order to avoid misclassifying terrain shadows as water it is necessary to raise the original Spectral Discriminant Line in the range $0 \leq x_4 \leq 3$.

This modification reduces the detection rate to at most 6.7% for the types of water found in the three scenes investigated in this study.

(2) All of the two-dimensional classifiers yield satisfactory results (i.e., $P_{12} \leq 0.1$ and $P_{21} \leq 0.1$).

(3) Classification based on a threshold value in Channel 4 alone does not yield satisfactory performance in the case of scenes having a low sun elevation angle and significant terrain relief.

(4) The classification system based on maximum benefit yields only slightly better performance than the LARS approach which assigns samples to the overall class of the most likely subclass.*

(5) The use of the Nonparametric density function yields only slightly better classification results than the conventional approach using the assumed Normal density function.*

(6) The classification system based on the full two-dimensional table yields only slightly better performance than the simplified approach using the Spectral Discriminant Line.

It should be emphasized that these specific conclusions are based on the particular data sets used in the analysis and different conclusions may apply for other data. The method of analysis, however, is applicable in all cases and yields the best discriminant region, together with measures of its effectiveness in the given application.

VII. GLOSSARY OF SYMBOLS

Symbol	Meaning
$\overline{B}$	Expected benefit.
N	Number of subclasses.
P_n	*A priori* probability of Subclass n.
$\tilde{P}_{ni}$	Probability of assigning Subclass n to Class i for $1 \leq n \leq N$ and $i = \begin{cases} 1 \text{ for Water} \\ 2 \text{ for Terrain Shadows} \end{cases}$
B_{ni}	Benefit obtained by assigning Subclass n to Class i for $1 \leq n \leq N$ and $i = \begin{cases} 1 \text{ for Water} \\ 2 \text{ for Terrain Shadows} \end{cases}$
D_n	Differential benefit defined by Eq. (3). The value is positive for water subclasses and negative for terrain shadows.
$p_n(X)$	Conditional probability density function for Subclass n.
X	Observation vector (x_1, x_4) composed of measured values in Channels 1 and 4.

*It should be emphasized that Conclusions 4 and 5 probably would not be true if the various subclasses were combined into the two major categories prior to classification.

Symbol	Meaning
$R_1(X)$	Region in observation space associated with Class 1.
$B(X)$	Local differential benefit defined by Eqs. (5) and (9).
$\hat{B}(X)$	Local differential benefit after smoothing according to Eq. (10).
$H(i,j)$	Two-dimensional filter used to smooth B(X) according to Eq. (10).
E	Classification Efficienty defined by Eq. (13); it is a measure of classifier effectiveness (and class separability). $0 \leq E \leq 1.0$.
P_{12}	Probability that a water pixel will be misclassified as terrain shadow.
P_{21}	Probability that a terrain shadow pixel will be misclassified as water.

