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# EFFECTS OF MISREGISTRATION ON MULTISPECTRAL RECOGNITION\*

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## ABSTRACT

Spatial misregistration of multispectral scanner data exists when two (or more) spectral band signals supposedly representing the same location are in fact data values generated from two (or more) overlapping or entirely different ground locations. Misregistration can be found to occur between scanner data channels due to inherent scanner system characteristics and between time periods of multitemporal data due to errors in spatial registration. A study was conducted at the Environmental Research Institute of Michigan to determine what effect these forms of spatial misregistration may have on the accuracy of recognition processing of agriculturally oriented scanner data. A technique was devised and used to measure misregistration along a scan line between channels of data. Misregistrations in excess of one pixel in magnitude were detected in the scanner system examined. This led to the implementation of both analytical and simulation techniques to determine what effect varying degrees of misregistration would have on recognition accuracy and subsequent proportion estimation. Analyses found misregistration to severely reduce the availability of field center pixels and subsequently to introduce significant errors in the classification accuracy and correct proportion estimation of a scene containing an inflated number of mixture pixels. The results of the analysis emphasized the need for the elimination of misregistration in multispectral scanner data and a need for an awareness of the deleterious effects of misregistration in processing results.

## INTRODUCTION

Multispectral recognition processing is carried out under the assumption that data in the various spectral bands are in perfect spatial registration. This is not always the case for data collected at a single time and is less likely to be true for data collected at more than one time, i.e., multitemporal data. This paper presents results of

an investigation that was undertaken to study the effects of such spatial misregistration on crop classification performance and proportion estimation, and provide quantitative estimates of them so that these effects of misregistration may be properly considered in future system designs and application developments. The analyses reported are based on satellite data from the LANDSAT and SKYLAB S-192 multispectral scanners.

Initially, a technique was devised to measure the amount of misregistration between channels of S-192 scanner data in conical format. The misregistration in this data was known to occur along the scan line only. Let  $f(t)$  and  $g(t)$  denote the continuous waveforms in the two channels over the interval  $(A,C)$ . The cross-correlation function  $r(t_0)$  is defined as

$$r(t_0) = \int_A^C f(t)g(t+t_0)dt \quad (1)$$

The amount of misregistration between the two channels can be estimated as the value of the parameter  $t_0$  which maximized the cross-correlation.

Making assumptions which allow the use of Shannon's sampling theorem<sup>1</sup>, the cross-correlation between each pair of channels was computed at fractional pixel increments of the parameter  $t_0$  in (1). The incremental value at which each function reached its maximum was taken as the measure of the amount of misregistration between the two channels. Results indicated that in some cases as much as a 1.13 pixel misregistration occurred between pairs of channels (SDO 10 and 19). Some of this measured misregistration was inherent due to differing sample rates among the channels. However, where this was not the case, the recommendation was made to use conical data and adjust for misregistration<sup>2</sup>. As an example, SDO 19 was shifted by one pixel before data were processed.

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LANDSAT data were also analyzed and no significant misregistration was found to occur between channels.

Multitemporal LANDSAT data sets, however, can be significantly misregistered, depending on the accuracy of spatial registration methods. Experience of investigators has shown that, unless special care is taken, spatial misregistration of one pixel and more will appear. It was of interest to measure to what degree misregistration could affect crop recognition performance.

#### APPROACH

Both analytical and simulation techniques were used to assess the effects of misregistration on classification accuracy and proportion estimation. The basic computational tools were programs which compute probabilities of detection and false alarm based on signatures (mean vectors and variance-covariance matrices) extracted from actual scanner data. Two simulation models were developed to generate signatures to represent signal distributions from misregistered pixels.

In order to facilitate the analysis of the effects of spatial misregistration, four categories of pixels were identified. As illustrated in Fig. 1, these are: (a) pure field center pixels which remain field center pixels when misregistered; (b) pure field center pixels for which those channels out of registration represent mixtures of two or more crop types; (c) mixture pixels for which channels out of registration represent different mixture proportions; and (d) mixture pixels for which those channels out of registration again represent pure field center values.

In the analysis of the effects of channel-to-channel spatial misregistration, pixels falling into category (a) were examined separately from those in (b), (c) and (d). That is, analysis of the effects of misregistration was broken into two principal efforts: (1) a study of the effects on pixels that are field center pixels both before and after the simulated misregistration and (2) a study of the effects on border and near-border pixels that are mixtures in some channel or sets of channels of more than one class after misregistration.

