Evaluation of Projection Errors using Commercial Satellite Imagery

Jim Bethel
Purdue University, School of Civil Engineering
Remote Sensing Seminar, 15 September, 2004
Outline of Presentation

• Traditional Mapping Polynomial, Image Warping, Rubber Sheetling Approach

• Description of Photogrammetry Approach: Physical Model and Replacement Model

• Example of Physical Model

• Example of Replacement Model

• Evaluation of Projection Errors Using Vendor Supplied Replacement Model

• Conclusions
For each point we create two equations. We need at least as many equations as unknowns. If more, then we use least squares. It is like a regression problem: linear, easy. But we are confounding the effects of sensor, platform motion, and terrain relief. What should be the order of the polynomial?
Graphical View of Rubber Sheet Transformation (2\textsuperscript{nd} order, 12-parameter)
Mapping Polynomials or Rubber Sheeting

If the terrain is flat, the sensor has narrow field of view, the sensor is nadir looking, and the ground sample distance is large, then you can get reasonable results using the approach of mapping polynomials.

The accompanying Quickbird image (0.61m pixel) shows the pitfalls of mapping polynomials when the above conditions do not apply. The two marked points have the same XY and they would get mapped into the same (row, col), but clearly that is wrong. You could expand the polynomial by adding some Z-terms. But that would not work. Modeling the actual physical imaging process is the only way.
Photogrammetric Approach to Image Geometry

Physical Sensor and Platform Models

- Sensor (Camera): aperture size, focal length, scanning elements, optical distortion
- Platform: orbit parameters: altitude, velocity, etc., orientation / attitude, rates and accelerations

Use Directly

Use Indirectly through Rational Polynomials
Use physical model **Directly**, vendor or manufacturer defines the generic model: equations and constants. For a particular image, numerical values come from

- **Vendor**: support data supplied with image, metadata, ephemeris data, etc.
- **User**: obtain numerical values using ground control points

**Advantage**: parameters have physical meaning, flexibility

**Disadvantage**: vendor may not want to share, each one different

Use physical model **Indirectly** via Rational Polynomial Coefficients (Replacement). For a particular image, numerical values come from

- **Vendor**: for certain products, vendor supplies 80 term RPC
- **User**: can obtain via regression using a dense 3D grid and corresponding image points, based on physical model

**Advantage**: same parameters for all sensors, easy for software applications

**Disadvantage**: no physical meaning to the coefficients
Physical Model
Schematic of telescope optics layout for modern remote sensing camera
Schematic of Spot Optics

From Pease, Satellite Imaging Instruments
Cutaway Drawing of Spot Sensor

Dimensional stability is very important to maintain good focus. The structural tubes are made from carbon fiber material with a small negative thermal expansion coefficient. The titanium fittings have a positive thermal expansion coefficient that just cancels the tubes. (That is good engineering!)

From Pease, Satellite Imaging Instruments
Attitude Sensing

Quickbird Assembly

Compare
Spot: 0.2 deg @ 820 km => 2870m
Quickbird: 3 sec @ 450 km => 7m
Terrestrial Photograph of Orion
Physically Based Model

Sensor parameters:
Focal length, principal point location, lens distortion, line rate, detector (pixel) size

Platform parameters:
Location X,Y,Z, time, attitude roll, pitch, yaw, kepler orbit elements \((a,e,i,W,w,n)\)

Relate ground point and image point by equations with the above *actual physical* parameters, rather than the generic \(a_0, a_1, a_2, \ldots\) parameters.
Development of the Condition Equations for a Space Based Pushbroom Camera (Using SPOT as an Example)
Development of SPOT Condition Equation – Good Model for Generic Pushbroom Camera from LEO

t_f: time at frame center, in header (metadata)
delta-t: delta-time from frame center, equals 0.001504 sec * line,

\[ t = t_f + \Delta t \]

orbit period, \( \Delta = 2\Delta \sqrt[3]{\frac{a^3}{GM_e}} \)