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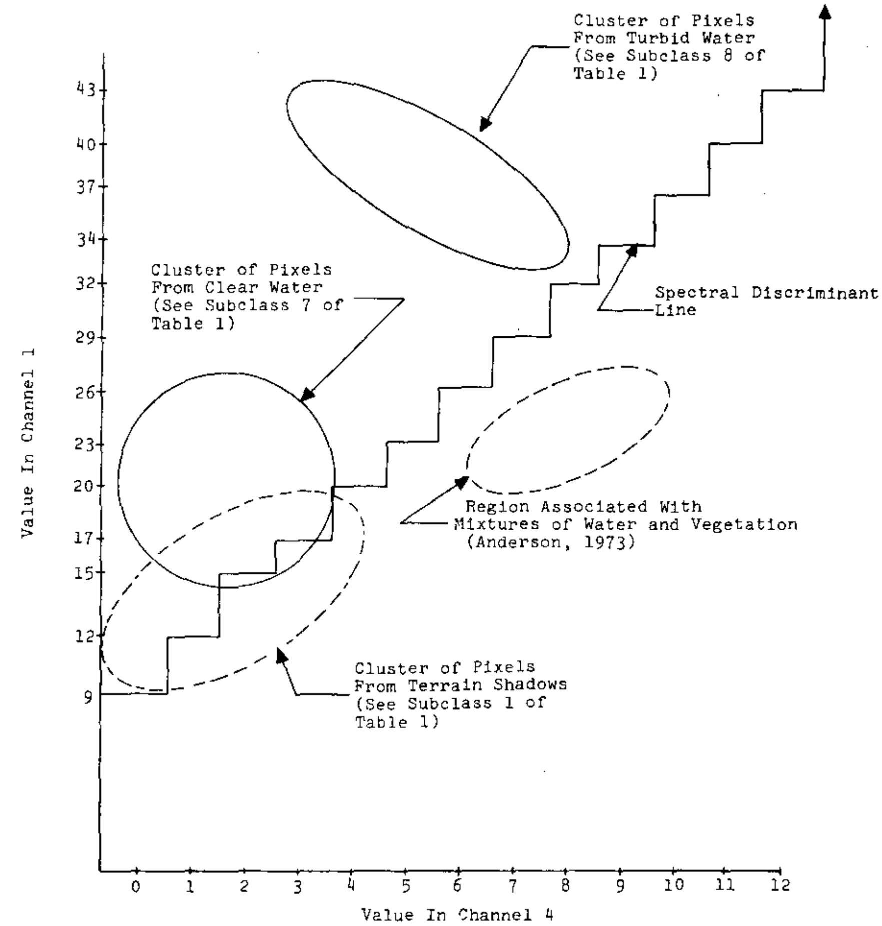

Fig. 1: Water Detection Based on ERTS Multispectral Scanner Channels 1 and 4.

Table 1 - Geographical and Statistical Description of Training Fields for Each Subclass

Subclass	Description	Scene	Lines	Samples	Pixels	M_1	M_4	σ_1	σ_4	ρ
1	Terrain Shadows	1191-15381	151-200	401-450	243	15.824	1.884	1.509	0.932	0.782
2	Terrain Shadows	1191-15381	201-250	551-600	183	14.685	1.194	2.334	1.013	0.825
3	Terrain Shadows	1191-15381	871-920	151-200	104	12.852	0.778	0.808	0.518	0.367
4	Large Reservoir	1191-15381	1351-1400	701-750	612	22.259	1.092	1.569	1.075	-0.384
5	Small Ponds	1191-15381	1801-1850	271-320	65	20.519	2.488	2.815	0.773	-0.257
6	Terrain Shadows	1191-15381	51-100	101-150	229	16.680	2.221	1.556	0.842	0.726
7	Sommerville Reservoir	1092-16305	900-925	201-225	649	19.890	0.996	1.006	0.577	0.097
8	Small turbid pond	1092-16305	791-840	426-475	22	36.709	5.906	1.848	0.803	-0.341
9	Small pond	1092-16305	1046-1095	401-450	11	20.452	2.091	0.914	0.900	0.176
10	Lake Houston	1073-16244	1511-1535	221-245	428	37.903	4.137	2.470	1.384	-0.106

