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# SEQUENTIAL CLASSIFICATION ALGORITHMS

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## I. ABSTRACT

In this paper, four versions of the sequential maximum likelihood algorithm have been employed to classify LANDSAT data & their performance is compared with that of the maximum likelihood classifier. The sequential maximum likelihood algorithm is a faster, but slightly less accurate version of the conventional maximum likelihood algorithm. In the sequential case, the number of features (MSS bands) used to classify a given pixel depends upon the particular band values of that pixel, since on the average, the number of bands used will be less than the maximum there will be a saving in computational time.

## II. INTRODUCTION

The sequential maximum likelihood algorithm (1) is a faster but slightly less accurate version of the conventional maximum likelihood algorithm. In the sequential case, the number of bands used to classify a given pixel depends upon the particular reflectance values of that pixel, whereas, in the conventional method all the bands are used for all the pixels. Because of the reduction in the average number of bands used, the computational load is also reduced and there is a saving in computer time.

## III. METHOD

In this study, four different versions of the sequential maximum likelihood algorithms are compared with each other as well as with the conventional maximum likelihood algorithm. A LANDSAT tape data was displayed and training samples belonging to five different categories were chosen. From the training sets, the statistics for each class, namely, the mean reflectances in the 4 bands as well as the covariance matrices were calculated. The training set itself was used for

classification. This enabled the classification to be calculated. In the following, the four versions of the algorithm are described and this is followed by a discussion of the results.

## IV. ALGORITHMS

### A. ALGORITHM-1

Let  $n$  be the total number of classes. Let  $m$  be the total number of bands. Let  $p_k(X/W_i)$ ,  $k=1, \dots, m$ ;  $i=1, \dots, n$ , be the conditional density of the  $i^{\text{th}}$  class using the first  $k$  bands.  $N$  is the number of classes under consideration at each instant.  $G'$  is the prespecified threshold.

Step 0:  $k=1$ ,  $N=n$ . Calculate  $p_i(X/W_i)$ ,  $i=1, \dots, N$ .

Step 1: Calculate  $\log_e [U_k(X/W_i)]$   
$$= \log_e \left[ p_k(X/W_i) / \prod_{j=1}^k p_k(X/W_j) \right]^{1/N}$$
  
for  $i=1, \dots, N$ .

Step 2: If, for any  $i$ ,

$$\log_e [U_k(X/W_i)] < \log_e [G'(1-k/m)]$$

then drop that class from further consideration. If all classes but one are dropped, then assign  $X$  to that class and stop. If all the classes satisfy the above inequality then assign  $X$  to the class for which  $\log_e [U_k(X/W_i)]$  is maximum among all the classes under consideration and stop. If the number of classes dropped during the present execution of step 2 is  $s$  then change  $N$  to  $N-s$ . Go to step 3.

Step 3: Change  $k$  to  $k+1$ . If  $k=m$ , go to step 4. Otherwise go to step 1.

Step 4: Calculate  $\log_e U_m(X/W_i)$  for  $i$  belonging to the set of classes under consideration. Assign  $X$  to the class for which  $\log_e U_m(X/W_i)$  is a maximum. Stop.

Note:  $G'$  should be greater than zero.

#### B. ALGORITHM-2

Algorithm-2 is got from algorithm-1 by replacing  $\log_e G'(1-k/m)$  by  $G'(1-k/m)$ .

#### C. ALGORITHM-3

Step 0:  $k=1, N=n$ . Calculate  $p_i(X/W_i)$ ,  $i=1, \dots, N$ .

Step 1: Calculate  $\log_e [U_k(X/W_i)]$   
 $= \log_e \left[ p_k(X/W_i) / \prod_{j=1}^N p_k(X/W_j) \right]^{1/N}$   
 for  $i=1, \dots, N$ .

Step 2: If, for any  $i$ ,

$$\log_e U_k(X/W_i) < \log_e G'(1-k/m)$$

then drop that class from further consideration. If all classes but one are dropped then assign  $X$  to that class and stop. If all the classes satisfy the inequality then assign  $X$  to that class for which  $\log_e U_k(X/W_i)$  is maximum among all the classes dropped during the present execution of step 2. If the number of classes dropped during the present execution of step 2 is  $s$ , then change  $N$  to  $N-s$ . If  $s=0$ , go to step 3. Else go to step 1.

Step 3: Increment  $k$ . If  $k=m$ , go to step 4. Otherwise, go to step 1.

Step 4: Calculate  $\log_e U_m(X/W_i)$  for  $i$  belonging to the set of classes under consideration. Assign  $X$  to the class for which  $\log_e U_m(X/W_i)$  is a maximum.

Note:  $G'$  should satisfy  $0 < G' < 1$ .