GMe = 398600.5E09 m³/s²

a_s = r_e + alt_s , 6378137m + 822000m = 7200137m

\[ \Delta = 2\Delta \sqrt[3]{\frac{7200137^3}{398600.5E09}} \]

\[ \Delta = 6080.259 \text{ min} \]

\[ \Delta = 101.338 \text{ sec} \]

t_p: time from ascending node to perigee

Must have approximations for \( \Delta, i, \theta, a, e \)

\[ e = \frac{\sqrt{a^2 b^2}}{a} \]
Condition Equation cont’d.

\[ t_p = \tan\left(\tan^{-1}\left(\frac{e\sqrt{1-e^2}}{1+e}\right) / 2\right) \]

\[ \dot{t}_p = t \quad \dot{t}_p \]

\[ M_n = \frac{2\dot{t}_p}{2}, \text{ mean anomaly} \]

\[ E = e \sin E + M_n, \text{ (kepler equation, } E : \text{ eccentric anomaly)} \]

Solve iteratively for \( E \)

\[ R_s = a(1 - e \cos E) \quad \text{; vector from earth center to satellite} \]

Construct \( M_b \) from 3 sequential rotations applied to XYZ (ECEF) to bring them parallel to xyz (instantaneous satellite system)
Condition Equation, cont’d.

The XYZ obtained in this way will be only approximately correct and we must allow for refinements, modeled as second order polynomials of time:

\[ X = X_0 + X_1 t + X_2 t^2 \]
\[ Y = Y_0 + Y_1 t + Y_2 t^2 \]
\[ Z = Z_0 + Z_1 t + Z_2 t^2 \]

Likewise the attitude (orientation) produced by the prior rotation matrix will be only approximately correct and we must allow for refinements to the attitude, again modeled as second order polynomials of time:

\[ \theta = \theta_0 + \theta_1 t + \theta_2 t^2 \]
\[ \phi = \phi_0 + \phi_1 t + \phi_2 t^2 \]
\[ \psi = \psi_0 + \psi_1 t + \psi_2 t^2 \]
\[ \omega = \omega_0 + \omega_1 t + \omega_2 t^2 \]
We must also account for a tilt or inclination of the camera. In the case of SPOT this is a cross track tilt (+/- 27 degrees) about the x (motion) axis, implemented by a stationary (but moveable) mirror:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta \\
\end{bmatrix}
\]

In the case of an agile spacecraft such as IKONOS or Quickbird, this pointing can be any arbitrary cross-track, in-track, or spin attitude, and thus requires 3 rotations:

\[
M_t = M_z (\square) M_y (\square) M_x (\square)
\]

Note that we are over parameterized with rotations here. You cannot carry all as unknowns. But it may be convenient to separate in this way to make if clear which physical effect the parameter refers to.
Collecting all of this into the collinearity condition equation:

\[
\begin{bmatrix}
0 \\
y \\
f
\end{bmatrix} = \begin{bmatrix}
M_t \\
M_a \\
M_b
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
R_s
\end{bmatrix} + \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

Combine terms, eliminate scale

\[
\begin{bmatrix}
0 \\
y \\
f
\end{bmatrix} = \begin{bmatrix}
U \\
V \\
W
\end{bmatrix}
\]

\[
0 = f \frac{U}{W}
\]

\[
y = f \frac{V}{W}
\]

\[
F_x = f \frac{U}{W}
\]

\[
F_y = y + f \frac{V}{W}
\]
We can also add some other inner orientation parameters such as lens distortion, principal point offset, etc.

So how many parameters do we have? There are 5 groups,

- **Orbit parameters**  $i, a, e, t_f$ (6)
- **Position corrections**  $X_0, X_1, X_2, Y_0, Y_1, Y_2, Z_0, Z_1, Z_2$ (9)
- **Attitude corrections**  $\psi_0, \psi_1, \psi_2, \psi_0, \psi_1, \psi_2, \psi_0, \psi_1, \psi_2$ (9)
- **Pointing**  $t$ (1)
- **Inner orientation**  $x_0, y_0, f, k_1$ (4)

Total here is 29, some will be held constant (maybe at zero), we may add some. Stochastic treatment is guided by redundancy, geometric strength of figure (parameters known to be highly correlated will probably not both be carried as unknowns), and by uncertainties
For SPOT we get an approximation of the off-nadir attitude from the angle readout of the mirror position. For Quickbird, we have the attitude described by quaternion elements, throughout the scene.
Depending on the source of information about ground control points, we may need to do some prior transformations such as,