Two techniques were employed in the analysis of the effects upon category (a) pixels. The first, an analytical technique, examined a simplified data structure studying the effects of misregistration within a context of two signatures with a common covariance. The second technique employed was one based on the simulation of the effects of misregistration. A simulation was also carried out in the analysis of the effects on pixels of categories (b), (c) and (d).

#### EFFECTS OF SPATIAL MISREGISTRATION ON ACCURACY OF RECOGNITION OF FIELD CENTER PIXELS

In an analytical analysis of the effects of spatial misregistration on field center recognition performance, two normal distributions with common covariance for an arbitrary number of channels of data were examined<sup>3</sup>. The conclusions of the analy-

sis were intriguing. Where 'common sense' might dictate the hypothesis that misregistration would

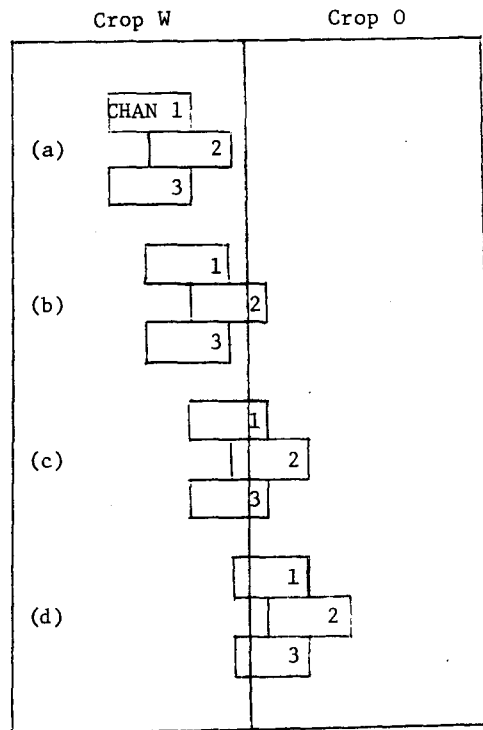


Figure 1. Illustration of 4 Resolutions Elements Misregistered Along The Scan Line One-Half Pixel In Channel 2 of Three Data Channels (Offset In The Vertical Direction For Illustrative Clarity)

hurt field-center recognition performance, the analysis indicated that, while this might frequently be the case, quite the opposite could be true. Under certain circumstances, misregistration could actually slightly improve results in the classification of field center pixels.

Since misregistration and between-channel correlation are closely related, the analysis examined error rate of classification as a function of correlation ( $\rho$ ). It was determined that a unique maximum error rate is reached at  $\rho_{crit}$  somewhere between  $-1 \leq \rho \leq 1$ . Figure 2 plots error rate  $\phi$  as a function of correlation  $\rho$  in a conceivable manner as determined by the analysis for the special case of two distributions having common covariance. Misregistering data will cause correlation to tend to zero, since the scene element measured in a misregistered channel will be spatially displaced from the scene element measured in the registered channels. Therefore, should the given correlation  $\Lambda$  between the two stated distributions lie in the range  $0 \leq \Lambda \leq \rho_{crit} \leq 1$  for perfectly registered data, then by misregistering the data the expected error rate would actually decrease in value.

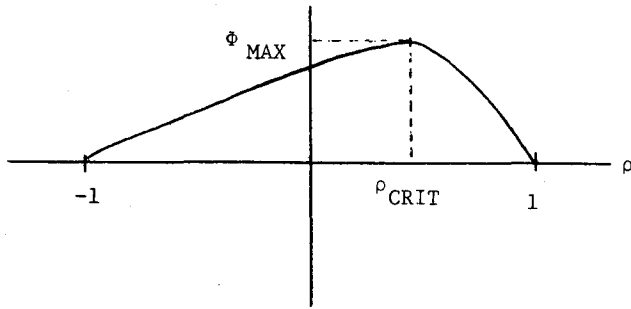


Figure 2. Error Rate of Recognition  $\phi$  as a Function of Correlation  $\rho$  in Field Centers, for Two Normal Distributions with Common Covariance Matrix