THE NUMBER OF PIXELS IN THIS SAMPLE= 243.0

PROB(CHAN 4,CHAN 1)X10**4

		0	1	2	3	4	5	6	7	8	9	10	11	12
	43	0	0	0	0	0	0	0	0	0	0	0	0	0
	42	0	0	0	0	0	0	0	0	0	0	0	0	0
	41	0	0	0	0	0	0	0	0	0	0	0	0	0
	40	0	0	0	0	0	0	0	0	0	0	0	0	0
	39	0	0	0	0	0	0	0	0	0	0	0	0	0
	38	0	0	0	0	0	0	0	0	0	0	0	0	0
	37	0	0	0	0	0	0	0	0	0	0	0	0	0
	36	0	0	0	0	0	0	0	0	0	0	0	0	0
V	35	0	0	0	0	0	0	0	0	0	0	0	0	0
A	34	0	0	0	0	0	0	0	0	0	0	0	0	0
L	33	0	0	0	0	0	0	0	0	0	0	0	0	0
U	32	0	0	0	0	0	0	0	0	0	0	0	0	0
E	31	0	0	0	0	0	0	0	0	0	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0
I	29	0	0	0	0	0	0	0	0	0	0	0	0	0
N	28	0	0	0	0	0	0	0	0	0	0	0	0	0
	27	0	0	0	0	0	0	0	0	0	0	0	0	0
C	26	0	0	0	0	0	0	0	0	0	0	0	0	0
H	25	0	0	0	0	0	0	0	0	0	0	0	0	0
A	24	0	0	0	0	0	0	0	0	0	0	0	0	0
N	23	0	0	0	0	0	0	0	0	0	0	0	0	0
N	22	0	0	0	0	0	0	0	0	0	0	0	0	0
E	21	0	0	0	0	41	0	0	0	0	0	0	0	0
L	20	0	0	0	41	41	0	0	0	0	0	0	0	0
	19	0	0	0	123	0	0	0	0	0	0	0	0	0
1	18	0	0	41	864	0	0	0	0	0	0	0	0	0
	17	0	0	123	1893	0	0	0	0	0	0	0	0	0
	16	82	823	1851	0	0	0	0	0	0	0	0	0	0
	15	205	699	1399	0	0	0	0	0	0	0	0	0	0
	14	246	1069	0	0	0	0	0	0	0	0	0	0	0
	13	82	288	0	0	0	0	0	0	0	0	0	0	0
	12	41	41	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0

VALUE IN CHANNEL 4

PROB(CHAN 1)X10**4 FOR CHAN 1 BETWEEN 9 AND 43

0	0	0	82	370	1315	2303	2756	2016	905
123	82	41	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0					

PROB(CHAN 4)X10**4 FOR CHAN 4 BETWEEN 0 AND 12

656	2920	3414	2921	82	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0					

Nonparametric

CHAN 1 MEAN= 15.824 CHAN 4 MEAN= 1.884

COVARIANCE MATRIX: K11= 2.277 K44= .869 K14=K41= 1.099

CH 1 STD. DEV.= 1.509 CH 4 STD. DEV.= .932 COR. COEF.= .782

NORMAL DENSITY FUNCTION

PROB(CHAN 4,CHAN 1)X10**4

		0	1	2	3	4	5	6	7	8	9	10	11	12
	43	0	0	0	0	0	0	0	0	0	0	0	0	0
	42	0	0	0	0	0	0	0	0	0	0	0	0	0
	41	0	0	0	0	0	0	0	0	0	0	0	0	0
	40	0	0	0	0	0	0	0	0	0	0	0	0	0
	39	0	0	0	0	0	0	0	0	0	0	0	0	0
	38	0	0	0	0	0	0	0	0	0	0	0	0	0
	37	0	0	0	0	0	0	0	0	0	0	0	0	0
	36	0	0	0	0	0	0	0	0	0	0	0	0	0
V	35	0	0	0	0	0	0	0	0	0	0	0	0	0
A	34	0	0	0	0	0	0	0	0	0	0	0	0	0
L	33	0	0	0	0	0	0	0	0	0	0	0	0	0
U	32	0	0	0	0	0	0	0	0	0	0	0	0	0
E	31	0	0	0	0	0	0	0	0	0	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0
I	29	0	0	0	0	0	0	0	0	0	0	0	0	0
N	28	0	0	0	0	0	0	0	0	0	0	0	0	0
	27	0	0	0	0	0	0	0	0	0	0	0	0	0
C	26	0	0	0	0	0	0	0	0	0	0	0	0	0
H	25	0	0	0	0	0	0	0	0	0	0	0	0	0
A	24	0	0	0	0	0	0	0	0	0	0	0	0	0
N	23	0	0	0	0	0	0	0	0	0	0	0	0	0
N	22	0	0	0	0	0	0	0	0	0	0	0	0	0
E	21	0	0	0	0	4	2	0	0	0	0	0	0	0
L	20	0	0	0	11	38	6	0	0	0	0	0	0	0
	19	0	0	10	153	119	4	0	0	0	0	0	0	0
1	18	0	2	176	637	119	1	0	0	0	0	0	0	0
	17	0	59	990	858	38	0	0	0	0	0	0	0	0
	16	5	449	1799	373	4	0	0	0	0	0	0	0	0
	15	59	1101	1056	52	0	0	0	0	0	0	0	0	0
	14	197	873	200	2	0	0	0	0	0	0	0	0	0
	13	210	223	12	0	0	0	0	0	0	0	0	0	0
	12	72	18	0	0	0	0	0	0	0	0	0	0	0
	11	8	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0