Map projection coordinates and orthometric (sea level) height

Geodetic coordinates and ellipsoid height, need info about geoid undulation

Geocentric, ECEF
Replacement Model
For the third order model, only terms with $i+j+k \leq 3$ are allowed. Those terms are shown below.

$$1, x, y, z, x^2, y^2, z^2, xy, xz, yz, x^2y, xy^2, x^2z, xz^2, y^2z, yz^2, x^3, y^3, z^3, xyz$$
Rigorous Sensor Model Parameter Estimation &
RPC Parameter Estimation

Estimate actual sensor parameters

Actual image points

Actual ground points

Estimate RPC parameters using the many fictitious ground and image points

Project fictitious ground points into image by rigorous parameters

Fictitious ground points within volume
Erdas Imagine / Orthobase support for IKONOS RPC data – note the \texttt{line\_numerator} coefficients go up to \#20, this implies a 3\textsuperscript{rd} order polynomial

| LINE\_OFF | +001384.62 pixels |
| SAMP\_OFF | +002492.12 pixels |
| LAT\_OFF  | +32 76260000 degrees |
| LONG\_OFF | -117 13290000 degrees |
| HEIGHT\_OFF | +0065 0000 meters |
| LINE\_SCALE | +002224.25 pixels |
| SAMP\_SCALE | +002805.25 pixels |
| LAT\_SCALE | +00.10360000 degrees |
| LONG\_SCALE | +000.07300000 degrees |
| HEIGHT\_SCALE | +0252 0000 meters |
| LINE\_NUM\_COEFF\_1 | -1.06791343419703E-03 |
| LINE\_NUM\_COEFF\_2 | +7.53256495448399E-01 |
| LINE\_NUM\_COEFF\_3 | +2.58335230123734E-01 |
| LINE\_NUM\_COEFF\_4 | -1.150012062519057E-02 |
| LINE\_NUM\_COEFF\_5 | +7.04274023833077E-04 |
| LINE\_NUM\_COEFF\_6 | +5.564515525173415E-04 |
| LINE\_NUM\_COEFF\_7 | +2.118277231062864E-04 |
| LINE\_NUM\_COEFF\_8 | +2.806916823545727E-04 |
| LINE\_NUM\_COEFF\_9 | -8.887709531793368E-05 |
| LINE\_NUM\_COEFF\_10 | -8.903699529178202E-06 |
| LINE\_NUM\_COEFF\_11 | +8.980707101284475E-06 |
| LINE\_NUM\_COEFF\_12 | -1.981967500179333E-05 |
| LINE\_NUM\_COEFF\_13 | +2.2605253903598E-05 |
| LINE\_NUM\_COEFF\_14 | -3.15085166750731E-05 |
| LINE\_NUM\_COEFF\_15 | -1.11938233066729E-05 |
| LINE\_NUM\_COEFF\_16 | +8.178251749507152E-06 |
| LINE\_NUM\_COEFF\_17 | +1.367311124259506E-06 |
| LINE\_NUM\_COEFF\_18 | -8.84392275521833E-06 |
| LINE\_NUM\_COEFF\_19 | +8.627476075161538E-06 |
| LINE\_NUM\_COEFF\_20 | -5.04088425564775E-08 |
| LINE\_DEN\_COEFF\_1 | +1.000000000000000E+00 |
| LINE\_DEN\_COEFF\_2 | +2.205536317487505E-04 |
| LINE\_DEN\_COEFF\_3 | +2.170377012059137E-03 |
| LINE\_DEN\_COEFF\_4 | +3.29016014535045E-04 |
| LINE\_DEN\_COEFF\_5 | -5.526445079601216E-05 |
| LINE\_DEN\_COEFF\_6 | -1.15163084496144E-05 |
| LINE\_DEN\_COEFF\_7 | +1.707180496103808E-05 |
| LINE\_DEN\_COEFF\_8 | +3.198248260257836E-05 |
| LINE\_DEN\_COEFF\_9 | -1.250347201134037E-05 |
| LINE\_DEN\_COEFF\_10 | -4.646410239602201E-06 |
| LINE\_DEN\_COEFF\_11 | -7.251784602539888E-09 |
| LINE\_DEN\_COEFF\_12 | -5.645760323463601E-09 |
| LINE\_DEN\_COEFF\_13 | -5.645760323463601E-09 |
| LINE\_DEN\_COEFF\_14 | -5.645760323463601E-09 |
Evaluation of Projection Errors Using Vendor Supplied RPC Rational Polynomial Coefficients
Control Point Projected to Incorrect Location in the Image – Suspect Similar Errors (magnitude & direction) Occur Everywhere in This Image
Error Pattern for This Image – Seems to be Common Bias plus Smaller Random Part