A simulation was made using LANDSAT multitemporal data over Fayette County, Ill., to test the hypothesis of the analytical analysis in a more realistic data processing situation. The simulation model assumed that between-channel correlation was a decreasing linear function of misregistration. To be specific, let a perfectly registered distribution  $S_R$  have mean  $A_R$  and covariance  $C_R$ . With some channel or channels misregistered,  $S_R$  would have the same mean vector  $A_R$  but a different covariance  $C_M$ . Any term of  $C_M$ , say  $c_{Mij}$  would be related to a term of  $C_R$  in the following manner:

$$c_{Mij} = c_{Rij} \text{ for } i=j$$

$$c_{Mij} = \beta c_{Rij} \text{ for } i \neq j, 0 < \beta < 1$$

where

' $\beta$ ' was dependent linearly on the degree of misregistration, i.e.,  $\beta = 1$  denotes no misregistration, and  $\beta = 0$  denotes misregistration by one or more pixels. Thus, if two channels  $i$  and  $j$  were misregistered by one-half pixel with respect to one another, then the covariance between  $i$  and  $j$ ,  $c_{Mij}$ , was simulated to be one-half the measured covariance between  $i$  and  $j$  in the registered signatures.

In eight-channel LANDSAT multitemporal data, channels representing the second-time period were simulated as being misregistered in varying degrees from those of the first-time period, using the above model, and the expected performance of the resultant misregistered field-center signatures was calculated. Table 1 indicates, as was determined analytically, that the effect of misregistration is not a significant factor of concern in the recognition of field center pixels which remain field center after misregistration. Other simulations of channel-to-channel misregistration indicate that this conclusion seemed to be independent of the number of channels misregistered.

Although the influence of misregistration on field-center pixel recognition accuracy was found to be insignificant, measurements made using S-192 data over a Michigan test site indicated that the availability of field center pixels was greatly

reduced as misregistration increased; see Table 2. A pixel sampled near a field boundary within the field may be misregistered in certain channels in such a way that the particular channel not in registration is actually focused on the field boundary causing the pixel to be a mixture of more than one agricultural class in that channel. Note, as an example in Table 2, that the availability of corn pixels that were pure corn pixels in every channel, was reduced by 85.3% from 3641 to 573 pixels with a misregistration of one pixel. Overall, 85.4% of all available pure field center pixels were lost due to misregistration of but one pixel. As previously mentioned, misregistrations of data of greater than one pixel have been detected. Conceivably a poorly registered multitemporal scene could contain no pixels that were pure field center of the same crop type in all channels and time periods. Two problems are incurred by the reduction in the availability of field center pixels: (1) statistics representing agricultural classes will have to be based on a small sample size of pure pixels drawn from the class, and (2) the number of boundary pixels or mixture pixels is inflated forcing a greater reliance on compensating errors in proportion estimation. The second problem was examined in detail and the remaining portion of this paper is a presentation of the analysis undertaken.

#### EFFECTS OF MISREGISTRATION ON THE ACCURACY OF RECOGNITION OF MIXTURE PIXELS

Once the extent to which misregistration increased the number of pixels that are mixtures of two or more ground covers was determined, concern shifted to the possible resulting deterioration of recognition accuracy. Analysis of these mixture pixels in fact did lead to the conclusions that (1) misregistration decreases the correct classification rate in the classification of data (here the expression correct classification is used in the sense that mixtures of two covers A and B are classified as either A or B), and (2) misregistration increases the false alarm rate. Thus, the availability of fewer field center pixels, coupled with the increased rate of classification errors among mixture pixels, greatly increases reliance on the compensation of errors for accurate proportion estimation. Overestimates of the proportions of certain crops were linked directly to misregistration.

A simulation technique was decided upon in order to quantify in some manner the effects of misregistration on the mixture elements of a data set. Signatures were adjusted and examined rather than the actual data in order to reduce the amount of processing required. Given a field center signature set from registered data, the problem became one of determining what manner signatures should be adjusted to account both for the presence of more than one material and the introduction of a known degree of misregistration.

The following model was developed to estimate the effects of misregistration on a signature representing a mixture distribution. Let  $\alpha_{wi}$  be the proportion of cover  $W$  present for each pixel in channel  $i$  and  $\alpha_{0i} = 1 - \alpha_{wi}$  the proportion of cover 0 present for each pixel in channel  $i$ . The

Table 1. Expected Field-Center Performance of LANDSAT Multitemporal Signatures for Varying Degrees of Misregistration

Degree of Misregistration (Time Period II)	$\beta$	Wheat	Corn	Tree	Water	Soy	Total
0 pixel	1.0	71.6	85.4	94.8	98.1	88.2	87.6
1/3 pixel	0.67	72.1	84.9	95.4	98.1	88.3	87.8
1/2 pixel	0.50	72.0	85.0	95.8	98.1	87.8	87.7
2/3 pixel	0.33	75.0	84.4	96.0	98.1	88.1	88.3
1 pixel	0.0	72.3	84.1	96.9	98.1	88.1	87.9