VALUE IN CHANNEL 4

PROB(CHAN 1)X10**4 FOR CHAN 1 BETWEEN 9 AND 43

0	0	8	90	445	1272	2268	2630	1945	935
286	55	6	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0					

PROB(CHAN 4)X10**4 FOR CHAN 4 BETWEEN 0 AND 12

551	2725	4243	2086	322	13	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0					

Normal

Fig. 2: Nonparametric And Normal Representations of Joint Conditional Probability Density Function For Subclass 1.

SUBCLASS	APRIORI PROBABILITY	DIFFERENTIAL BENEFIT
1	.160	-10.0
2	.160	-10.0
3	.160	-10.0
4	.080	30.0
5	.040	100.0
6	.160	-10.0
7	.080	30.0
8	.040	100.0
9	.040	100.0
10	.080	30.0

MAXIMUM BENEFIT FOR CLASS 1 = 19.200
MAXIMUM BENEFIT FOR CLASS 2 = -6.400

TWO-DIMENSIONAL CASE
EXPECTED BENEFIT X 10**4

VALUE IN CHANNEL 1	0	1	2	3	4	5	6	7	8	9	10	11	12
43	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	55	167	110	0	0	0	0	0	0	0
41	0	0	0	448	167	0	0	0	0	0	0	0	0
40	0	0	0	503	784	0	0	0	0	0	0	0	0
39	0	0	0	616	3595	3555	167	0	0	55	55	0	0
38	0	0	223	4269	4214	5843	9198	0	0	0	-86	0	0
37	0	0	0	1178	897	4084	3916	5507	110	55	110	0	0
36	0	0	0	335	674	1983	167	167	110	55	0	0	0
V 35	0	0	0	0	223	1926	1871	110	55	110	0	0	0
A 34	0	0	0	0	0	55	110	2039	110	55	0	0	0
L 33	0	0	0	0	0	55	0	3636	110	0	0	0	0
U 32	0	0	0	0	0	0	0	0	55	0	0	0	0
E 31	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0
I 29	0	0	0	0	0	0	0	0	0	0	0	0	0
N 28	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0
C 26	38	650	612	0	0	0	0	0	0	0	0	0	0
H 25	155	76	612	1228	0	0	0	0	0	0	0	0	0
A 24	743	2078	4121	612	650	-116	0	0	0	0	0	0	0
N 23	4039	4939	1619	273	117	76	0	0	0	0	0	0	0
N 22	2846	5598	6212	5175	117	0	0	0	0	0	0	0	0
E 21	700	2435	9579	883	524	0	0	0	0	0	0	0	0
L 20	2145	21986	8204	7920	5122	0	0	0	0	0	0	0	0
19	479	1631	5823	2861	0	0	0	0	0	0	0	0	0
1 18	479	1598	149	-677	0	0	0	0	0	0	0	0	0
17	-32	-278	119	-1575	0	0	0	0	0	0	0	0	0
16	-305	-2884	-6663	0	0	0	0	0	0	0	0	0	0
15	-745	-4804	-7007	0	0	0	0	0	0	0	0	0	0
14	-1423	-5735	0	0	0	0	0	0	0	0	0	0	0
13	-4238	-9374	0	0	0	0	0	0	0	0	0	0	0
12	-1403	-3798	0	0	0	0	0	0	0	0	0	0	0
11	-153	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0

VALUE IN CHANNEL 4

Fig. 3a: Local Differential Benefit and Decision Line For Case 7.