Errors from control checkpoints

Random vectors

Common bias vector

Error vectors decomposed into bias and random parts
Conjecture: The bias is itself a random vector that is consistent within an image but different between images (corollary: the biases between “same orbit” images might be correlated)
Graphical Depiction of Errors from Three Images
(Conjecture – verify with actual images & GCPs)
Another Way to Visualize an Individual Control Point Error, Decomposed into a Bias and a Random Component
For a single image with GCPs, can estimate bias and random part this way

\[
\begin{align*}
\mathbf{e}_x &= \mathbf{e}_{bias} \cos \theta + \mathbf{r}_x \\
\mathbf{e}_y &= \mathbf{e}_{bias} \sin \theta + \mathbf{r}_y
\end{align*}
\]

\[
\begin{align*}
\mathbf{a} &= \mathbf{e}_{bias} \cos \theta \\
\mathbf{b} &= \mathbf{e}_{bias} \sin \theta
\end{align*}
\]

Want to estimate one lambda and theta for each image, making the a,b substitution allows one to use a linear estimation model
Simulation of RPC projection errors using conjectured model

\[ P = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} \]

\( \square_{bias} = 5 \)
\( \square_{random} = 2 \)
\( \square_{12} = 1.00 \)
\( \square_{14} = \square_{24} = 0.5 \)
\( \square_{13} = \square_{23} = \square_{43} = 0.0 \)
The Study
(Supported by NGA)
Purdue, IKONOS, stereo pair (in 2 segments)
Indianapolis, Quickbird, single image
GPS Survey, 4 receivers, 8 sessions, NGS control and photo-ID points for imagery evaluation
Selected occupations of NGS control points and photo ID points.
GPS network, from Pinnacle Adjustment Software, 15 photo ID points to use for evaluation of IKONOS imagery.
Error Vectors of Thailand Imagery 1 (Left)

Error Vectors Scattering Plot (Scale of Magnitudes is 50:1)

- Actual Ground Control Points
- Projected Ground Control Points
Error Vectors of Thailand Imagery 1 (Left)
Error Vectors of Fallon Imagery (Left)
Error Vectors of Fallon Imagery (Left)
Error Vectors of Purdue Imagery (Left)
(Left and Right refer to a stereo pair)
Error Vectors of Purdue Imagery (Left)
Summary of Results for 70 Images, for 2 sensors, each with 2 classes, bias and random errors corresponding to 40% confidence region

<table>
<thead>
<tr>
<th>Satellite(^1)</th>
<th>Tilt Angle(^2)</th>
<th>(e_b^{11}) (Pixel)</th>
<th>(e_r^{12}) (Pixel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IKONOS</td>
<td>Small</td>
<td>4.15</td>
<td>0.95</td>
</tr>
<tr>
<td>IKONOS</td>
<td>Large</td>
<td>6.49</td>
<td>0.96</td>
</tr>
<tr>
<td>QuickBird</td>
<td>Small</td>
<td>11.21</td>
<td>1.27</td>
</tr>
<tr>
<td>QuickBird</td>
<td>Large</td>
<td>17.57</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Conclusions

• The results are surprisingly good considering that we are doing “direct geopositioning”, i.e. RPC’s come from the satellite navigation data (i.e. no control points used in projection – only for checking)

• Furthermore, by far, most of the error is in the common bias term, which means if you introduce one high quality control point, and augment the RPC’s with shift terms, you are down in the 1-2 pixel error range

• We have recommended a method of error propagation using these eb/er terms in image space, current Eb/Er terms from the NITF standard have ambiguous definition and are not applied uniformly.