Table 2. Display of the Number of Pure Field-Center Pixels Available for Varying Degrees of Misregistration

	TOTAL PIXELS INCLUDING MIXTURES	NO MISREGISTRATION		ONE-HALF PIXEL MISREGISTRATION		ONE-PIXEL MISREGISTRATION	
		#	%	#	%	#	%
CORN	3641	1526	41.9	1054	28.9	537	14.7
BRUSH	820	341	41.6	227	27.7	117	14.3
TREE	49	175	35.7	105	21.4	41	8.4
GRASS	2922	1250	42.8	896	30.7	491	16.8
BARE	653	222	34.0	140	21.4	55	8.4
STUBBLE	1081	391	36.2	247	22.8	100	9.3
OTHER	706	296	41.9	209	29.6	119	16.9
T O T A L	10313	4201	40.7	2878	27.9	1460	14.2

misregistered mean vector  $A_m$  then can be expressed in each channel  $i$  as:

$$A_{Mi} = \alpha_{wi} A_{wi} + (1 - \alpha_{wi}) A_{oi} \quad (2)$$

A term  $c_{Mi,j}$  of the variance-covariance matrix can be expressed as:

$$c_{Mi,j} = \min(\alpha_{wi}, \alpha_{wj}) * c_{wi,j} + \min(\alpha_{oi}, \alpha_{oj}) * c_{oi,j} \quad (3)$$

whenever  $i = j$ , the variance term is given by

$$c_{Mi,i} = \alpha_{wi}^2 * c_{wi,i} + (1 - \alpha_{wi})^2 * c_{oi,i} \quad (4)$$

Letting  $\sigma_i^2$  represent the channel  $i$  variance, with appropriate subscripts, we have

$$\sigma_{Mi}^2 = \alpha_{wi}^2 \sigma_{wi}^2 + \alpha_{oi}^2 \sigma_{oi}^2 \quad (5)$$

This last expression is equivalent to the mixture variance estimation model, previously developed and implemented at the Environmental Research Institute of Michigan<sup>4</sup>.

Equation (3) describes in full the estimated covariance between any two channels of data that are being simulated under the stated model. Diagonal terms of the variance-covariance matrix (the channel variances) are described by Eq. (5). Let us here consider the correlation terms between channels in an attempt to more fully describe and justify the underlying assumptions made in arriving at this simulation model.

Figure 3 displays a possible configuration of the composite signal received by six different channels (or sets of channels in the case of multitemporal data) while focusing on a single resolution element. Figure 3(a) indicates that all six channels are focused on precisely the same location, a borderline resolution element of wheat. This indicates a perfectly registered vector of signals. Figure 3(b) indicates a vector wherein channels 3, 5, and 6 are misregistered and actually viewing mixtures of wheat and other.

Correlation terms between channels 1, 2, and 4 remain identical in Fig. 3(b) to their calculated value for the case shown in Fig. 3(a). It is also

easy to see that the cross correlation between channels 3 and 5 is identical to the mixture covariance estimation model: whenever  $\alpha_{wi} = \alpha_{wj}$ , Eq. (3) becomes

$$c_{mi,j} = \alpha_{wi} * c_{wi,j} + \alpha_{oi} * c_{oi,j} \quad (6)$$

which is ERIM's mixture model (the off-diagonal equivalent of Eq. (5)).

(a) Perfectly Registered (b) Misregistered

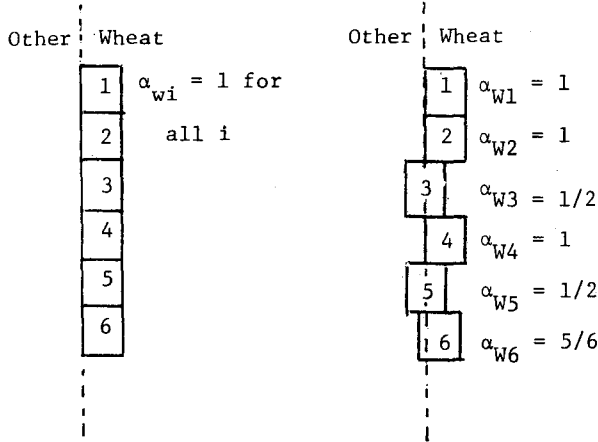


Figure 3. An Example of Channel Misregistration For A Single Resolution Element

However, whenever  $\alpha_{wi} \neq \alpha_{wj}$  as is the case, for example, in channel 1 versus 3 or 3 vs. 6, Eq. (3) addresses situations not previously considered by the mixture model. Assumptions made in the evaluation of these covariance terms are to be more fully discussed, cross-hatched) between the two channels in the wheat