DECISION BASED ON MAXIMUM BENEFIT

VALUE IN CHANNEL 1	0	1	2	3	4	5	6	7	8	9	10	11	12
43	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	1	1	1	0	0	0	0	0	0	0
41	0	0	0	1	1	0	0	0	0	0	0	0	0
40	0	0	0	1	1	0	0	0	0	0	0	0	0
39	0	0	0	1	1	1	1	0	0	1	1	0	0
38	0	0	1	1	1	1	1	0	0	0	2	0	0
37	0	0	0	1	1	1	1	1	1	1	1	0	0
36	0	0	0	1	1	1	1	1	1	1	0	0	0
V 35	0	0	0	0	1	1	1	1	1	1	0	0	0
A 34	0	0	0	0	0	1	1	1	1	1	0	0	0
L 33	0	0	0	0	0	1	0	1	1	0	0	0	0
U 32	0	0	0	0	0	0	0	0	1	0	0	0	0
E 31	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0
I 29	0	0	0	0	0	0	0	0	0	0	0	0	0
N 28	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0
C 26	1	1	1	0	0	0	0	0	0	0	0	0	0
H 25	1	1	1	1	0	0	0	0	0	0	0	0	0
A 24	1	1	1	1	1	2	0	0	0	0	0	0	0
N 23	1	1	1	1	1	1	0	0	0	0	0	0	0
N 22	1	1	1	1	1	0	0	0	0	0	0	0	0
E 21	1	1	1	1	1	0	0	0	0	0	0	0	0
L 20	1	1	1	1	1	0	0	0	0	0	0	0	0
19	1	1	1	1	0	0	0	0	0	0	0	0	0
1 18	1	1	1	2	0	0	0	0	0	0	0	0	0
17	2	2	1	2	0	0	0	0	0	0	0	0	0
16	2	2	2	0	0	0	0	0	0	0	0	0	0
15	2	2	2	0	0	0	0	0	0	0	0	0	0
14	2	2	0	0	0	0	0	0	0	0	0	0	0
13	2	2	0	0	0	0	0	0	0	0	0	0	0
12	2	2	0	0	0	0	0	0	0	0	0	0	0
11	2	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0

VALUE IN CHANNEL 4

Fig. 3b: Two-Dimensional Observation Space Partition For Case 7.

CLASS	K_{11}	K_{44}	K_{14}	M_1	M_4	P_i	T
1	2.277	.869	1.099	15.824	1.884	.160	9.200
2	5.446	1.026	1.950	14.685	1.194	.160	9.200
3	.653	.268	.154	12.852	.778	.160	9.200
4	2.463	1.156	-.647	22.259	1.092	.240	9.200
5	7.921	.597	-.559	20.519	2.488	.400	9.200
6	2.421	.709	.951	16.680	2.221	.160	9.200
7	1.012	.333	.057	19.890	.996	.240	9.200
8	3.417	.645	-.507	36.709	5.906	.400	9.200
9	.835	.810	.145	20.452	2.091	.400	9.200
10	6.102	1.818	-.354	37.903	4.137	.240	9.200

