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MACHINE ESTIMATION OF TIMBER VOLUMES FOR USE IN SAMPLING SURVEYS

A Method for High Flight and Space Imagery, Interface Considerations, and Results

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I. ABSTRACT

The digital timber volume estimation method was developed primarily for use with Landsat MSS scanner data. The technique makes use of a vector field clustering algorithm, nearest neighbor classification, and regression analysis. When such a technique is to be applied to reduce the cost for a given level of precision in a forest inventory, the interface between sampling methods and the digital estimation model must be considered. The candidate models and sampling methods must be evaluated using test data as closely related to actual circumstances as possible.

II. INTRODUCTION AND OBJECTIVES

This paper describes results of a Landsat investigation¹ concerning development of a multi-stage forest inventory system. In such a system, aerial and space platform imagery as well as ground data can be used in a sampling framework through which estimates of timber volumes are obtained.

The image interpretation models used in this forestry context are quantitative rather than qualitative, unlike models used in disciplines such as geology or agriculture. They are set apart by the fact that the estimates need not approach a "99 percent" accuracy to be useful, as they are only inserted as auxiliary data in a sampling survey for the purpose of reducing the overall sampling effort required to achieve a specified level of precision.

The quality of the interpretation model, therefore, should be dictated by overall survey economic considerations, i.e., by finding answers to the following complex question: What interpretation models and techniques contribute most to the sampling efficiency and at what cost, and what are the appropriate sampling techniques and methods?

Considering the variety of interpretation methods, imagery types, instrumentation, as well as the many different sampling techniques and the manner in which they may be combined in a multi-stage sample survey, it is not hard to grasp that this question may remain unanswered for many years to come, if not forever. Hence, the best one can

do is select an approach with which to experiment and theorize in the realm of the possible.

III. TEST DATA

A. "Ground Truth"

We at EarthSat were fortunate to have available for experimentation results of an extensive commercial timber inventory conducted just before the event of Landsat-1. The volume estimates of such an inventory cannot be called ground truth, and in the context of our study this term is actually misleading, even though the estimates are partly based on ground truth in the form of tree dendrometry.

Rather than comparing machine interpretations with ground truth, we were interested in finding out what can be gained in a forest inventory if space imagery is included as the first level of the first stage in a multistage forest inventory. Thus, instead of ground truth, we were in need of test populations for which the attributes one level removed from the space level were known. The estimates of the commercial survey satisfied this need exceedingly well. In addition, we used two other types of second level estimates: (1) volume estimates obtained by human interpretation of U2 photographs, and (2) estimates obtained from logging records and inventories conducted prior to the EarthSat timber survey. We will refer to the different types of second level attributes as the ES, U2, and SP volume estimates, respectively.

B. Test Site

From the area covered by the commercial survey, we selected as our test site the mountainous terrain of the Trinity Alps in California with elevations ranging from 2,000 to 8,000 feet. The timber cover in this area consists of a mixture of red and white fir, sugar and ponderosa pine, and douglas fir, in proportions which vary with elevation.

Alternate square miles in a checkerboard pattern are owned by the Southern Pacific Land Company and the Federal Government. We selected these square-mile "sections" (or fractions thereof) and

their associated timber volumes as the basic unit in our experiment.

C. Imagery

NASA provided us with MSS images in bands 5 and 7 of the test site as well as with computer compatible tapes. In particular, we used frame E1094-18224 of which band 7 has the appearance of a radar image due to a low sun angle because of the late fall date. We also obtained high flight 1:125,000 scale U2 color infrared photographs.

To locate our sections on the imagery, we extended the previously developed 1:40,000 scale precision image annotation system to the U2 photos. Control for photo orientation and location was obtained by executing a photogrammetric block adjustment.

A special generalized precision annotation system was developed for the Landsat images. A report on this system and the accuracies obtained with it can be found in van Roessel and Langley².

The reason for exercising this much care to locate the sections on the various types of imagery was of course to prevent negative results which could possibly arise from unintentional comparison of different pieces of land on different types of imagery. With our precision annotation techniques there was no question of the correct piece of land being interpreted.