Figure 4(a) displays the areas associated with the various components of Eq. (3). Note that  $\alpha_{wj} = \min(\alpha_{wi}, \alpha_{wj})$  gives the proportion of overlap (area hatched) and  $\alpha_{oi} * c_{oi,j}$  is the contribution of the other field. Hence  $\alpha_{wj} * c_{wi,j}$  is the contribution of  $c_{wi,j}$  to the constructed covariance term  $c_{mi,j}$ . Similarly,  $\alpha_{oi} = \min(\alpha_{oi}, \alpha_{oj})$  is the proportion of the other field that is common to both channels i and j (area

variance of 'other' in channels i and j. Hence, where there is no overlap (unshaded area), the cross correlation is assumed to be negligible and therefore zero.

The two basic assumptions made in the derivation of the covariance estimation model are that (1) within the same field, the correlation between two ground samples within a given channel drops off rapidly as the distance between the samples increases<sup>5,6</sup> and (2) signals from different crops are totally uncorrelated. Figure 4(b) illustrates the second assumption. Here the correlation  $c_{Mi,j} = 0$ .

The above model was used in an analysis of effects of channel-to channel misregistration of Skylab S-192 data on mixture pixels. A subset of five signatures from the agricultural scene of S-192 data was used as the basis for the simulation. These signatures represented the ground covers: corn, tree, grass, bare

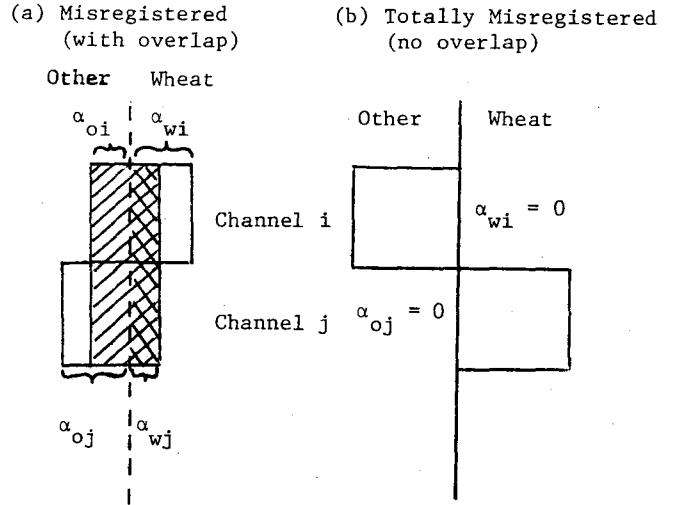


Figure 4. A Misregistration Configuration In Two Channels For A Single Resolution Element

soil and brush. Misregistration between channels was reduced to  $\leq .38$  pixels prior to signature extraction (an average misregistration between channels of .15 pixels), and it was assumed for purposes of simulation that the data the signatures were generated from were perfectly registered.

From these initial signatures, many signatures were generated representing a variety of distributions as affected by varying degrees of misregistration. Once field center signatures were calculated, new distributions representing mixtures of all permutations of two ground covers for varying proportions were simulated as follows. Let  $\alpha_{iw}$  and  $\alpha_{io}$  be the proportions of distributions W and O in the ith channel used to simulate a mixture of ground covers W and O. For perfectly registered signatures,  $\alpha_{io}$  was set at 2/3, 1/3, and 0 for every channel i. However for misregistered signatures, the channels out of registration would be in different proportions. For example, if a signature was misregistered by 1/2 a pixel the proportion of cover type W would be  $\alpha - 1/2$ . Hence any field-center pixels in the registered case within 1/2 pixel of the boundary would become mixture pixels in the misregistered case. (In effect, there would be fewer field center pixels.) Therefore signatures representing mixtures of misregistered distributions were simulated with proportions of  $\alpha_{iw}$  and  $\alpha_{io}$  in the registered channels i and  $(\alpha_{jW} - \beta)$  and  $(\alpha_{jO} + \beta)$  in the misregistered channels j, where  $\beta$  is the degree of misregistration.