NUMBER OF CLASSES CONSIDERED= 10

CLASSIFICATION BASED ON LARS CLASSIFIER

	43	0	10	10	10	10	10	10	0	0	0	0	0	0
	42	0	10	10	10	10	10	10	10	0	0	0	0	0
	41	0	10	10	10	10	10	8	10	0	0	0	0	0
	40	0	10	10	10	10	8	8	8	0	0	0	0	0
	39	0	10	10	10	10	8	8	8	10	0	0	0	0
	38	0	10	10	10	10	8	8	8	0	0	0	0	0
	37	0	10	10	10	10	8	8	8	8	0	0	0	0
	36	0	10	10	10	10	8	8	8	8	0	0	0	0
V	35	0	10	10	10	10	8	8	8	8	0	0	0	0
A	34	0	10	10	10	10	8	8	8	8	0	0	0	0
L	33	0	0	10	10	10	10	6	8	8	0	0	0	0
U	32	0	0	10	10	10	10	8	8	0	0	0	0	0
E	31	0	0	0	10	10	10	0	0	0	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0
I	29	0	0	5	0	0	0	0	0	0	0	0	0	0
N	28	0	5	5	5	0	0	0	0	0	0	0	0	0
	27	4	5	5	5	0	0	0	0	0	0	0	0	0
C	26	4	5	5	5	0	0	0	0	0	0	0	0	0
H	25	4	4	5	5	5	0	0	0	0	0	0	0	0
A	24	4	4	5	5	5	0	0	0	0	0	0	0	0
N	23	4	4	5	5	5	0	0	0	0	0	0	0	0
N	22	4	4	5	5	9	0	0	0	0	0	0	0	0
E	21	7	7	9	9	9	0	0	0	0	0	0	0	0
L	20	7	7	9	9	9	0	0	0	0	0	0	0	0
	19	7	7	9	5	5	0	0	0	0	0	0	0	0
1	18	7	7	5	5	5	0	0	0	0	0	0	0	0
	17	0	2	6	6	5	0	0	0	0	0	0	0	0
	16	2	2	1	5	5	0	0	0	0	0	0	0	0
	15	2	1	1	5	5	0	0	0	0	0	0	0	0
	14	2	3	3	5	5	0	0	0	0	0	0	0	0
	13	3	3	3	5	5	0	0	0	0	0	0	0	0
	12	3	3	0	5	0	0	0	0	0	0	0	0	0
	11	2	3	0	0	0	0	0	0	0	0	0	0	0
	10	2	0	0	0	0	0	0	0	0	0	0	0	0
	9	2	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

VALUE IN CHANNEL 4

Fig. 4: Observation Space Partition Based on Most-Likely Subclass.

Subclass	A Priori Probability
4	0.08
5	0.04
7	0.08
8	0.04
9	0.04
10	0.08

$$\frac{10^4}{\sum_{n\epsilon 1} P_n} \cdot \sum_{n\epsilon 1} P_n p_n(X)$$

		0	1	2	3	4	5	6	7	8	9	10	11	12
	43	0	0	0	0	0	0	0	0	0	0	0	0	0
	42	0	0	0	5	16	10	0	0	0	0	0	0	0
	41	0	0	0	43	16	0	0	0	0	0	0	0	0
	40	0	0	0	48	75	0	0	0	0	0	0	0	0
	39	0	0	0	59	344	216	16	0	0	5	5	0	0
	38	0	0	21	409	403	187	260	0	0	0	0	0	0
	37	0	0	0	112	86	142	126	155	10	5	10	0	0
	36	0	0	0	32	64	66	16	16	10	5	0	0	0
V	35	0	0	0	0	21	60	55	10	5	10	0	0	0
A	34	0	0	0	0	0	5	10	71	10	5	0	0	0
L	33	0	0	0	0	0	5	0	99	10	0	0	0	0
U	32	0	0	0	0	0	0	0	0	5	0	0	0	0
E	31	0	0	0	0	0	0	0	0	0	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0
I	29	0	0	0	0	0	0	0	0	0	0	0	0	0
N	28	0	0	0	0	0	0	0	0	0	0	0	0	0
	27	0	0	0	0	0	0	0	0	0	0	0	0	0
C	26	3	20	16	0	0	0	0	0	0	0	0	0	0
H	25	14	7	16	33	0	0	0	0	0	0	0	0	0
A	24	68	190	140	16	20	3	0	0	0	0	0	0	0
N	23	370	334	69	25	10	7	0	0	0	0	0	0	0
N	22	260	434	257	162	10	0	0	0	0	0	0	0	0
E	21	64	223	293	41	27	0	0	0	0	0	0	0	0
L	20	196	1315	439	269	183	0	0	0	0	0	0	0	0
	19	44	149	202	127	0	0	0	0	0	0	0	0	0
1	18	44	159	20	74	0	0	0	0	0	0	0	0	0
	17	3	0	71	129	0	0	0	0	0	0	0	0	0
	16	0	0	16	0	0	0	0	0	0	0	0	0	0
	15	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0

VALUE IN CHANNEL 4

Fig. 5a: Probability Density Function For Overall Water Class With Typical Spectral Discriminant Line.