IV. MACHINE ESTIMATION TECHNIQUE

In this paper we will concentrate on the machine oriented timber volume estimation technique, although we also experimented with human interpretation models. The machine system consists of two parts; namely, the training subsystem and the timber volume estimation subsystem

A. Training Subsystem

This part is applied to training areas in which timber volume estimates are available. The following six major programs are used.

1. Feature Extraction. The Landsat CCT images are blocked up into 8x8 pixel elements which are called "intels." For each intel, one tone and one contrast measure for each spatial band can be extracted. To compute these values we use a fast Walsh transform algorithm. Final output of this program is a list of $2 \times N \times K$ features where N is the number of channels used and K is the number of intels.

2. Feature Modification and Display. With this program, features may be combined in the form of differences or ratios, and histograms and scattered diagrams may be plotted.

3. Unsupervised Classification. We selected G. A. Butler's³ vector field approach for a clustering algorithm to first perform unsupervised classification. With this non-statistical method a

"gravitational" field is generated in the n -dimensional feature space by using the generalized Newtonian formula, $F = M_1 M_2 S^{-r}$ where r is called the "field strength." A gradient searching technique is used to look for nodes or centers of zero gravity in this vector field. These nodes are representative of the cluster centers. This technique allows one's perspective of the data to range from locally sensitive to globally sensitive by just varying one parameter: the field strength.*

A major variation on Butler's original technique is that we use the "chain" algorithm⁴ to thin out the feature space before clustering, thereby reducing the large number of computations that must otherwise be made. The link points in the chain are taken as points in the thinned feature space, to which non-zero masses are assigned according to the number of original points associated with each link point. The same chain algorithm is also used to define a small set of starting points from which the gradient search for the zero gravity centers is started.

4. Nearest Neighbor Classification. After the cluster centers are found, the intels can be classified according to the clusters with which they are associated. Roese⁵ has used a maximum likelihood procedure assuming a multivariate normal distribution for his clusters.

We do not make any distributive assumptions, however, and use simple nearest neighbor classification according to the Euclidean distance for each of the unthinned points. Before classifying, the n -dimensional feature space is standardized.

5. Class Area per Section Calculation. A classification image in the form of 8x8 blocks of class numbers corresponding to the original intels is generated from the nearest neighbor results. With the precision annotation system, section boundaries are then superimposed on this classification image and the class symbols within each section are tallied using an "in or out" algorithm. These tally figures are converted into proportions of classes per section.

6. Regression. Next, a regression is performed in which the dependent variable is the "known" timber volume for the section and the independent variables are the class percentages. This regression is based on the following model:

$$V_j - \bar{V} = \beta_1 p_{1j} + \beta_2 p_{2j} + \dots + \beta_i p_{ij} + \epsilon_j \dots (1)$$

where V is the timber volume per square mile for parcel j , \bar{V} is the average volume over all parcels, p_{ij} is the class proportion of class i in parcel j , and ϵ_j is an error term. We call the beta's in this model Differential Volume Levels. Note that, although \bar{V} is redundant, it is included in the model so that the beta's will represent class differences around a mean volume rather than absolute volume figures. This is to facilitate statistical testing of their significance.

B. Timber Volume Estimation Subsystem

The main outputs from the training phase are the cluster center coordinates and the differential volume levels. These are the controlling parameters for the estimation process, in which extracted features from the unknown area are entered into the nearest neighbor classifier. The class areas per section are then determined and they are multiplied with the differential volume levels and added to the mean volume to yield the estimate for the section.

V. SAMPLING METHODS

As possible first-stage sampling methods in a multistage design to receive the machine-produced estimates as auxiliary data, we considered: (1) variable probability sampling, (2) regression sampling, and (3) stratified sampling with proportional allocation.

As a measure of the decrease in variance (or the gain in precision) due to the auxiliary data, we adopted the formulation presented by Zarkovic⁶ in a slightly different form; namely, the variance of the estimator for simple random sampling V_{srs} , minus the variance of the estimator under consideration, expressed as a percentage of V_{srs} .