Once the simulated signatures were calculated, a program was used to compute the expected performance matrix for a linear rule classifier at a 0.001 probability of falsely rejecting a pixel from the multivariate Gaussian distributions. A Monte-Carlo technique was used to generate samples. The performance matrix gives the probability that pixels from each given signature distribution will be classified into each recognition class based on the best linear decision boundaries ('best' in the least average probability of misclassification sense) between

recognition classes. Analysis consisted of the study of the expected performance curve as a function of the location of the pixel across a field boundary. The study focused on the analysis of three basic problems: (1) the effect of misregistration on the classification of a mixture pixel of two ground covers; (2) the effect of misregistration on the false alarm rate of any given crop among mixtures of two other ground covers; and (3) the effect of misregistration on proportion estimation.

Figures 5, 6, and 7 and many others like them were used in the analysis. These figures contain curves that display the expected performance of pixels of the types labeled at the top of the graphs, as a function of the proportion present of each of the two possible crop types. In a sense one could envision, as an aid in studying these graphs, a pixel moving across a fixed field boundary and at various locations the expected probability of that pixel's classification would be calculated. Note in each of the graphs a zone representing pure field center pixels in the registered case has been labeled as well as an area representing mixtures of varying degrees. The width of these zones is exactly one pixel and the field boundary would appear as drawn. The right-hand corner of a pixel placed on this grid would lie at the labeled mixtures proportion.

Figure 5 displays three graphs, one for each degree of misregistration of the three SDOs considered (SDOs 2, 12, 17) plotting the expected probability of classifying brush and brush-grass mixtures as brush (the solid line) or grass (the dashed line). In Fig 5(a), on top, one notes that in the area designated brush, these field center pixels are for the most part classified as brush. As the mixture of brush and grass becomes predominantly grass, the performance curve increases for grass and decreases for brush. Also, note in Fig. 5(a) that, at the border (1/2, 1/2) mixture pixels are in proportion one-half grass and one-half brush and are called brush or grass 70% of the time. These pixels are therefore classified as neither brush nor grass 30% of the time. As misregistration is introduced (compare Figs. 5(a), (b), and (c)), field center brush pixels are not classified as brush with as much consistency. The expected performance for those pixels most near the border deteriorates from around 78% to 42% correct for one-half pixel misregistration, and down to 15% for one pixel misregistration. The indication in these figures as well as many others studied is that misregistration does affect the classification of near-border and border pixels significantly.

Figure 6 is a display of the effect of three channels of misregistration on the corn false alarm rate among bare soil and bare soil-brush mixture pixels. It indicates that a large percentage of these mixture pixels would be misclassified as corn. Should a trend in this direction be established, corn would be overestimated since errors may not compensate.

Figure 7 is presented to show that the trend does establish itself and that the errors introduced are not strictly compensatory for proportion estimation, especially when misregistration is

introduced in the scene. Noting an increased rate of corn false alarms among brush-grass pixels, these would necessarily have to be compensated for by a decrease in the correct classification of corn or mixtures of corn-other pixels. Figure 7 is a graph of the expected probability of "correct" classification of two ground covers as labeled as a function of the mixture proportion. The solid line indicates the amount of brush-grass correctly classified. With more misregistration there are more false alarms particularly of corn, as previously noted. However, the correct classification of corn, corn-grass or corn-brush pixels does not correspondingly decrease, indicating that corn may be overestimated.

The grass false alarm rate was, similar to corn, noted to rise as misregistration increased for this simulation. This was of particular interest since both these classes were overestimated when the actual data was processed.

Figure 8 is a diagram indicating why misregistration may have caused corn and grass to have been overestimated. The line connecting the means of the indicated signatures A and B is the mixture line, that is, the means of any mixture of A and B would fall on this line. Misregistration, however, would cause the mixture means to move anywhere within the indicated rectangle.

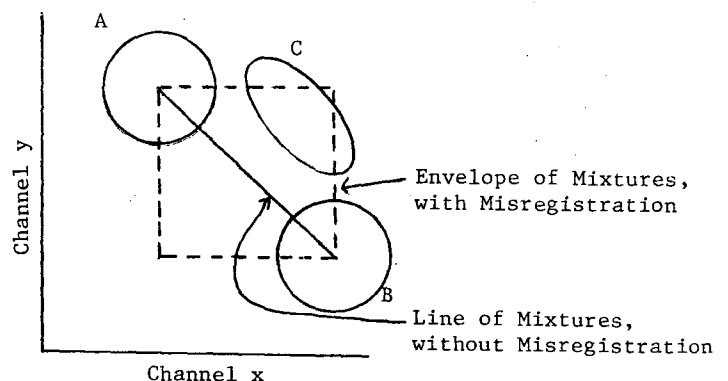


Figure 8. Illustration Of The Effects Of Misregistration On The Spectral Domain Of Mixture Pixels

The implication is that a greater number of mixtures of A and B would be drawn near to distribution C and one would expect to find with misregistration a higher false alarm rate of crop C among mixtures of A and B than without misregistration.