Subclass	A Priori Probability
1	0.16
2	0.16
3	0.16
6	0.16

$$\frac{10^4}{\sum_{n\epsilon 2} P_n} \cdot \sum_{n\epsilon 2} P_n p_n(X)$$

		0	1	2	3	4	5	6	7	8	9	10	11	12
	43	0	0	0	0	0	0	0	0	0	0	0	0	0
	42	0	0	0	0	0	0	0	0	0	0	0	0	0
	41	0	0	0	0	0	0	0	0	0	0	0	0	0
	40	0	0	0	0	0	0	0	0	0	0	0	0	0
	39	0	0	0	0	0	0	0	0	0	0	0	0	0
	38	0	0	0	0	0	0	0	0	0	0	13	0	0
	37	0	0	0	0	0	0	0	0	0	0	0	0	0
	36	0	0	0	0	0	0	0	0	0	0	0	0	0
V	35	0	0	0	0	0	0	0	0	0	0	0	0	0
A	34	0	0	0	0	0	0	0	0	0	0	0	0	0
L	33	0	0	0	0	0	0	0	0	0	0	0	0	0
U	32	0	0	0	0	0	0	0	0	0	0	0	0	0
E	31	0	0	0	0	0	0	0	0	0	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0
I	29	0	0	0	0	0	0	0	0	0	0	0	0	0
N	28	0	0	0	0	0	0	0	0	0	0	0	0	0
	27	0	0	0	0	0	0	0	0	0	0	0	0	0
C	26	0	0	0	0	0	0	0	0	0	0	0	0	0
H	25	0	0	0	0	0	0	0	0	0	0	0	0	0
A	24	0	0	0	0	0	24	0	0	0	0	0	0	0
N	23	0	0	0	0	0	0	0	0	0	0	0	0	0
N	22	0	0	0	0	0	0	0	0	0	0	0	0	0
E	21	0	0	0	0	32	0	0	0	0	0	0	0	0
L	20	0	0	0	23	111	0	0	0	0	0	0	0	0
	19	0	0	35	107	0	0	0	0	0	0	0	0	0
1	18	0	21	78	502	0	0	0	0	0	0	0	0	0
	17	10	43	371	937	0	0	0	0	0	0	0	0	0
	16	47	450	1136	0	0	0	0	0	0	0	0	0	0
	15	116	750	1095	0	0	0	0	0	0	0	0	0	0
	14	222	896	0	0	0	0	0	0	0	0	0	0	0
	13	662	1464	0	0	0	0	0	0	0	0	0	0	0
	12	219	593	0	0	0	0	0	0	0	0	0	0	0
	11	24	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0

VALUE IN CHANNEL 4

Fig. 5b: Probability Density Function For Overall Terrain Shadows Class With Typical Spectral Discriminant Line.

Table 2: Summary of Spectral Partitions Obtained for Various Combinations of Conditions.