Using Zarkovic's expressions modified in this manner, the percentage gain for each method is:

1. Variable probability sampling.

$$\Delta G_{vps} = \left\{ \frac{N}{n} \sum_{i=1}^N Y_i^2 \left(1 - \frac{\bar{Y}}{X_i} \right) \times 100\% \right\} / V_{srs} \quad \dots(2)$$

where N is the total number of units in the population, n is the sample size and X_i is the first level auxiliary variable, and Y_i is the second level attribute.

2. Regression sampling.

$$\Delta G_{reg} = \rho^2 \times 100\% \quad \dots(3)$$

where ρ is the population correlation coefficient.

3. Stratified sampling.

$$\Delta G_{strat} = \frac{\sum_{h=1}^L W_h (\bar{Y}_h - \bar{Y})^2 \times 100\%}{V_y^2} \quad \dots(4)$$

where W_h (N_h/N) and \bar{Y}_h are the stratum proportions and the stratum mean for the h^{th} stratum of L strata, respectively.

Using the existing volume estimates of the previously described test populations as second level attributes (Y_i 's) and the machine estimates as first stage attributes (X_i 's), we were able to compute the

percentage gain criteria for each sampling method, and thus evaluate the machine interpretation as well as the sampling methods.

VI. INTERFACE CONSIDERATIONS

One of our concerns was the interface between sampling survey methods and interpretation models. We asked ourselves the following questions:

1. Is the interpretation model statistically significant?
2. If so, what does the model contribute to the survey efficiency, and how can this contribution be predicted?
3. What sampling method is most robust in coping with training to application site deterioration?

To evaluate the statistical significance of our models, we used the following regression statistics: (1) the multiple correlation coefficient; (2) an F statistic to test the hypothesis that the differential volume levels $\beta_1 = \beta_2 = \beta_i = 0$; namely,

$$F_{p-1;v} = \frac{SS(R/bo)/(p-1)}{s^2} \quad \dots(5)$$

where $SS(R/bo)$ is the sum of squares attributable to p volume levels, s is the standard error of estimate, and $v = N-p$ (Draper and Smith⁷, p. 64); and (3) t statistics for the individual beta's to test the significance of their difference from zero.

To evaluate the model contribution to survey precision, we used the relative gain criteria described in the previous paragraph. We realized, however, that the significant deterioration in performance may occur when the model is applied to a much larger area, such as our testing area, which may or may not include all or part of the original training site.

Some insight into this deterioration was obtained by formulating a theoretical model concerning variable probability sampling. This sampling method has recently been applied in multistage forest inventories with a space stage.^{8,9}

Due to constraints in image processing and availability of test data, selected test and training sites cannot be considered random samples from a population. Thus, one cannot assume any functional distribution for the sample units which comprise these sites. A way out of such a situation was suggested by Cochran¹⁰; namely, the finite training-test population is regarded as drawn at random from a super population for which certain properties are assumed. In our case this solution was applied as follows:

1. The training-test site combination is considered as drawn from a finite super population for which we assume that the first level auxiliary

values are linearly related to the second level population attributes: $Y_i = a + bX_i + e_i$ where we assume that

$$E(a) = \alpha, \text{Var}(a) = V_a^2, E(b) = \beta, \text{Var}(b) = V_b^2,$$

$$E(e_i) = \epsilon_i, \text{Var}(e_i) = V_{e_i}^2$$

2. The super population of all training site combinations can in turn be considered as a sample drawn from an infinite super population for which we assume that

$$E(e_i/X_i) = 0, \text{Var}(e_i/X_i) = \gamma X_i^g$$

With these assumptions we can now derive an expression for the expected value of the gain in precision for variable probability sampling conditional on the first super population.

$$E(G/sp_1) = \frac{N^2}{n} \left\{ (\beta^2 + V_b^2) V_x^2 + (\alpha^2 + V_a^2) \text{cov} \left(\frac{1}{X_i}, X_i \right) + 2a \text{cov} \left(\frac{\epsilon_i}{X_i}, X_i \right) + 2b \text{cov} (\epsilon_i, X_i) + \text{cov} \left(\frac{V_{e_i}^2 + \epsilon_i^2}{X_i}, X_i \right) \right\} \dots(6)$$