#### CONCLUSIONS

It has been determined through a simulation of the effects of misregistration on recognition performance that several problems result. Although there seems to be no serious effect in the correct classification of field center pixels that remain field center pixels after misregistration, it was found that even for small degrees of misregistration (up to one pixel) the availability of pure field center pixels is greatly reduced. Analysis of mixture pixels lead to the conclusions that (1) misregistration increases the error rate in the classification

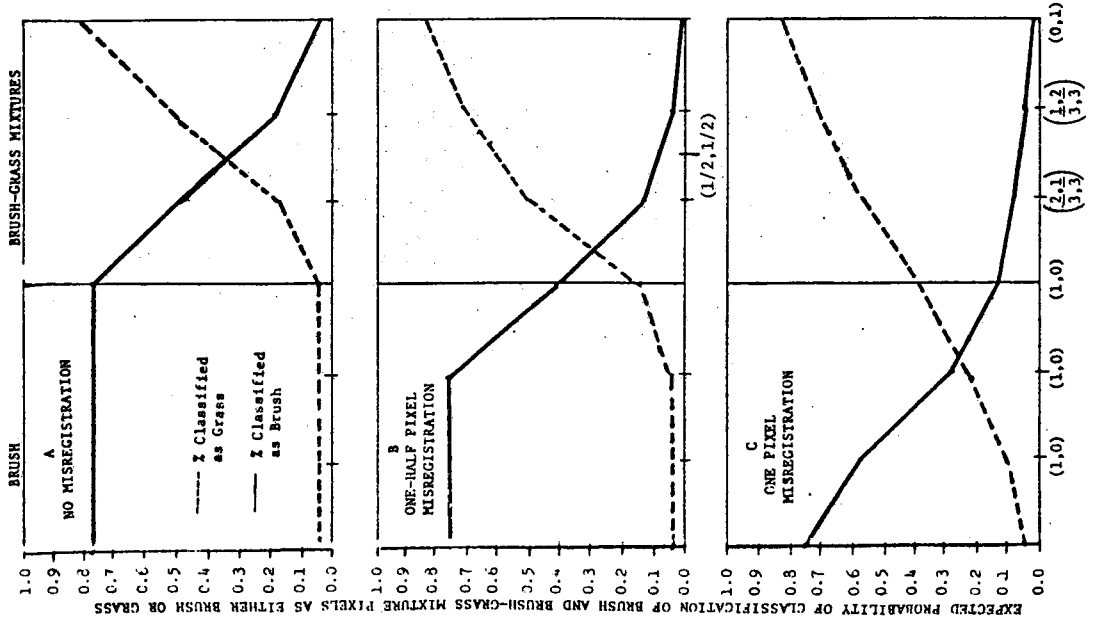


FIGURE 5. EXPECTED CLASSIFICATION PERFORMANCE OF BRUSH IN BRUSH-GRASS MIXTURE PIXELS. THREE CHANNELS MISREGISTERED.