#### D. ALGORITHM-4

Algorithm-4 is got from algorithm-3 by replacing  $\log_e G'(1-k/m)$  by  $G'(1-k/m)$ .

Note:  $G' < 1$ .

#### V. CHOICE OF $G'$

To determine the value of  $G'$  to be used for any of the above algorithms, that particular algorithm should be applied to the training set. The accuracy of classi-

fication for various values of  $G'$  is determined and depending upon the accuracy required a value of  $G'$  is chosen.

#### VI. RESULTS OF COMPARATIVE STUDY

One hundred and ninety-six training samples belonging to five different classes were classified using all the above algorithms and also the conventional maximum likelihood algorithm.

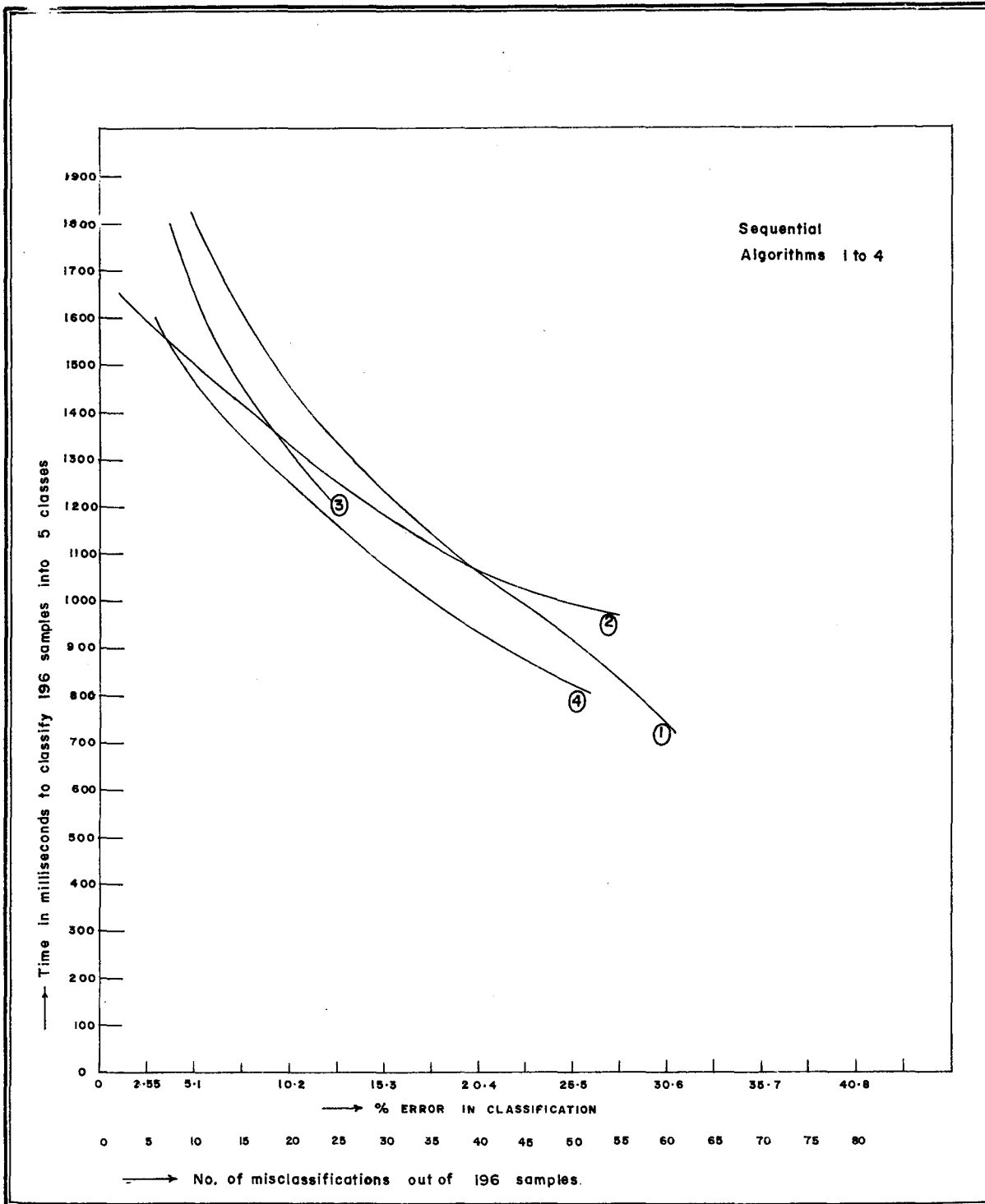
Using different values of  $G'$ , graphs connecting the accuracy of classification and the time required for classification have been plotted. It can be seen that algorithms 2 and 4 perform better than the maximum likelihood algorithm. Among the two algorithms, 4 seems to be better.

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