The expected value of this expression, given the properties for the second super population, is as follows:

$$E(E(G/sp_1)) = \frac{N^2}{n} \left\{ (\beta^2 + V_b^2) V_x^2 + (\alpha^2 + V_a^2) \text{cov} \left(\frac{1}{X_i}, X_i \right) + \text{cov} \left(\frac{V_{e_i}^2 + \gamma X_i^g}{X_i}, X_i \right) \right\} \dots(7)$$

If the test population is confined to the training site, then $V_a^2, V_b^2, V_{e_i}^2$ can be assumed zero, as there is only one training site combination. Also, $\alpha = 0$, since the model is restrained to go through the origin for the entire test population.

Under these conditions (7) reduces to:

$$E(E(G/sp_1)) = \frac{N^2}{n} \left\{ \beta^2 V_x^2 + \alpha^2 \left(1 - \frac{\bar{X}}{N} \sum_{i=1}^N \frac{1}{X_i} \right) + \gamma \text{cov} (X_i^{g-1}, X_i) \right\} \dots(8)$$

(An identical expression is found in Murthy¹¹.) From this expression it can be seen that the expected gain is always positive, since

$$\gamma \text{cov} (X_i^{g-1}, X_i) > -\beta^2 V_x^2, \dots(9)$$

as γ is assumed positive.

Therefore, if the precision gain is computed on the training site data, we can expect that the

variable probability gain will always be positive. However, when the test population is not identical to the training population, the gain will only be positive if

$$\text{cov} \left(\frac{V_{e_i}^2 + \gamma X_i^g}{X_i}, X_i \right) > (\alpha^2 + V_a^2) \left(\frac{\bar{X}}{N} \sum_{i=1}^N \frac{1}{X_i} - 1 \right) - (\beta^2 + V_b^2) V_x^2 \dots(10)$$

From this expression we can see, for instance, that the larger the variation of the intercept V_a^2 , the less likely it is that this condition will hold.

The important conclusion is that, computed from training site data, G_{vps} will always be positive, but as soon as the results are applied elsewhere, degradation of the expected gain may occur,

even to the point where the method with the auxiliary variable may be worse than simple random sampling. Whether this is also true for other sampling

methods in the context of this model remains to be investigated, but our experimental results indicate that the variable probability sampling is especially susceptible to this problem and is the least robust of the methods investigated in this study, even though potential gains may be greater under favorable conditions.

VII. RESULTS

A. Optimization of the Volume Estimation System

As our training areas we selected two contiguous blocks of 8x8-square-mile sections which were located in a much larger 1,500-square-mile area from which the test populations were selected. The first training area contained a water body, whereas the second area consisted of forested land only.

To optimize the timber volume estimation system in terms of the features extracted from the digital images, we performed a 2⁴ factorial experiment for the first test area, from which we concluded that intel contrast did not significantly contribute to the model. Therefore, this factor was dropped and subsequently we only performed a 2³ experiment for the second test area. The results without the contrast factor are shown in Table 1.

Table 1. Results of 2³ Factorial Experiment to Define Optimum Feature Combinations

Training Area 1						
Run Comb.	R	F	F _{0.95}	ΔG _{vps}	NV	Effect
(1)	.32	1.49	1.99	8.5	10	33.3
a	.32	2.33	2.17	9.5	7	4.3
b	.40	11.74	2.76	15.4	3	4.8
ab	.54	16.45	2.53	34.7	4	4.0
c	.56	10.84	2.25	40.9	6	21.0
ac	.62	9.02	2.04	44.9	9	-3.1
bc	.67	10.40	1.99	53.7	10	1.3
abc	.66	9.96	1.99	50.8	10	-2.2

Training Area 2						
Run Comb.	R	F	F _{0.95}	ΔG _{vps}	NV	Effect
(1)	.32	1.49	2.04	8.5	10	21.4
a	.44	3.60	2.37	17.9	5	6.3
b	.25	.96	2.37	5.0	5	0.9
ab	.60	3.88	2.04	25.1	9	0.3
c	.56	5.33	2.25	27.5	6	14.6
ac	.63	3.52	1.95	30.1	11	-8.5
bc	.63	3.21	1.92	32.2	12	-1.0
abc	.60	3.88	2.04	25.1	9	-5.1