Method Used for Classification					Different Representations of Partitions as Described in Section 4														
Case	Classification Criterion	Statistical Representation	A Priori Probabilities	Differential Benefits	Value Which Channel 1 Must Equal or Exceed for Water When Channel 4 Equals:													Value Which Channel 4 Must Equal or Exceed for Water
					0	1	2	3	4	5	6	7	8	9	10	11	12	
0	Original Spectral Line	–	–	–	9	12	15	17	20	23	26	29	32	34	37	40	43	–
1	Maximum Benefit	Nonparametric	Independent of Subclass	Independent of Subclass	18	18	19	19	20*	25*	26*	29*	32*	34*	39	40*	43*	2
2	Maximum Benefit	Normal	Independent of Subclass	Independent of Subclass	17	18	19	20	20	21	26*	29*	31*	34*	37*	40*	43*	2
3	Most Likely Subclass	Normal	Independent of Subclass	Independent of Subclass	17*	18	19	20	20	23*	26*	29*	32*	34*	37*	40*	43*	–
4	Maximum Benefit	Nonparametric	Dependent on Subclass	Independent of Subclass	18	18	19	20	22	25*	26*	29*	32*	34*	39	40*	43*	4
5	Maximum Benefit	Normal	Dependent on Subclass	Independent of Subclass	17	18	19	20	21	23*	26*	29*	32*	34*	37*	40*	43*	4
6	Most Likely Subclass	Normal	Dependent on Subclass	Independent of Subclass	17*	18	19	20	21	23*	26*	29*	32*	34*	37*	40*	43*	–
7	Maximum Benefit	Nonparametric	Dependent on Subclass	Dependent on Subclass	18	18	17	19	20*	25*	26*	29*	32*	34*	37*	40*	43*	0
8	Maximum Benefit	Normal	Dependent on Subclass	Dependent on Subclass	17	18	18	19	11*	14*	26*	29*	31*	34*	37*	40*	43*	0
9	Most Likely Subclass	Normal	Dependent on Subclass	Dependent on Subclass	17*	18	18	18	13*	23*	26*	29*	32*	34*	37*	40*	43*	–
10	Maximum Benefit Smoothed According to Eq. 10	Nonparametric	Independent of Subclass	Independent of Subclass	18	18	19	19	19	19	26*	29*	31*	31*	33*	36*	43*	2

*In cases where the Spectral Discriminant Line is not uniquely determined, the value selected is the one most nearly equal to the original line.

Table 3: Summary of Performance Results Obtained for Various Combinations of Conditions.

Method Used for Classification					Performance Measures Described in Section 5								
Case	Classification Criterion	Statistical Representation	A Priori Probabilities	Differential Benefits	Two-Dimensional Table			Two-Channel Spectral Line			Channel 4 Threshold		
					E (%)	P_{12} (%)	P_{21} (%)	E (%)	P_{12} (%)	P_{21} (%)	E (%)	P_{12} (%)	P_{21} (%)
0	Original Spectral Line	–	–	–	–	–	–	–	–	–	–	–	–
1	Maximum Benefit	Nonparametric	Independent of Subclass	Independent of Subclass	95.77	3.26	3.29	95.69	3.33	3.29	62.43	38.97	44.83
2	Maximum Benefit	Normal	Independent of Subclass	Independent of Subclass	95.07	4.37	2.69	95.07	4.37	2.69	62.43	38.97	44.83
3	Most Likely Subclass	Normal	Independent of Subclass	Independent of Subclass	95.07	4.37	2.69	95.07	4.37	2.69	–	–	–
4	Maximum Benefit	Nonparametric	Dependent on Subclass	Independent of Subclass	97.00	6.53	0.79	96.94	6.70	0.79	73.24	70.41	1.99
5	Maximum Benefit	Normal	Dependent on Subclass	Independent of Subclass	96.61	6.20	1.58	96.61	6.20	1.58	73.24	70.41	1.99
6	Most Likely Subclass	Normal	Dependent on Subclass	Independent of Subclass	96.61	6.20	1.58	96.61	6.20	1.58	–	–	–
7	Maximum Benefit	Nonparametric	Dependent on Subclass	Dependent on Subclass	94.97	2.25	7.78	94.91	2.32	7.90	74.93	0.00	100.00
8	Maximum Benefit	Normal	Dependent on Subclass	Dependent on Subclass	94.88	2.90	4.54	94.88	2.90	4.54	74.93	0.00	100.00
9	Most Likely Subclass	Normal	Dependent on Subclass	Dependent on Subclass	94.57	2.16	9.56	94.57	2.16	9.56	–	–	–
10	Maximum Benefit Smoothed According to Eq. 10	Nonparametric	Independent of Subclass	Independent of Subclass	95.69	3.13	3.53	95.64	3.13	3.54	62.43	38.97	44.83