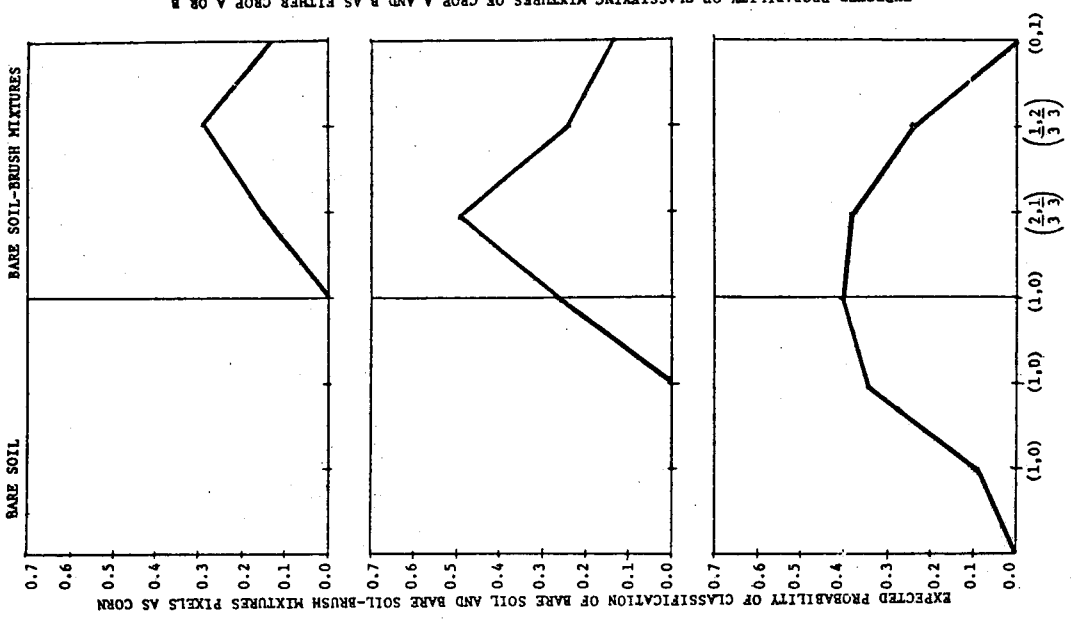


FIGURE 6. MIXTURE PROPORTION (BARE SOIL-BRUSH) FOR REGISTERED PIXELS. CORN FALSE ALARMS AMONG BARE SOIL AND BARE SOIL-BRUSH MIXTURE PIXELS. THREE CHANNELS MISREGISTERED.

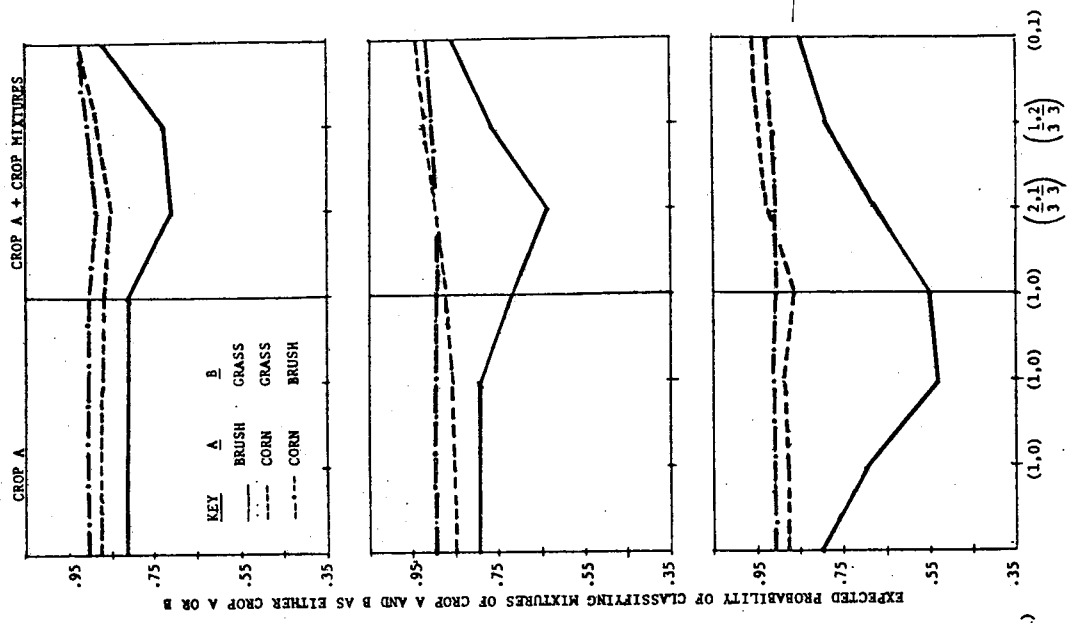


FIGURE 7. MIXTURE PROPORTIONS (BRUSH-GRASS) FOR PERFECTLY REGISTERED PIXELS. EXPECTED CLASSIFICATION OF MIXTURE PIXELS OF CROPS 'A' AND 'B' AS EITHER CROP AND OR CROP B.



of data and (2) misregistration increases the false alarm rate. Furthermore, the general belief that mixture pixels and misregistration effects will result in errors that are compensating over a scene was brought into question. It is strongly suggested that further scanner systems and data preparation algorithms and procedures be designed to take every precaution to minimize spatial misregistration in order to optimize the accuracy of the information extracted when performing automatic multispectral earth resource survey.

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