The basic factors in this experiment were:
 (1) X and Y attributes applied at random, thus results are entirely due to chance; a: tone value band 5; b: tone value band 7; c: the difference between the tone values for bands 5 and 7. A run combination of ab, for instance, means that a two-dimensional feature space was used with the tone value of band 5 on one axis, and the tone value of band 7 on the other axis.

The major conclusions that can be drawn from these experiments are:

1. With the exception of the runs in which the X and Y attributes were combined at random, all models were significant at the 95 percent level (compare F with F_{0.95}).
2. One effect stood out above all others; namely, the difference between the tone values of bands 5 and 7. The values for this effect have been circled in Table 1.
3. The results for the second training area were not as good as those for the first one. The reason for this difference is that the first training area contained a water body. Water is easily interpreted, and must have zero timber volume.

B. Training Results with the Optimized Estimation System

After we had established the desirable features with the described experiment, we trained the system using both 64-square-mile training areas, using as features the tone values for bands 5 and 7 and their difference. We selected a clustering of seven classes for the final result. Four of these classes were highly significant. Differential volume levels and t statistics for each of these classes as well as their significance probabilities are presented in Table 2.

Table 2. Volume Levels and t Statistics for Final Training

Class	Differential Volume Level (1000 Bd. Ft./ Sq. Mile)	t	Significance Probability
1	6,489	4.02	>0.9995
2	3,284	5.19	>0.9995
3	1,015	0.93	<0.90
4	-545	-0.78	<0.90
5	-638	-0.47	<0.75
6	-1,557	-2.12	>0.975
7	-3,420	-5.91	>0.9995

The F value for this training model was 15.44, at the 0.995 significance level to be compared with F_{7,128} = 3.09. Thus, the hypothesis that the volume levels were zero could be safely rejected. Moreover, the obtained F value satisfies a criterion mentioned by Draper and Smith⁷, namely: for a useful prediction model the F value should be larger than four times the selected percentage point value (15.44 > 12.36). Estimated gains in precision for the three sampling methods are shown in Table 3.

Table 3. Gains in Precision for Combined Training Areas

Sampling Method	Relative Gain (percent)
Stratified Sampling	26.7
Regression Sampling	43.4
Variable Probability Sampling	43.9

With these results it seemed that all necessary prerequisites for a successful application of the system in a much larger testing area were present.

C. Test Results with the Optimized Volume Estimation System

Within the 1,500-square-mile area, 138 parcels (square-mile sections or parts thereof) were selected for the test populations. Not all of these parcels were covered by the U2 high flight imagery. U2 estimates were obtained for a subset of 95 parcels, which we will call the U2 set, as

compared to the full set. Of the full set 118 parcels were located outside the original training areas. For the U2 set we could use SP, ES, and of course U2 estimates. For the full set we could use only ES or SP estimates. Designating the digital volume estimates as MSS estimates, we will describe the test populations by the attribute combinations as MSS-SP, MSS-ES, and MSS-U2. When working with the training areas, we always used full one-square-mile sections, each covering approximately 640 acres. The parcels of the test populations, however, were often only fractions of these square miles and could be as small as 40 acres. Taking this into consideration, we expected that if a correction for area was not made, this factor could be responsible for a certain percentage of the gain in precision, since total volume per parcel is naturally strongly correlated with the area of the parcel. Therefore, in addition to using the total parcel volume as the population attributes, we also used the mean volume/square mile.

Table 4 contains the results of our testing effort. The gains for the three sampling methods are listed in this table for each of the attribute combinations, for both the U2 and the full set, and for both total and mean volume.

Table 4. Test Results for the MSS Digital Interpretation System
(Sample Unit: SP Parcel)

Attribute Combinations	Rel. Gain in Precision (percent)			Statistical Significance		
	Strat	Regr	Var Prob	F	F _{0.95}	r
TOTAL VOLUME						
<u>Full Set</u>						
MSS-ES	32.5	33.9	34.3	70.0	3.90	0.58
MSS-SP	25.3	27.9	32.3	52.6	3.90	0.53
<u>U2 Set</u>						
MSS-U2	47.0	35.8	38.2	51.9	3.96	0.60
MSS-ES	28.0	25.4	24.2	31.8	3.96	0.50
MSS-SP	20.8	23.4	26.2	28.5	3.96	0.48
MEAN VOLUME						
<u>Full Set</u>						
MSS-ES	18.8	13.3	5.6	20.9	3.90	0.36
MSS-SP	13.0	13.7	13.2	21.6	3.90	0.37
<u>U2 Set</u>						
MSS-U2	30.3	17.7	13.0	20.0	3.96	0.42
MSS-ES	17.8	6.8	4.2	6.8	3.96	0.26
MSS-SP	12.0	11.0	11.0	11.5	3.96	--

MSS = MSS digital interpretation system estimates
 U2 = High flight U2 model interpretation estimates
 ES = EarthSat estimates obtained by combining interpretation of 1:40,000 panchromatic aerial photographs with SP parcel records
 SP = Estimates obtained from SP parcel records
 Strat = Stratified sampling (proportional allocation)
 Regr = Regression sampling
 Var Prob = Variable probability sampling

In contrast to the training areas, the F statistics and correlation coefficients in this table are now related to the assumption of a linear relationship between the population attributes.

The conclusions that can be drawn from the testing results in Table 4 are the following:

1. The F statistics show that all linear relationships are significant at the 95 percent level (compare F with F_{0.95}). All of the relationships, with the exception of the MSS-ES and MSS-SP combinations of the U2 set for the mean volume, are also significant at the 0.999 level. Thus, there is very little doubt regarding the statistical significance of the applied results of the volume estimation technique.

2. The mean volume figures represent the pure contribution of the digital volume estimation system. These gains are significantly lower than the total volume figures. Thus, area does play an important role in the reduction of the variance.

3. We obtained the highest gains for the MSS-U2 test populations. One of the explanations for this difference may be that a combination of U2 and SP estimates were used for the training. However, a more compelling reason is probably found in the notion that high timber volumes such as those occurring in old growth stands simply escape detection on small-scale images. The characteristics seen on these images are spatial ones and timber volume is only partly related to spatial arrangement of the tree crowns.

4. The results for the U2 set were not as high as those for the full set. A probable reason for this difference is that 12 parcels of the full set which are not included in the U2 set were situated in the training area.

5. The MSS-ES test population gave higher gains on the average than the MSS-SP one. This confirms that the old SP estimates were inferior to the newer commercial survey estimates.

6. A one-way analysis of variance performed on the total volume results for the three sampling methods shows that there is no significant difference between sampling methods for these results (F_{2,12} = 0.08). Since the volume area relationship is one which necessarily goes to the origin, variable probability sampling performs very well when area is an important factor.

The same analysis of variance performed on the mean volume figures does show a significant difference for the sampling methods (F_{2,12} = 12.76). Thus, when considering the pure contribution of the digital system, these methods rank according to their gains as follows: (1) stratified, (2) regression, and (3) variable probability.

7. In comparison with the training site results, it seems that there is a significant decrease in gain from training to test site.

VIII. FINAL REMARKS

Although the test results were all statistically significant, it seems that the relatively small gains attributable to the digital interpretation system (about 13 percent) do not warrant the cost of commercial use of digital MSS volume estimates under the present configuration. However, one should not lose sight of the fact that positive results were obtained in a difficult, mountainous area such as the Trinity Alps in California. If we were to make another effort, several improvements could be made such as the use of a smaller intel size. It also seems likely that in the future one will not be restricted by the relatively coarse resolution of the Landsat MSS scanner.

Another instructive result of the investigation is that caution is in order when applying low correlation digital estimates of high flight images in the sampling survey. Simple stratified sampling seems preferable to other methods in terms of gain, robustness, and ease of use, when area is not an important factor.

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